Electromagnetic and Thermal Model Parameters of Oil-filled Transformers

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Abstract: The study of predictive control of temperature in oil-filled transformers is performed in this work. In order to study transformers thermal loss of life, complex models taking into account electrical and thermal characteristics are required. Moreover, the precision of thermal models is dependent upon the exactitude of the parameters. The work presented in this article shows that, through electromagnetic similitude laws, for an homogeneous series of transformers with different rated powers, the main parameters required for the thermal model can be obtained. Different methodologies to estimate thermal parameters with data from standardised heat-run tests are compared. The foremost advantage of our alternative methodology is its compactness, since parameters are obtained only from the knowledge of transformer rated power. Theoretical results were compared with data from transformer manufacturers and the good agreement between both validates theoretical results is accomplished.
Key-Words: Electromagnetic similitude laws, Oil-filled transformers, Temperature control, Thermal parameters

1 Introduction

Due to the widespread and easily use of computer calculations, numerical models are fundamental tools for a great number of subjects under study. Many parameters can intervene on transformer thermal model, depending upon models refinement [1],[2]. Electrical parameters such as load and no-load losses, can be directly determined from transformer data sheet and standardised tests. Thermal parameters such as the transformer thermal time constant and the oil temperature rise must be determined from specific tests and, usually, are not referred on data sheets [3]. Electrical parameters are of much precise determination than thermal parameters. This work concerns the estimation of structural parameters of transformer thermal model, based upon electromagnetic similitude laws and real standardised transformer characteristics. According to International Standards classification, a distribution transformer presents a maximum rating of 2500 kVA and a high-voltage rating limited to 33 kV; within such a large power range, design and project problems for the lower to the higher power transformers, are quite different. For studying a large power range of transformers, for which only the main characteristics are known, one can use the model theory; this method is largely established. "The most practicable way of determining the characteristics of apparatus embodying non-linear materials such as magnetic core ones, is usually experimental; analysis, while often valuable, is largely empirical and must therefore be verified by actual experimental data. By the use of model theory, however, the experimental data obtained on one unit, can be made to apply to all geometrically similar units, regardless of size, provided certain similarity conditions are observed" [4],[5],[6]. General similitude relationships for main characteristics of ONAN (O*il* N*atural* A*ir* N*atural*) cooled transformers within a power range from 25 kVA to 2500 kVA will be deduced. Transformer main characteristics that will be studied are: no-load magnetic losses, short-circuit Joule losses, transformer total mass, transformer oil mass. Similitude relationships will allow the definition of these characteristics as functions of transformer apparent rated power.

2 Similitude Relationships for Electromagnetic Parameters

Similitude relationships will be established with the help of a generic transformer linear dimension, represented by l. It will be consider that this linear dimension, l_i, of an i transformer from the studied power range, will be related to the same linear dimension, l_j, of other *j* transformer of the same range, through an geometric relation of the form:

$$l_i = kl_j , \quad (1)$$

being *k* a constant (scale factor).

Transformer main characteristics that will be studied are: no-load magnetic losses, P_o, short-circuit Joule losses, P_{cc}, transformer total mass, M_T, transformer oil mass, M_o, main thermal time constant, τ_0. Similitude relationships will allow the definition of these characteristics as functions of transformer rated power, S_R.

Unless particular conditions specified, general assumptions on next expression derivation are:

i) frequencies involved in time varying characteristics are sufficiently low so that state can be considered quasi-stationary.

ii) materials are magnetically, electrically and thermally homogeneous.

iii) magnetic flux density is sinusoidal time-varying, always perpendicular to the core section and uniform at any cross section

2.1 Rated Power

There is considered an elementary electromagnetic circuit, with an winding of n_w turns and an iron core where a sinusoidal varying magnetic flux density B is assumed. Neglecting the voltage drop due to winding resistance, the RMS value of the induced voltage per winding turn on terminals, U_e, is given by [4],[5],[6]:

$$U_e = \frac{1}{\sqrt{2}} \omega \mathrm{B}_{Max} \mathrm{A}_c , \quad (2)$$

where: U_e induced voltage 9RMS value) per winding turn [V], B_{Max} maximum magnetic flux density value on magnetic circuit [T], ω angular frequency [rad.s^{-1}], A_C core cross-section [m^2].

Also, the rated RMS value of the winding current, IR, can be defined as:

$$\mathrm{I}_R = \mathrm{A}_e J_R , \quad (3)$$

with: I_R rated current (RMS value) [A], J_R rated current density (RMS value) [Am^{-2}], $\mathrm{A_e}$ winding turn cross-section [m^2]

From (2) and (3), the rated power at terminals 1-2, denoted by S_R, will be given by:

$$S_R = \frac{1}{\sqrt{2}} \omega \mathrm{B}_{Max} \mathrm{A}_c n_w A_e J_R . \quad (4)$$

Using the linear dimension *l*, and considering that frequency, as well as the number of winding turns are invariant, expression (4) can be written as:

$$S_R \alpha l^4 \mathrm{B}_{Max} J_R . \quad (5)$$

Expression (5) means that, for a given pair of B_{Max} and J_R values, the rated power will increase proportionally to the fourth power of the transformer linear dimension.

2.2 Mass and Volume

For the *Mass* and *Volume study*, the transformer will be considered as an homogeneous body with an equivalent volumic density, m_{veq}. Mass, is, therefore, traduced by:

$$M = m_{veq} V , \quad (6)$$

with: *M* transformer mass [kg], m_{veq} mas per unit volume [kg.m^{-3}], *V* transformer volume [m^{-3}] and thus, in terms of linear dimensions, both *M* and *V* will be proportional to the third power of transformer linear dimension

$$M, V \alpha\, l^3 . \quad (7)$$

2.3 Joule Power Losses without Skin Effect

In the absence of current harmonics, losses due to transformer variable load are essentially due to the flowing of the current through winding DC resistance, also referred as Joule losses, P_{winDC}.

According to [16], these losses can be determined from a transformer short circuit test, under rated current. Due to their reduced value under this situation, one can neglect magnetic power losses on core and so, short-circuit power losses will be given, essentially, by Joule losses on windings. Under rated current it will be:

$$P_{cc} \approx P_{winDC} = R \cdot I_R^2 = \frac{1}{\gamma_w} \frac{l_w}{A_e} I^2{}_R , \quad (8)$$

with: γ_w electrical conductivity of windings material [Ω^{-1} m^{-1}], l_w windings wiring length [m].

On (8) derivation one is not taking into account losses due to skin effect. This effect arises in conductors carrying alternating currents and can be traduced by a non-uniform current density caused by the varying magnetic field produced within the conductor by its own current, as well as by its neighbouring conductors. When the load current of a transformer increases, this usually give rise to an increase of eddy and hysteresis losses, even without a change in the core magnetic flux, due to this skin effect—these losses are called *stray load losses*.

Stray load losses increase with the frequency of the current and with the size of the conductors. To reduce these losses, similarly to the core lamination, also, in properly designed transformers, large section conductors are subdivided into several conductors of small section, insulated from each other and suitable transposed throughout the windings, so that skin effect is minimised. For the purpose of this similarity study, stray losses will be neglected. Attending to (3) expression (8) can be rewritten as:

$$P_{cc} \approx \frac{1}{\gamma_w}\frac{l_w}{A_e}\left(A_e J_R\right)^2 . \qquad (9)$$

For this similitude study, a constant ambient temperature scenario can be assumed, and so the resistivity of the windings material can be considered a constant value, resulting, for the short-circuit power losses, the expression:

$$P_{cc} \propto J_R{}^2 l^3 . \qquad (10)$$

Expression (10) means that, for a given value of current density, load losses will increase with the third power of the core linear dimensions.

2.4 No-load Power Losses

Under transformer no-load situation, the losses that occur in the material arise from two causes:
i) the tendency of the material to retain magnetism or to oppose a change in magnetism, often referred to as magnetic hysteresis
ii) the RI^2 heating which appears in the material as a result of the voltages and consequent circulatory currents induced in it by the time variation of the flux.

The first of these contributions to the energy dissipation is known as *hysteresis power losses*, P_H, and the second, as eddy *current power losses*, P_E, at a constant industrial frequency. Attending to the general approach of this study and to their reduced value under no-load operation, Joule power losses due to magnetisation current will be neglected, as well as any other additional power losses. According to [3], eddy current power losses can be traduced by:

$$P_E = \frac{\omega^2\gamma_c}{24}\varepsilon^2 B_{Max}{}^2 V_{core} , \qquad (11)$$

with: γ_c electrical conductivity of magnetic sheets (Fe-Si) per unit volume [Ω^{-1} m^{-3}], ε thickness of magnetic sheets [m], V_{core} effective core volume [m^3].

The thickness of the core sheets will be consider constant, within the analysed power range, and therefore:

$$P_E \propto B_{Max}{}^2 l^3 . \qquad (12)$$

For the hysteresis losses on a magnetic circuit of volume Fin which the magnetic flux density is everywhere uniform and varying cyclically at a frequency co, the empirical Steinmetz expression [3], will be considered:

$$P_H = \frac{\omega}{2\pi} k_H V B_{Max}{}^{\nu} , \qquad (13)$$

with: k_H hysteresis coefficient (material characteristics), ν empirical Steinmetz exponent (it can vary from 1,6 to 2,5).

For the usual Fe-Si sheets, one can consider that ν =2 and thus (13) can be rewritten as:

$$P_H \propto B_{Max}{}^2 l^3 . \qquad (14)$$

Attending to (12), the proportionality relationship for no-load power losses will be given by:

$$P_0 \propto B_{Max}{}^2 l^3 . \qquad (15)$$

Expression (15) traduces the proportionality of no-load power losses with the third power of transformer linear dimension (volume) for each given value of magnetic flux density. Table 1 regroups the basic similitude relationships deduced on previous paragraphs and which will be developed on next sections.

Table 1: Basic similitude relationships.

$S_R \propto l^4 B_{Max} J$	$P_{CC} \propto l^3 J_R$	$P_0 \propto l^3 B_{Max}{}^2$	$M, V \propto l$

Apart from *Mass* and *Volume* all these transformer characteristics depend upon B_{Max} and J_R evolutions within the considered power range; these evolutions will be analysed on next section.

3 Thermal Parameters

The linear first order thermal model presented in International Standards and derived on [9], is considered a reference; to use it, knowledge of transformer main thermal time constant, τ_0, as well as final top-oil temperature rise under rated load, $\Delta\Theta_o$, is needed. Usually, these two parameters are determined using data from a heat-run test, although estimation with data from the cooling curve is also possible [8], as well as on-line estimation from a monitoring system. Several methodologies can be found to estimate these two parameters from test data [3], [7], [8], and [9]. Experimental constrains for their application are different for each methodology (the required time duration for the test, the necessity of equidistant measured values), graphical and numerical methodologies lead to different results and, some of them, do not allow estimation of parameters uncertainty.

3.1 Similitude Relationships

In agreement with the thermal model of the homogeneous body, the final temperature rise, $\Delta\Theta_f$, is dependent upon the total power losses generated inside the body, P_{loss}, the external cooling surface, A_s and also upon the heat transfer coefficient, h_{cr}, as derived on:

$$\Delta\Theta_f = \frac{P_{loss}}{h_{cr} A_S}. \qquad (16)$$

All losses in electrical power apparatus are converted into heat and insulation materials are the ones that suffer most from overheating; on windings insulation materials, overheat will slowly degrading materials thermal and chemical insulation properties and on oil, overheat will produce chemical decomposition, degrading its dielectric strength. Since heating, rather than electrical or mechanical considerations directly, determines the permissible output of an apparatus, design project includes heating optimisation. Which means that each transformer will be designed to heat just the maximum admissible value, under normal rated conditions. The maximum safe continued load is the one at which the steady temperature is at the highest safe operating point. Reference [8] considers an hot-spot temperature of 98°C, for an ambient temperature of 20°C. On a transformer, all the power losses are due to summation of constant voltage magnetic losses and variable current winding losses. Let total losses, under rated load, denoted by P_{lossR}, be approximated by:

$$P_{lossR} = P_{CC} + P_0 \qquad (17)$$

Considering (16) and (17) and attending to similitude expressions for load and no-load losses, top-oil final temperature rise under rated load, $\Delta\Theta_{ofR}$, will be:

$$\Delta\Theta_{ofR} \propto \left(J_R^{\,2} + B_{Max}^{\,2}\right) l \qquad (18)$$

Considering B_{Max} and J_R are constant values, final transformer temperature rise would increase with the first power of linear dimension:

$$\Delta\Theta_{ofR} \propto l \qquad (19)$$

If only B_{Max} is a constant value and J_R values, final temperature rise will still increase with transformer size. Therefore, regardless which hypothesis is consider, the final transformer temperature rise, will always be:

$$\Delta\Theta_{ofR} \propto l^{\phi} \qquad (20)$$

with an ϕ value equal or greater than the unity.

One could then conclude that final temperature rise of transformers would always rise with its linear dimension. In practice this fact does not occur because transformers refrigeration system is improved as rated power increases, by increasing the external cooling surface through corrugation. The effect of refrigeration improvement can be traduced by an equivalent refrigeration rate, $(h_{cr} A_s)_{eq}$, which increases with the third power of the linear dimension l.

$$\left(h_{cr} A_S\right)_{eq} \propto l^3 \qquad (21)$$

Under these conditions, equation (19) can be rewritten as:

$$\Delta\Theta_f = \frac{P_{lossR}}{(h_{cr} A_S)_{eq}} = ct. \qquad (22)$$

This expression, however, can not be validated with data since neither $\Delta\Theta_{ofR}$ nor $(h_{cr} A_s)_{eq}$ values are available on transformer data sheets. According to the thermal model of an homogeneous body, the thermal time constant, τ_0, can be given by:

$$\tau = c_m M \frac{\Delta\Theta_f}{P_{loss}} \qquad (23)$$

On the lack of transformer thermal capacity knowledge, cm, one of the approximate methods suggested by IEC 76-2 to estimate the transformer main thermal time constant, is based upon information available on transformer rating plate, this expression is reproduced on:

$$\tau_0 = \frac{5M_T + 15M_0}{P_{loss}} \Delta\Theta_{of} \qquad (24)$$

where M_T and M_o represent the transformer total and the oil masses, respectively.

Expression (24) derives from the assumption that, within an homogeneous transformer series, there is a constant proportion between transformer total mass and oil mass; coefficients affecting M_T and M_o reflect this assumed proportionality as well as different thermal capacities for each part. A similar relationship is suggested by [17]. Remark should be made that this is an approximate formula, and therefore, resulting values will carry inherent errors. As an illustrative example is presented, relatively to an ONAN 160 kVA distribution transformer, 20/0.4 kV rated voltage, whose main time constant was estimated from two different methods. Since available data included transformer characteristics, oil mass, total mass and also the heating test from the manufacturer, main thermal time constant was estimated through heating test data, according to [7] proposed procedures. Extrapolation of all the points from the heating curve, led to a thermal time constant value of 1.9 hour; extrapolating only the upper 60% part of the heating curve, a more accurate value would be obtained [7] and that was 1.8 hour. On the other hand, using expression (24) the resulting value was 1.5 hour, which traduces the approximately character of this expression. Usually, distribution transformers catalogues do not include thermal time constant values; nevertheless, they are of primordial importance in loss of life expectancy studies. In order to validate similitude expressions, values obtained through expression (24) will be used.

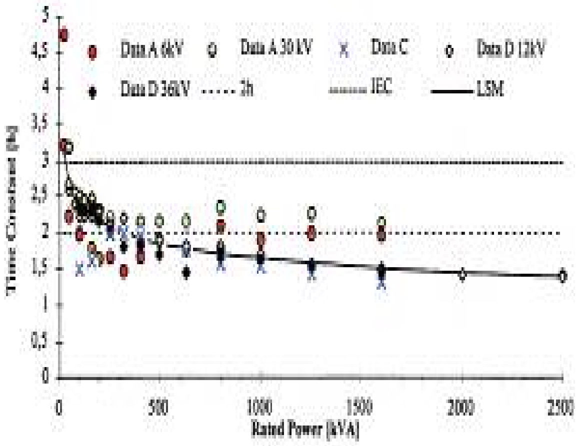


Fig.1: Thermal time constants based on expression (24)

Since available data includes M_T, M_o and P_{loss} rated values, the thermal time constant, under rated losses, τ_0, was determined, assuming that final top-oil temperature rise, $\Delta\Theta_{of}$, was 60K for all transformers. This temperature rise is the maximal admissible value for top-oil temperature rise of oil-immersed transformers referred to steady state under continuous rated power [8]. With this assumption, the resulting τ_0 values will correspond to an overestimation and, therefore, transient hot spot temperatures will be underestimated, as well as consequent loss of life. Results are represented on Figure 1. To describe the evolution of transformer thermal time constant with rated power, the following generic expression was assumed:

$$\tau_{pu} = s^{\varsigma} \qquad (25)$$

With the LSM fitting method, the obtained mean value of the ς estimator leads to:

$$\tau_{pu} = s^{-0.143} \qquad (26)$$

with $\sigma_{\hat{\varsigma}}$ = 0.016 and the 95% confidence interval limited by [- 0.174; - 0.111].

Reference [8] proposes 3 hours for the thermal time constant value to be used on loss of life calculations, provided no other value is given from the manufacturer. Attending to (24) and to the fact that the maximum admissible $\Delta\Theta_{of}$ value was assumed, the proposed value of 3 hours is of difficult justification. International guides are often referred as conservative ones; however, for loss of life considerations, a conservative value for transformer thermal time constant should not be a maximum value but, on the opposite, a minimum one. According to this study, which is based on expression (24), if a fixed value had to be assumed for the thermal time constant of distribution transformers, this value would be approximately 2 hours. From expression (23), considering approximation (17), and introducing similitude expressions for M_T, P_o and P_{cc}, the resulting similitude expression for transformer thermal time constant, under rated conditions, is:

$$\tau_0 \propto \frac{l^3}{l^{3\beta} + l^3} \qquad (27)$$

or, in terms of rated power:

$$\tau_0 \propto \frac{S_R^{\,6/(5+3\beta)}}{S_R^{\,6/(5+3\beta)} + S_R^{\,6/(5+3\beta)}} \qquad (28)$$

Considering B_{Max} and J_R constant values for the transformer homogeneous series (β=1):

$$\tau_0 \propto const. \qquad (29)$$

This result agrees with International Standards since they propose a fixed value of 3 hours for the thermal time constant of all distribution transformers [7]. Considering J_R evolution ($\mu_{\hat{\beta}}$=1.021), thermal time constant evolution with rated power would be represented by:

$$\tau_0 \propto \frac{S_R^{\,0.744}}{S_R^{\,0.760} + S_R^{\,0.744}} \qquad (30)$$

This expression is represented on Figure 2. The scatter diagrams of Figure 1 and Figure 2 evidence a considerable dispersion of values for thermal time constant. Recalling that these thermal time constant values were not obtained from catalogue data, but through expression (24), this variance can be explained either by the approximate character of the expression, either by the high variance values of total and oil masses, already verified when analysing these transformer characteristics. Regardless the hypotheses of J_R variation, constant or slightly increasing with transformer rated power, the conclusion regarding thermal time constant is similar: from similitude relationships the thermal time constant of distribution transformers are close to 2 hours.

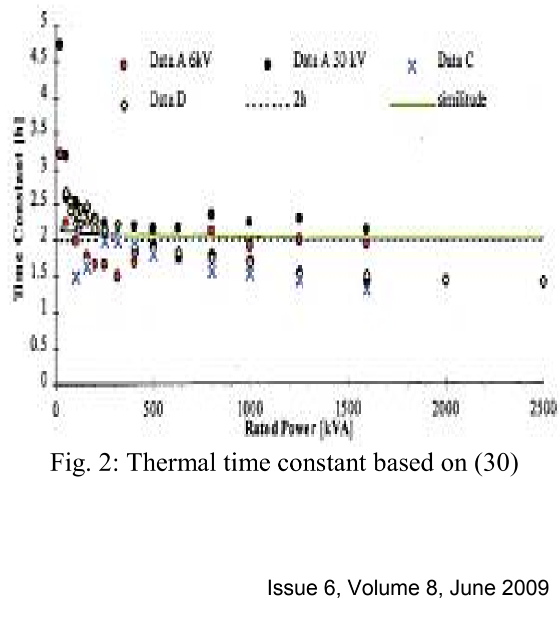


Fig. 2: Thermal time constant based on (30)

3.2 Thermal Parameters Estimation Tests

In this section transformer thermal time constant and final top-oil temperature rise under rated load, will be estimated. International Standards methodologies and methodology proposed in [9], will be applied to a single set of values from a simulated heat run test, so that "correct" parameter values are known in advance and results from different methodology can be compared [15].

3.2.1 International Standards Methodology

Existing methodologies can be classified into numerical and graphical ones. Both assume that the temperature rise, relatively to ambient temperature, of such a process can be approximated to a first order exponential process and therefore described by an increasing time exponential function:

$$\Delta\Theta_0(t) = \Delta\Theta_{of}\left(1 - e^{-t/\tau_0}\right) \tag{31}$$

where $\Delta\Theta_{of}$ denotes the final steady-state temperature rise of top-oil [K].

Method known as "three points method", [12], [13],[14] (TPM) derives directly from application of (31) to three equidistant data values $(t_1, \Delta\Theta_{o1})$, $(t_2, \Delta\Theta_{o2})$ and $(t_3, \Delta\Theta_{o3})$ such that $t_3 = t_2 + \Delta t = t_1 + 2\Delta t$. It results

$$\Delta\Theta_{of} = \frac{\Delta\Theta_{02}{}^2 - \Delta\Theta_{01}\Delta\Theta_{03}}{2\Delta\Theta_{02} - \Delta\Theta_{01} - \Delta\Theta_{03}}$$

and

$$\tau_0 = \frac{\Delta t}{\ln\dfrac{\Delta\Theta_{02} - \Delta\Theta_{01}}{\Delta\Theta_{03} - \Delta\Theta_{02}}}. \tag{32}$$

Other method recommended by [7] is the "least square method" (LSM) based upon the minimisation of square errors between data values and theoretical heating function (31). In practice, due to the complexity and non-linearity of thermal exchange, the transformer heating process is governed by more than one thermal time constant, [7], [8], possibly time or temperature dependent. Therefore, more accurate values are obtained by applying methodologies to the final part of the heating curve, when the effect of smaller thermal time constants (windings) is negligible, prevailing the effect of larger one, τ_0. For this reason, and according to [7], successive estimates by the TPM should converge and, to avoid large random numerical errors, time interval Δt should be of the same magnitude as τ_0 and $\Delta\Theta_{o3}/\Delta\Theta_{of}$ should not be less than 0.95, which, assuming (31) model, is equivalent to:

$$t_3 \geq 3\tau_0 \tag{33}$$

Similarly, the LSM should be applied only for the 60% upper part of the heating curve. Constrains for the TPM application are the necessity of equidistant measured data values and the time duration of the test given by (33). Criterion to terminate the heat run test is [7]: *to maintain the test 3 more hours after the rate of change in temperature rise has fallen below 1K per hour, and take the average of last hour measures as the result of the test*. For long term tests, such as the required by [4], invariant process conditions are of difficult sustenance namely: the constancy in transformer losses (voltage, current, $\cos\varphi$) and thermal exchange (ambient temperature, wind, sun).

3.2.2 Alternative Method

Reference [9] proposes a new method to estimate $\Delta\Theta_{of}$ and τ_0. Since (31) linearization, by a simple mathematical transformation [10],[11], is not possible for unknown $\Delta\Theta_{of}$ and τ_0 parameters and truncated data, an approximation of (31) by a polynomial function is proposed:

$$1 - e^{-t/\tau_0} \approx \left(\frac{t}{\tau_0}\right) \Big/ \left[1 + \left(\frac{t}{\tau_0}\right)/6\right]^3 \tag{34}$$

The exponential function is a majoring of the polynomial function being the systematic error, ε_S, one commits with this approximation a function of the ratio t/τ_0. This systematic error can be measured through:

$$\varepsilon_S = \frac{1 - e^{-t/\tau_0}}{(t/\tau_0)/\left[1 + (t/\tau_0)/6\right]^3} - 1 \tag{35}$$

A majoring of this systematic error, ε_M is:

$$\varepsilon_M = (t/\tau_0)^3/216. \tag{36}$$

Inserting approximation (35) into (31), one obtains:

$$f(\Delta\Theta(t), t) = a + bt \tag{37}$$

being *f* a generic non-linear function and:

$$a = \left[\frac{\tau_0}{\Delta\Theta_{of}}\right]^{\frac{1}{3}} \text{ and } b = \frac{1}{6}\left[\frac{1}{\tau_0{}^2\Delta\Theta_{of}}\right]^{\frac{1}{3}} \tag{38}$$

Therefore, linear regression methods can be used to obtain estimators of *a* and *b*, which, from a statistical point of view are random variables [1]. From estimators of *a* and *b*, $\Delta\Theta_{of}$ and τ_0 estimators can be derived as follows:

$$\Delta\hat{\Theta}_{of} = \frac{1}{6\hat{a}^2\hat{b}} \text{ and } \hat{\tau}_0 = \frac{\hat{a}}{6\hat{b}} \tag{39}$$

This methodology allows the determination of parameters variability from an estimator variability; according to recent usual recommendations, [17], the variation coefficients of the parameters, denoted by $CV_{\Delta\theta_f}$ and CV_τ, can be approximately evaluated by uncertainty propagation of corresponding variances:

$$CV_{\Delta\Theta f} \approx \sqrt{4(CV_a)^2 + (CV_b)^2} \quad \text{and}$$

$$CV_{\tau_0} \approx \sqrt{(CV_a)^2 + (CV_b)^2} \qquad (40)$$

Concerning the test duration, this methodology reduces the test duration required by [7] because relatively accurate values for the parameters can be estimated only from the beginning of the exponential trajectory, with $t<2\tau_0$. This alternative methodology will be referred as L*imited* P*eriod* M*ethodology* (LPM). From the basics of linear regression, a minimum of two data values (N=2) is required to estimate parameter values. However, and with the usual assumption that residuals are normally distributed, its second moment (variation) estimation do involves the calculus of a *t*-Student distribution with N-2 degrees of freedom. Therefore, although N=2 allows the parameters estimation, the corresponding variability determination requires $N \geq 3$ [1],[2],[14]. Moreover the initial pair of measurements (t=0; $\Delta\Theta_{o0}$=0) can not be part of the measurements set; the function to which linear regression is applied is, itself, a function of the ratio $t/\Delta\Theta_o$ and thus, initial pair of measurements would lead to a mathematical in determination.

3.2.3 Simulated Case Studies

In order to evaluate the accuracy of the concurrent methodologies, the data set of the heat run test was simulated. With such a procedure, correct values of parameters $\Delta\Theta_{of}$ and τ_0 are known in advance and therefore, errors of estimators given by the two methodologies can be evaluated. Following the first order model of International Standards, data for the simulated heat run test was assumed to follow a deterministic single exponential function, representing transformer thermal behaviour from no-load to rated load. To represent the uncertainties of the measuring process an additive perturbation such as random gaussian white noise with a null mean and variance σ^2, generated with a Monte Carlo method [13],[14],[16], was considered:

$$\Delta\Theta_0(t) = \Delta\Theta_{of}\left(1 - e^{\frac{-t}{\tau_0}}\right) + \mathrm{N}(0,\sigma) \qquad (41)$$

For a distribution transformer rated 630 kVA, 10 kV/400 V, considered values for parameters are: $\Delta\Theta_{of}$=55 K and τ_0=2h. Test data was generated up to t_{max}=12 h and with a time step Δt_{meas}=0.25 h. Four data sets were generated considering realistic o values and Table 2 specifications. Sample lengths are N=100 thus Monte Carlo inherent errors are lower than σ.

Table 2: Case studies specifications.

Specifi-cation	σ [K]	Equidistant measurements	Truncation	t_{max}/τ_o
Set n°1	0.5	Equidistant.	0- 12 h	6
Set n°2	1	Equidistant	0 - 8 h	4
Set n°3	1	Non-Equidistant	0 - 3 h	1,5
Set n°4	1	Non-Equidistant	1 - 4 h	2

Table 3: International Standards methodology results (TPM and LSM).

	Set n°1		Set n°2		Set n°3		Set n°4	
	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0
TPM	55.0	n.c.	n.c.	n.c.	-	-	-	-
LSM	55.3	2.03	56.0	2.15	48.5	1.53	50.3	1.63

Simulated data referred as Set n°3 and set n°4 are represented on Figure 3. Both time scale t and reduced time scale t/τ_0 are represented. Set n°l specifications are almost ideals since it is the most favourable for Standards methodology; white noise is of reduced variation and measurements are performed at equidistant intervals. Set n°2 is more realistic; it is similar to n°l but with a doubling white noise variation. Set n°3 presents the same level of white noise as set n°2 but measurements are not equidistant and data series was truncated on its high limit, drastically reducing test duration.

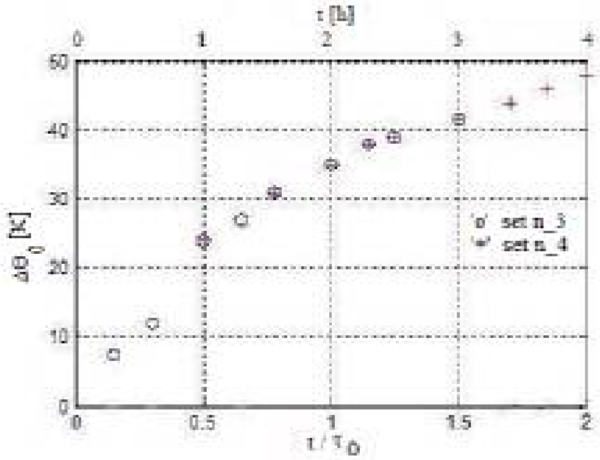


Fig. 3: Heat-run test data, set n^{0}3 and set n°4.

Set n°4 is similar to set n°3 except for truncation limits; data set window was shifted one hour later.

3.2.4 Results for International Standards Methodologies

These results are resumed on Table 3. Set n° 1 is the only one fulfilling [7] criterion to end the test at 11 hours ($\approx 5.5\,\tau_0$). The TPM did not converge (n.c) for τ_0 estimation on set n°l, Figure 4, nevertheless, conditions stated by [7] are fulfilled since time interval $\Delta\, t$ between Θ_{o1}, Θ_{o2} and Θ_{o3} is of the same magnitude as τ_0 and represented values fulfil the condition $\Delta\Theta_{o3}/\Delta\Theta_{of}$<0.95. It did not converge either for $\Delta\Theta_{of}$ or τ_0 on set n°2.

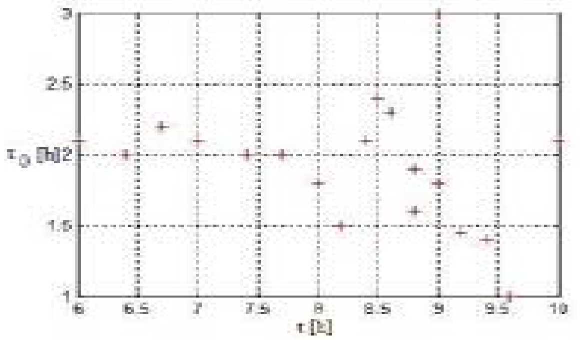
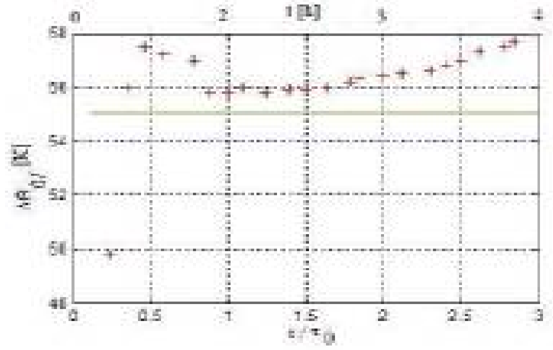


Fig. 4: Estimated τ with (54) and data set n°l (TPM).

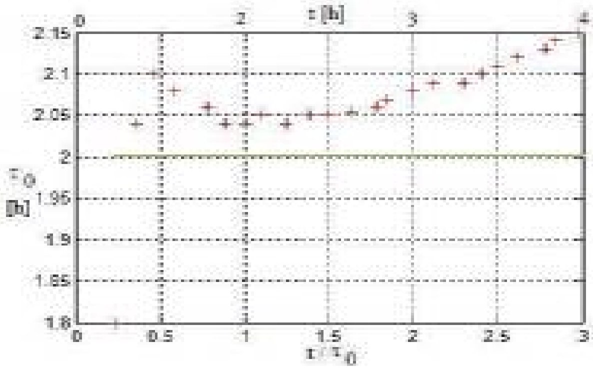
This methodology can not be applied on sets n°3 and 4, since data measurements are not equidistant. LSM provide admissible results for all tests; however its accuracy is reduced for set n°4, to which corresponds a very short test duration.

3.2.5 Results for Alternative Methodology

Since the systematic error of LPM is dependent upon the ratio t/τ_0, most relevant results for each of the four considered sets are represented in a graphical form. Figure 5 to Figure 8 represent successive estimates of parameters, as a function of increasing cumulative data from tests [18],[19]. Exact values of the parameters to be estimated are also represented as dotted lines.

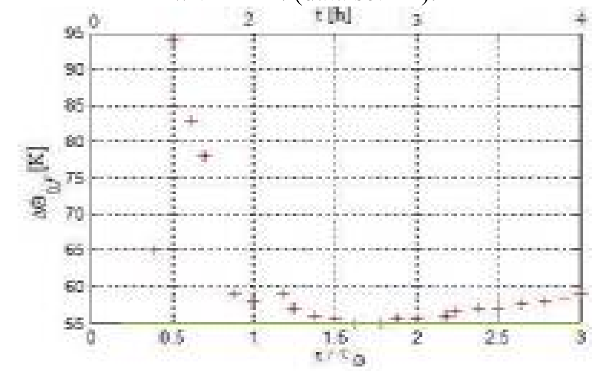


a)

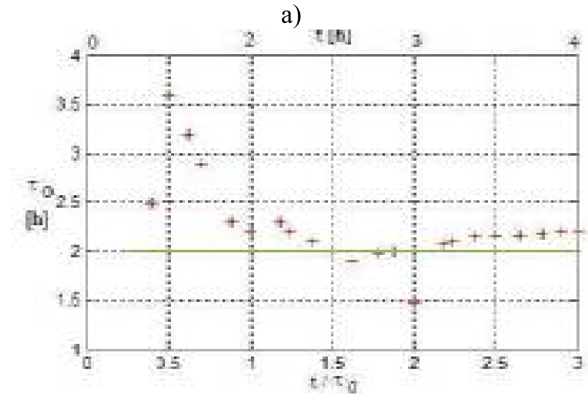


b)

Fig. 5: Average value of $\Delta\Theta_f$ (a) and τ (b) estimated with LPM. (data set n°l).

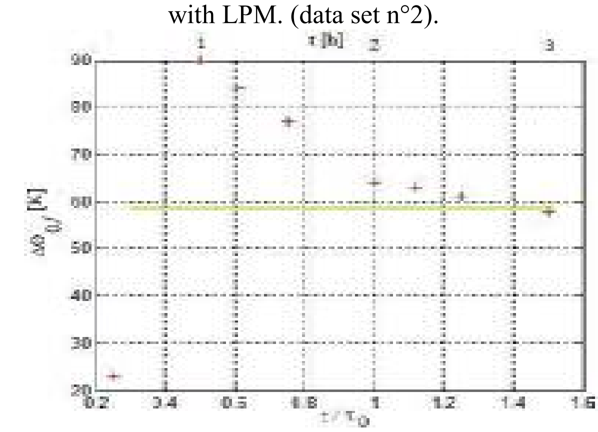


a)

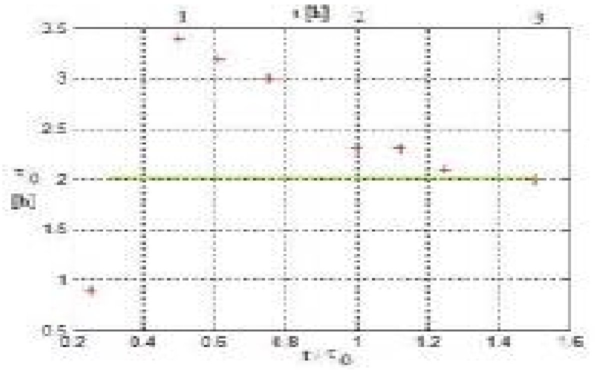


b)

Fig. 6: Average value of $\Delta\Theta_f$ (a) and τ (b) estimated with LPM. (data set n°2).

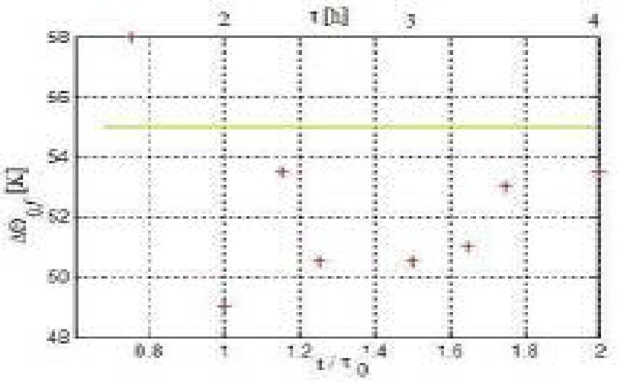


a)

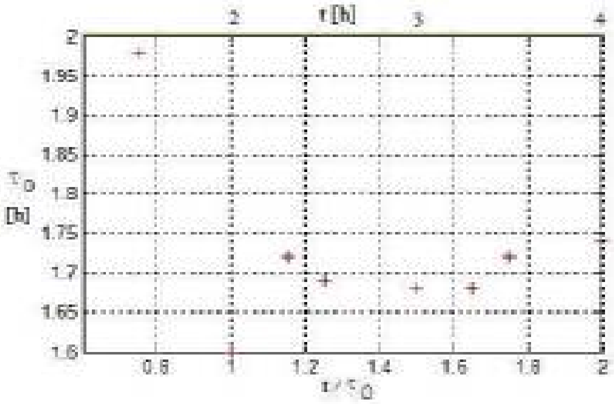


b)

Fig. 7: Average value of $\Delta\Theta_f$ (a) and τ (b) estimated with LPM. (data set n°3).



a)



b)

Fig. 8: Average value of $\Delta\Theta_f$ (a) and τ (b) estimated with LPM. (data set n°4).

3.2.6 LPM Previous Considerations and Efficiency Criterion

The approximation of the increasing exponential function (31) by a polynomial function, (19), gives rise to a systematic error of LPM, which is given by (32). This error and its majoring (21) are represented in Figure 9 as a function of the ratio t/τ_0.

In order to reduce this error, data to apply LPM must belong to the lower part of the heating curve (reduced t/τ_0 values). This error explains the increasing time drift of estimated parameter values for high t/τ_0 values, most visible on Figure 9. This mathematical constrain is traduced by an economical advantage since the duration of the required transformer heat-run tests is substantially reduced relatively to International Standards requirements. From the linear regression theory, however, to parameters estimated with a reduced number of data measurements, a high variability coefficient is associated [1].

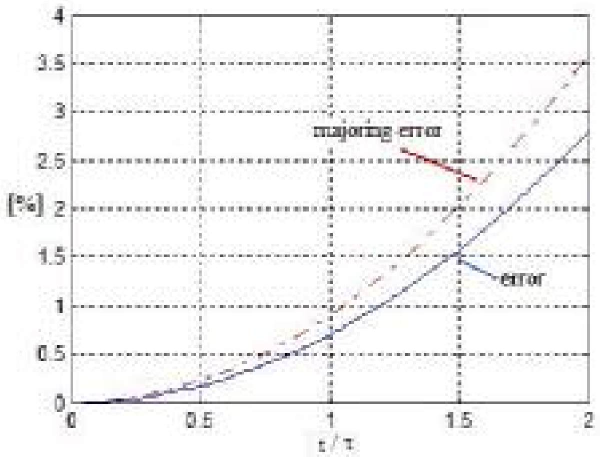


Fig. 9: LPM systematic error, ε_S and its majoring, ε_M

The first estimated parameters represented on Figure 5 to Figure 8 ($0<t/\tau_0<1$) do present a high error; however, to these values great variability coefficients are associated which, traduced by the corresponding 95% confidence interval, will include the exact $\Delta\Theta_{of}$ and τ_0 values. It is not the purpose of any methodology to estimate parameters with such a high variability, corresponding to unrealistic situations. Therefore, a compromise must be achieved between a sufficient number of data measurements but within a t/τ_0 interval constrained by the systematic error represented on Figure 9. This work proposes that approximately 10 measurements (N=10), in a range below 1.5 t/τ_0, must be considered. Comparison of results obtained with data sets n°3 and n°4 will exemplify the importance of this upper limit. While set n°3, by respecting this observation constraint (upper limit is 1.5 t/τ_0), gives very good results, set n°4, with a similar observation window length but shifted one hour (upper limit is $2t/\tau_0$), evidences a degradation of results. Taking into account previous considerations and results (Figure 5 to Figure 8) it is possible to propose a simple criterion for obtaining an accurate set of ($\Delta\Theta_{of}$, τ_0) estimators. After Figure 5 to Figure 8, one realises that best

($\Delta\Theta_{of}, \tau_0$) estimators are obtained within the range τ_0 to $2\tau_0$ and thus on the vicinity of $1.5\tau_0$. A-priori, τ_0 is unknown, and thus, so are τ_0 and ε_M. Therefore, estimates of these values (denoted by $t/\hat{\tau}_0$ and $\hat{\varepsilon}_M$) should be determined, at each instant, using the correspondent τ_0 estimation (denoted by $\hat{\tau}_0$). On Table 3 to Table 6, information concerning observed data (*t* and N), $\Delta\Theta_{of}$ and τ_0 estimators (mean and variation coefficients) and t/τ_0 and ε_M estimators, is regrouped. Due to the non-linear transformation used by LPM (37), statistical errors, CV, simultaneously depend upon N and σ (measurements variability) which, *a-priori*, are unknown parameters. A quantitative quality criterion is of difficult establishment due to errors dependence upon unknown parameters such as σ and τ_0. Therefore, an heuristic qualitative criterion is proposed, as following: to consider approximately 10 successive measurements and determine respective $\Delta\hat{\Theta}_{of}$ and $\hat{\tau}_0$ values, within a range 0< $t/\hat{\tau}_0$<1.5. A reasonably accurate set of ($\Delta\Theta_{of}, \tau_0$) estimators is obtained for $t/\hat{\tau}_0$~1.5. If $t/\hat{\tau}_0$ range can not be fulfilled (which is the case of set n°4), estimators corresponding to the lowest $t/\hat{\tau}$ values, should be considered. Application of this qualitative criterion leads to the conclusion that best bidimensional estimators ($\Delta\Theta_{of}, \tau_0$) are obtained for N=12 (on set n°l), N=12 (on set n°2), N=9 (on set n°3) and N=4 (on set n°4). These values are represented on bold face font on Tables 4 to 7.

Table 4: LPM results for Set n°1.

Data		$\Delta\hat{\Theta}_{of}$		$\hat{\tau}_0$		LPM	
t[h]	N	μ[° C]	*CV[%]*	μ[h]	CV[%]	$t/\hat{\tau}_0$ [%]	$\hat{\varepsilon}_M$ [%]
2.00	8	55.65	1.31	2.02	1.29	0.99	0.45
2.25	9	55.85	1.05	2.03	1.03	1.11	0.63
2.50	10	55.77	0.85	2.03	0.83	1.23	0.86
2.75	11	55.83	0.70	2.03	0.68	1.35	1.15
3.00	**12**	**55.85**	**0.59**	**2.03**	**0.57**	**1.48**	**1.49**

Table 5: LPM results for Set n°2.

Data		$\Delta\hat{\Theta}_{of}$		$\hat{\tau}_0$		LPM	
t[h]	N	μ[° C]	CV[%]	μ[h]	CV [%]	$t/\hat{\tau}_0$ [%]	$\hat{\varepsilon}_M$ [%]
2.00	8	56.97	14.09	2.07	13.87	0.97	0.42
2.25	9	58.11	11.29	2.11	11.09	1.07	0.56
2.50	10	56.19	9.01	2.03	8.80	1.25	0.88
2.75	11	55.63	7.35	2.00	7.15	1.38	1.20
3.00	**12**	**54.75**	**6.18**	**1.97**	**5.97**	**1.55**	**1.69**

Table 6: LPM results for Set n°3.

Data		$\Delta\hat{\Theta}_{of}$		$\hat{\tau}_0$		LPM	
t[h]	N	μ[° C]	CV[%]	μ[h]	CV[%]	$t/\hat{\tau}_0$ [%]	$\hat{\varepsilon}_M$ [%]
2.00	6	62.89	16.63	2.32	16.44	0.86	0.30
2.25	7	61.49	11.78	2.26	11.59	1.00	0.46
2.50	8	57.79	9.60	2.09	9.38	1.20	0.79
3.00	**9**	**55.56**	**7.51**	**1.99**	**7.26**	**1.51**	**1.59**

Table 7: LPM results for Set n°4.

Data		$\Delta\hat{\Theta}_{of}$		$\hat{\tau}_0$		LPM	
t[h]	N	μ[° C]	CV[%]	t[h]	N	μ[° C]	CV[%]
3.00	3	49.12	10.38	3.00	3	49.12	10.38
3.25	**4**	**52.15**	**3.90**	**3.25**	**4**	**52.15**	**3.90**
3.50	5	50.45	3.00	3.50	5	50.45	3.00
4.00	6	50.49	1.99	4.00	6	50.49	1.99

3.2.7 Comparative Analysis

Table 8 regroups International Standards (Table 3 for TPM and LSM) and LPM (Table 6 to Table 7) methodologies results giving the estimated parameter errors, as percentage values of correct ones $\Delta\Theta_{of}$=55 K and τ_0=2 h. The duration of the test to achieve corresponding results is also represented (t_{max}). For LPM, values after the §3.2.6 criterion are represented.

Table 8: Parameter errors [%] for concurrent methodologies.

	Set n°l		Set n°2		Set n°3		Set n°4	
	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0	$\Delta\Theta_{of}$	τ_0
International Standards Methodology								
t_{max}	11 h		8h		3h		4h	
TPM	0.0	n.c.	n.c.	n.c.				
LSM	0.19	0.51	1.81	7.00	-11.81	-24.00	8.73	-18.7
Alternative Methodology LPM for $1 < t/\hat{\tau}_0 < 1.5$								
t_{max}	3h		3h		3h		3.25 h	
	1.55	1.51	-0.49	-2.51	1.03	-0.51	5.19	-14.4

International Standards methodologies (TPM and LSM) give very good estimations for set n°l but they require 11 hours of run test, while LPM methodology provides sufficiently accurate values after 3 hour of testing. For set n°2, LPM provides better estimators and after, approximately, less than 1/2 of the test duration required by International Standards (TPM and LSM). For set n°3, estimations given by LPM are clearly better than those provided by International Standards (LSM) for the same test duration. Although data of set n°4 does not fulfil LPM requirements, it provides better estimators than LSM and with reducer test duration.

4 Conclusions

In order to study transformers thermal loss of life, complex models taking into account electrical and thermal characteristics are required. Moreover, the precision of thermal models is dependent upon the exactitude of the parameters. The work presented in this article shows that, through electromagnetic similitude laws, for an homogeneous series of transformers with different rated powers, the main parameters required for the thermal model are achieved. The foremost advantage of this methodology is its compactness, since parameters are obtained only from the knowledge of transformer rated power. Theoretical results were compared with data from transformer manufacturers and the good agreement between both validates theoretical results. Due to data variation one can not conclude whether, within the considered power range, the rated current density should be considered constant or not; due to data variation, results from both hypotheses are satisfactory. As will be studied on future, the exactitude of thermal parameters "thermal time constant" and, mainly, "final temperature rise", is determinant on thermal model accuracy. Usually, these parameters are obtained from standardised heat-run tests and their correct measurement is of difficult precision due to data measurement variability. In this article, an easy and efficient method to estimate these thermal parameters, as well as the corresponding using criteria, were proposed. This robust methodology presents advantages relatively to the standardised methodologies, since it allows a considerably reduction on test duration, and provides results which are always physically acceptable and with measurable precision.

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