Steady State Modelling of Induction Motor Operating With Unbalanced Supply System

KANWARJIT SINGH SANDHU, VINEET CHAUDHARY

Electrical Engineering Department
National Institute of Technology Kurukshetra
Haryana-136119
INDIA
sandhukjs@rediffmail.com, vineetvceb@gmail.com

Abstract: - Three phase supply system is found to be quite balanced in both magnitude and displacement at the generation levels but it is not so at distribution end. Voltage unbalance which is a common and global phenomenon is found to be very effective in deteriorating the performance of electrical apparatus at user end. Such environment badly affects the performance of induction motors, an important class of electric machines which is widely used in industrial. Present paper is an attempt to tackle such operations with specific recommendations & proposals. Symmetrical component approach is adopted to estimate the performance of a three-phase induction motor operating with unbalanced supply system. Further MATLAB/SIMULINK & PSIM environments have been used simultaneously for simulation purpose. A new approach is proposed to rerate the motor operating with voltage unbalance.

Key-Words: - Abnormal operation, Three Phase Induction Motors, Rerating, Steady state Analysis, Unbalanced Supply.

1. Introduction

In the present developing scenario, induction motor is an important class of electric machines which is widely used in industrial, commercial and domestic applications. Even scientists have found these machines capable of generating electrical power [1-4]. Due to many attractions such as simple rugged and inexpensive construction, reduced maintenance, and excellent operating characteristics induction motors are found to be very popular in industries. As a rough estimate nearly 80% of world industrial motors are three-phase phase induction motors. These motors are designed and manufactured with intentions that their operation with balanced supply system results in to rated output. Voltage unbalance which is a common phenomenon is observed almost everywhere in a three phase system across the world. Although three phase voltage supply is quite balanced in both magnitude and displacement at the generation and transmission levels. It exits at utilization end due to the followings;

- Unequal distribution of single-phase loads in a three phase supply system.
- Asymmetry of transmission line and transformer winding impedances.
- Open delta transformer connections.
- Defective transformers in power system network.
- Blown out fuses in on capacitor banks.
- Time varying operation of single phase loads.
- Traction loads.
- Rural electric power system.
- Adjustable speed drives (ASD) applications.

The voltage unbalance has many ill-effects on supply system as well as on the consumer. Absence of voltage unbalance on a distribution system is almost impossible due to randomness of the connections and disconnection of single phase loads, uneven distributions of single-phase loads on the three phases and inherent asymmetry of power system. However, there are utility system level mitigation techniques as well as plant level mitigation techniques that can be used to improve the voltage unbalance and its effects.

Strictly speaking voltage unbalance can be considered as an irregularity of power supply system. The influence of unbalanced voltages on the performance of motor was first studied by Reed and Koppman in1936 [5]. Further in 1956 Williams [6] proved that an induction motor operation with unbalanced voltage is undesirable. Studies conducted by Gafford et al [7] come up in 1959 concerning temperature rise of an induction motor and it was pointed out that due to this overheating premature ageing of insulation takes place and the life of motor is reduced. In 1963 Berndt et al [8] presented a method for derating of an induction motor operating with unbalance supply. In 1975 R. F. Woll [9] provided a simple and brief method in order to study the impact of unbalanced voltages on the losses and its negative effects on the insulating material of induction motor. In 1985 Cummings [10] study provided methods for protection of the motor and adjusting the relays settings against unbalance. It was observed that most of the research was focused on voltage unbalance caused by under-voltage unbalance. In fact, over-voltage unbalance often occurs in off-peak periods in several countries. Further the impact of voltage unbalance on the performance has been described qualitatively, not quantitatively or numerically.

Therefore, the subject has been revisited in resent years and Kersting [11] has discussed the effect of 0-5% unbalance on the losses of induction motor. Analysis of induction motor in phase frame eliminates the necessity of transforming all known quantities in to symmetrical components. In 2000 Wang [12] studied the influence of voltage unbalance upon the study state performance of an induction motor analytically. It has been shown that voltage unbalance may cause the motor line currents to be very unbalanced. In 2001 Wang [13] evaluated the effects of voltage unbalance on induction motor by using complex voltage unbalance factor (CVUF) that consist magnitude as well as angle to fully describe the voltage unbalance phenomenon. In 2002 Pillay and Hafmann [14] had examined the derating of an induction machine when supplied by unbalanced voltages in combinations with over- and under-voltages by using electrical and thermal model. They concluded that the difference in definitions do not result in significant difference when operated by unbalanced supplies in the 5% range. In 2004 Faiz et al [15] analyzed the different definitions given by standards and suggested that the available definitions of voltage unbalance are not comprehensive and complete, therefore the results by these definitions are not reliable. In 2005 Faiz et al [16] proposed that inclusion of phase angle in addition to voltage unbalance factor (VUF) give more accurate result. He introduced a method to determine the derating factor precisely using the complex unbalance factor (CVUF), in order to evaluate this factor, the machine is loaded such that the current do not exceed the rated value, derating factor is then computed as the ratio of the machine output power under balanced supply conditions to that under balanced condition. In 2006 Faiz et al [17] investigated that, for the same voltage unbalance, derated motor may have higher efficiency than non-rated motor.

Present paper describes the approach based upon symmetrical component theory to analyze the operation of 3-phase induction motor, operating with unbalanced supply system. PSIM and MATLAB/SIMULINK are also used for the simulation under such operations. Comparison of results proves the effectiveness of methodologies. A new proposal has been suggested for rerating of induction motor.

2. Types of Voltage Unbalance

There may be different type of unbalance in a supply system and exists definite possibility of voltage variations above and below the rated value. Thus voltage unbalance can be classified into over-voltage unbalance (OVU) under-voltage unbalance (UVU) unbalance. OVU is a condition when the three phase voltages are not equal to each other, in addition positive sequence component is greater than the rated value while UVU is a condition where the three phase voltages are not equal to each other, and in addition the positive-sequence component is lesser than the rated value. Unbalanced supply systems may be put under any one of the following categories.

2.1. Single Phase Under-Voltage Unbalance

The single phase under voltage unbalance situation arises when there is a large single phase load in the system and it does not have enough compensation, the voltage in concerned phase will be lower than other two phases.

2.2. Two Phase Under-Voltage Unbalance

Two phase under-voltage unbalance occurs when the two of the three phases have heavy load and do not have enough compensation, in this situation those two phases will have higher voltage drop than the third phase.

2.3. Three Phase Under-Voltage Unbalance

This type of condition arises when the loads of three phases are all too heavy and not balanced. In this situation the voltage in all three phases will be lower than the rated voltage and three-phase under voltage unbalance arises. This is the most common unbalanced created by the unequal loading in a three phase supply system.

2.4. Single Phase Over-Voltage Unbalance

To maintain a system voltage at rated value, capacitors are normally used to compensate system reactive power. If one of the three phase voltages is over-compensated, the voltage of this phase will be higher than the rated value, the single phase over-voltage unbalance occurs.

2.5. Two Phase Over-Voltage Unbalance

If two of the three phases are over compensated, the voltages of these two phases will be higher than the rated value. This condition called two phase over-voltage unbalance.

2.6. Three Phase Over-Voltage Unbalance

Three phase over-voltage unbalance is a condition when three phases are over-compensated to different degrees, then all the three-phase voltages will be higher than the rated value and not equal.

2.8. Single Phase Angle Displacement Unbalance

If the three phase voltages are balanced, the angle displacement between them should be equal to 120^0.Let one of the phase is taken as reference, if one of the other two phase angles is deflected, unequal displacement in single phase angle occur

2.9. Two Phase Angle Displacement Unbalance

Similar to single phase angle unbalance, if the other two phase angles are both deflected, then unequal angle displacement in two phase angles occurs.

In actual practice more than these eight types of voltage unbalance can be present in a three phase supply system. They may be the combination of first six magnitude unbalances and last two phase angle unbalances. In all unbalances discussed above three-phase under voltage and three-phase over voltage unbalance is very common voltage unbalance in supply.

3. Unbalance Definition

The two general definitions for voltage unbalance as described are;

3.1. NEMA or IEEE Definition

The voltage unbalance percentage (VUP) at the terminal of a machine as given by the National Electrical Manufacturer Association Motor and Generator Standard (NEMA MGI) used in most studies is:

$$\%VUP = \frac{\max imum \quad voltage \quad deviation \quad from \quad average \quad line \quad voltage}{average \quad line \quad voltage} \times 100\%$$

3.2. IEC or Symmetrical Component Definition

The voltage unbalance factor (VUF) is defined by International Electro technical Commission (IEC) as the ratio of negative-sequence voltage component to the positive-sequence voltage component.

$$\%\,VUF = \frac{negative-sequence \quad voltage \quad component \quad magnitude}{positive-sequence \quad voltage \quad component \quad magnitude} \times 100\%$$

$$= \frac{|\mathrm{V_n}|}{|\mathrm{V_p}|} \times 100\%$$

Where Vn and Vp are the magnitude of negative and positive sequence voltage components respectively are obtained by symmetrical component transformation.

An extension of VUF is the complex voltage unbalance factor (CVUF) that is defined as the ratio of negative-sequence voltage phasor to positive-sequence voltage phasor. The CVUF is a complex quantity having the magnitude and angle. Appropriately, complex voltage unbalance factor (CVUF) can be written as

$$\mathrm{CVUF} = \frac{\mathrm{V_n}\angle\theta_\mathrm{n}}{\mathrm{V_p}\angle\theta_\mathrm{p}} = \mathrm{K_u}\angle\theta_\mathrm{u}\,;$$

Where, $K_u = \left|\frac{V_n}{V_P}\right|$ is VUF,

and

$\angle\theta_u = \angle\theta_n - \angle\theta_p$, phase angle by which V_n leads V_p.

4. Steady State Analysis

Steady state analysis of an induction motor operating with unbalanced voltage supply is possible using symmetrical component approach. This requires the development of positive- and negative-sequence equivalent circuit representation as shown in Fig .1.

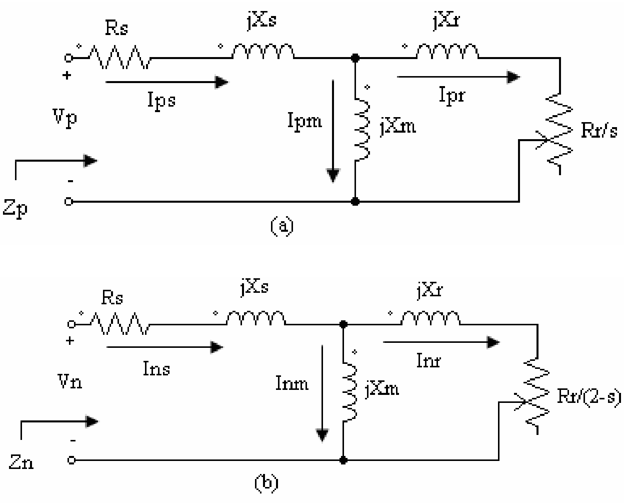


Fig.1 Per-phase equivalent circuit representation

Where, per phase values are defined as;

Vp positive-sequence voltage,

Vn negative-sequence voltage,

Rs stator resistance,

Xs stator reactance,

Rr rotor resistance referred to stator,

Xr rotor reactance referred to stator,

Xm magnetizing reactance,

Zp positive-sequence impedance of motor,

Zn negative-sequence impedance of motor,

Ips stator positive-sequence current phasor,

Ipr rotor positive-sequence current phasor,

Ins stator negative-sequence current phasor,

Inr rotor negative-sequence current phasor,

s operating slip of the motor

Let V_a, V_b and V_c be the phase voltages of a motor. The corresponding zero-, positive and negative-sequence components $(V_0, V_p \text{ and } V_n)$ of the voltages are given by

$$\begin{bmatrix} V_0 \\ V_p \\ V_n \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Where, $a = 1\angle 120^\circ$

Analysis of equivalent circuit gives;

$$Z_i = R_s + jX_s + \frac{(jX_m)\left(\frac{R_r}{s_i} + jX_r\right)}{\frac{R_r}{s_i} + j(X_m + X_r)} \tag{1}$$

For positive-sequence impedance, $i = p$ & $(s_p = s)$

For negative-sequence impedance, $i = n$ & $(s_n = 2 - s)$.

The Positive– and Negative-sequence stator and rotor are;

$$I_{ps} = \frac{V_p}{Z_p} \tag{2}$$

$$I_{pr} = I_{ps} \times \frac{(jX_m)}{\frac{R_r}{s} + j(X_m + X_r)} \tag{3}$$

$$I_{ns} = \frac{V_n}{Z_n} \tag{4}$$

$$I_{nr} = I_{ps} \times \frac{(jX_m)}{\frac{R_r}{(2-s)} + j(X_m + X_r)} \tag{5}$$

Assuming that the machine in delta or ungrounded Wye connected;

$$\left.\begin{aligned} I_{as} &= I_{ps} + I_{ns} \\ I_{bs} &= a^2 I_{ps} + aI_{ns} \\ I_{cs} &= aI_{ps} + a^2 I_{ns} \end{aligned}\right| \tag{6}$$

Similarly the rotor and magnetizing branch currents for three phases of induction motor can be easily calculated by transformation.

$$\begin{aligned} I_{ar} &= I_{pr} + I_{nr} \\ I_{br} &= a^2 I_{pr} + aI_{nr} \\ I_{cr} &= aI_{pr} + a^2 I_{nr} \end{aligned} \tag{7}$$

$$\begin{aligned} I_{am} &= I_{pm} + I_{nm} \\ I_{bm} &= a^2 I_{pm} + aI_{nm} \\ I_{cm} &= aI_{pm} + a^2 I_{nm} \end{aligned} \tag{8}$$

The motor input power and power factor can be expressed in terms of symmetrical components of the voltage and currents as

Input active power;

$$(P_{in}) = \mathrm{Re}\left[3\left(V_p \cdot I_{ps}^{\;*} + V_n \cdot I_{ns}^{\;*}\right)\right]$$

Input reactive power;

$$(Q_{in}) = \mathrm{Im}\left[3\left(V_p \cdot I_{ps}^{\;*} + V_n \cdot I_{ns}^{\;*}\right)\right]$$

Power factor $(p.f.) = \cos\left[\tan^{-1}\left(\frac{Q_{in}}{P_{in}}\right)\right]$

Where $(*)$ indicates the conjugate value.

As the currents for the stator and rotor phases are known the stator copper losses and rotor copper losses can be calculated as

Stator copper losses $= \left(I_{as}^{\;2} + I_{bs}^{\;2} + I_{cs}^{\;2}\right)R_s$

Rotor copper losses $= \left(I_{ar}^{\;2} + I_{br}^{\;2} + I_{cr}^{\;2}\right)R_r$

Total copper losses

$$= \left(I_{as}^{\;2} + I_{bs}^{\;2} + I_{cs}^{\;2}\right)R_s + \left(I_{ar}^{\;2} + I_{br}^{\;2} + I_{cr}^{\;2}\right)R_r$$

In case core loss and mechanical losses are negligible, output power due to positive- and negative-sequence component may be obtained as;

$$P_p = 3I_{pr}^{\;2}\left(\frac{1-s}{s}\right)R_r$$

$$P_n = 3I_{nr}^{\;2}\left(\frac{s-1}{2-s}\right)R_r$$

Where as net output is;

$$P_{out} = P_p + P_n$$

Where, P_n is negative at normal slip because rotor rotates in opposite direction of the magnetic field produced by negative-sequence component. Torques produced by positive- and negative-sequence component is;

$$T_p = \frac{P_p}{\omega_m} = \frac{P_p}{\omega_s(1-s)} = \frac{3I_{pr}^2 R_r}{s\omega_s}$$

$$T_n = \frac{P_n}{\omega_m} = \frac{P_n}{\omega_s(1-s)} = \frac{3I_{nr}^2 R_r}{(2-s)\omega_s}$$

Where, ω_m is angular speed of the rotor, and ω_s is synchronous speed. There for motors net output torque is given as;

$$T = T_p + T_n = \frac{3R_r}{\omega_s}\left(\frac{I_{pr}^2}{s} - \frac{I_{nr}^2}{2-s}\right) \qquad (9)$$

Efficiency of the motor is defined as;

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

Simulation of 3-phase induction motor under unbalanced supply system is possible by using MATLAB/Simulink or PSIM software, as shown in Fig.-2 and Fig.-3 respectively

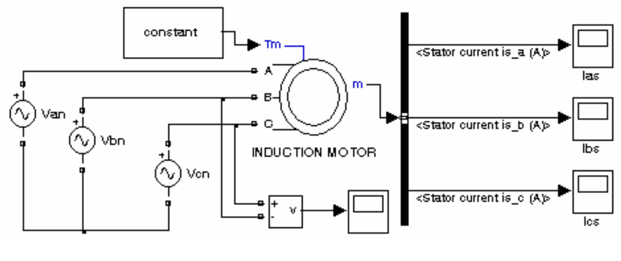


Fig.2 MATLAB Simulation

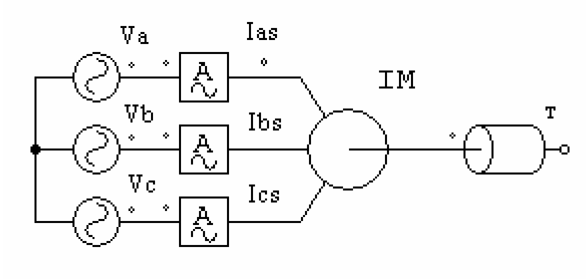


Fig.3 PSIM Simulation

5. Rerating Of Motor

In order to maintain the temperature/heating of the motor under unbalanced operation, William [2] developed the following expression for rerating ;

$$\text{Rerating} = \sqrt{\left(1 - \left(\frac{I_{ns}}{I_r}\right)^2\right)} \times \text{(name plate rating)}$$

Where 'I_r' is rated current of the motor.

However it has been felt that the motor losses which are primarily responsible for overheating must be maintained corresponding to full load operation under with balanced supply system.

Further using equations (1) to (9), for a known value of Vp and Vn (i.e. for given voltage unbalance), total losses of the machine and torque developed may be represented as a function of operating slip as;

Losses=function (slip) (10)

Torque=function (slip) (11)

Equation (10) gives the value of operating slip to maintain the full load losses and this may be used in (11) to estimate the rerated torque. This approach as proposed may be helpful to control the heating of motor.

6. Results and Discussions

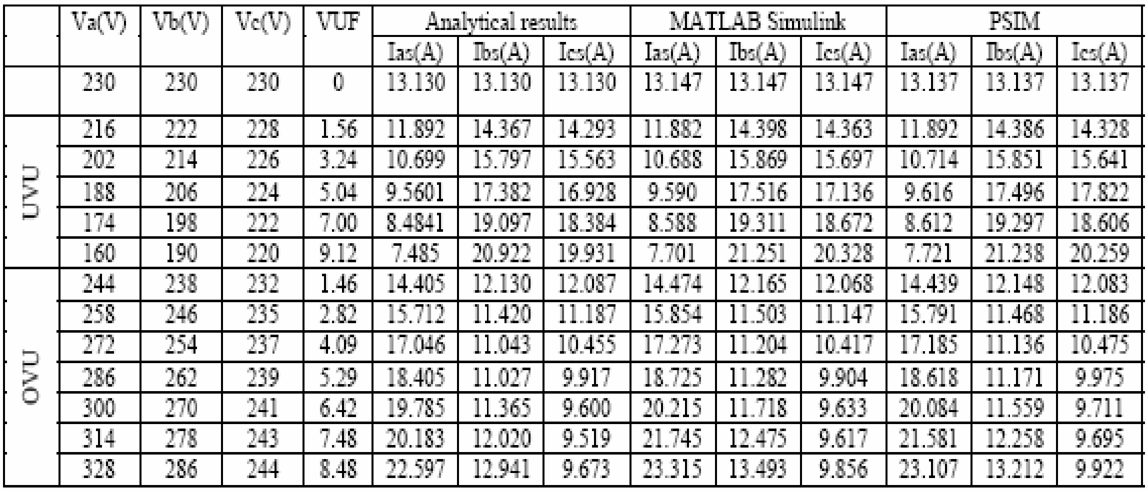
Table 1 shows a comparison of analytical, MATLAB and PSIM simulations. Results as obtained on an induction motor [Appendix 1], using symmetrical component approach (analytical results), MATLAB/SIMULINK & PSIM are found to be in good agreement for both types of unbalance i.e. under voltage and over voltage unbalance. This confirms the validity of approach adopted to estimate the performance of induction motor operating with unbalanced supply system.

Table 2 gives the performance comparison of machine with and without rerating. It is observed that without rerating losses which are primarily responsible for heating increases with an increase in unbalance factor.

Fig. 4 to Fig. 7 describes the effects of voltage unbalance on total losses, output torque, efficiency and operating power factor of the machine. Study leads to the following observations.

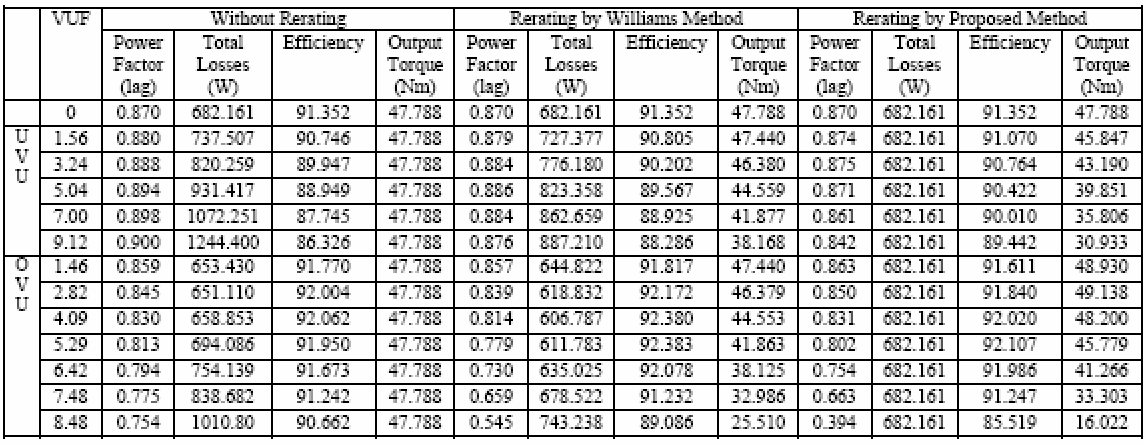
- In order to maintain the thermal limits of motor, there is a need to reduce the mechanical load for any degree of unbalance.
- Current in one phase rises rapidly as compared to other two phases, thus resulting in to non uniform heating.
- Without rerating losses varies non-linearly with VUF i.e drastic overheating and efficiency falls sharply.
- Losses that are the indication of heat are maintained in proposed methodology.
- Rerated torque is more for normal range of OVU.

TABLE 1:- COMPARISON OF ANALYTICAL AND SIMULATED RESULTS



	Va(V)	Vb(V)	Vc(V)	VUF	Analytical results			MATLAB Simulink			PSIM		
					Ias(A)	Ibs(A)	Ics(A)	Ias(A)	Ibs(A)	Ics(A)	Ias(A)	Ibs(A)	Ics(A)
	230	230	230	0	13.130	13.130	13.130	13.147	13.147	13.147	13.137	13.137	13.137
UVU	216	222	228	1.56	11.892	14.367	14.293	11.882	14.398	14.363	11.892	14.386	14.328
	202	214	226	3.24	10.699	15.797	15.563	10.688	15.869	15.697	10.714	15.851	15.641
	188	206	224	5.04	9.5601	17.382	16.928	9.590	17.516	17.136	9.616	17.496	17.822
	174	198	222	7.00	8.4841	19.097	18.384	8.588	19.311	18.672	8.612	19.297	18.606
	160	190	220	9.12	7.485	20.922	19.931	7.701	21.251	20.328	7.721	21.238	20.259
OVU	244	238	232	1.46	14.405	12.130	12.087	14.474	12.165	12.068	14.439	12.148	12.083
	258	246	235	2.82	15.712	11.420	11.187	15.854	11.503	11.147	15.791	11.468	11.186
	272	254	237	4.09	17.046	11.043	10.455	17.273	11.204	10.417	17.185	11.136	10.475
	286	262	239	5.29	18.405	11.027	9.917	18.725	11.282	9.904	18.618	11.171	9.975
	300	270	241	6.42	19.785	11.365	9.600	20.215	11.718	9.633	20.084	11.559	9.711
	314	278	243	7.48	20.183	12.020	9.519	21.745	12.475	9.617	21.581	12.258	9.695
	328	286	244	8.48	22.597	12.941	9.673	23.315	13.493	9.856	23.107	13.212	9.922

TABLE 2:- PERFORMANCE COMPARISON



	VUF	Without Rerating				Rerating by Williams Method				Rerating by Proposed Method			
		Power Factor (lag)	Total Losses (W)	Efficiency	Output Torque (Nm)	Power Factor (lag)	Total Losses (W)	Efficiency	Output Torque (Nm)	Power Factor (lag)	Total Losses (W)	Efficiency	Output Torque (Nm)
	0	0.870	682.161	91.352	47.788	0.870	682.161	91.352	47.788	0.870	682.161	91.352	47.788
UVU	1.56	0.880	737.507	90.746	47.788	0.879	727.377	90.805	47.440	0.874	682.161	91.070	45.847
	3.24	0.888	820.259	89.947	47.788	0.884	776.180	90.202	46.380	0.875	682.161	90.764	43.190
	5.04	0.894	931.417	88.949	47.788	0.886	823.358	89.567	44.559	0.871	682.161	90.422	39.851
	7.00	0.898	1072.251	87.745	47.788	0.884	862.659	88.925	41.877	0.861	682.161	90.010	35.806
	9.12	0.900	1244.400	86.326	47.788	0.876	887.210	88.286	38.168	0.842	682.161	89.442	30.933
OVU	1.46	0.859	653.430	91.770	47.788	0.857	644.822	91.817	47.440	0.863	682.161	91.611	48.930
	2.82	0.845	651.110	92.004	47.788	0.839	618.832	92.172	46.379	0.850	682.161	91.840	49.138
	4.09	0.830	658.853	92.062	47.788	0.814	606.787	92.380	44.553	0.831	682.161	92.020	48.200
	5.29	0.813	694.086	91.950	47.788	0.779	611.783	92.383	41.863	0.802	682.161	92.107	45.779
	6.42	0.794	754.139	91.673	47.788	0.730	635.025	92.078	38.125	0.754	682.161	91.986	41.266
	7.48	0.775	838.682	91.242	47.788	0.659	678.522	91.232	32.986	0.663	682.161	91.247	33.303
	8.48	0.754	1010.80	90.662	47.788	0.545	743.238	89.086	25.510	0.394	682.161	85.519	16.022

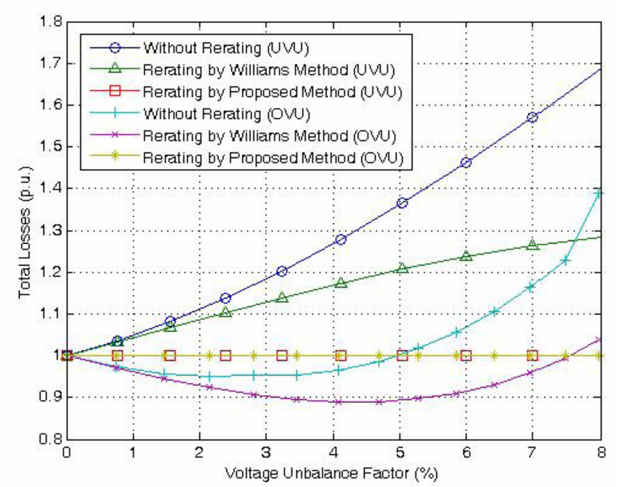


Fig. 4 Variation of losses with VUF

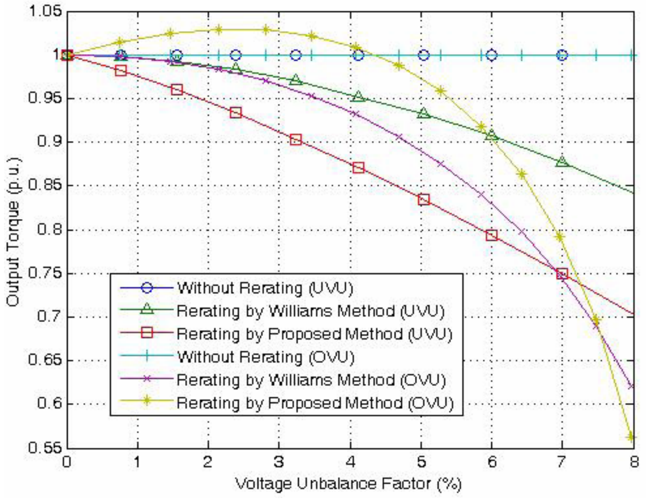


Fig. 5 Variation of output torque with VUF

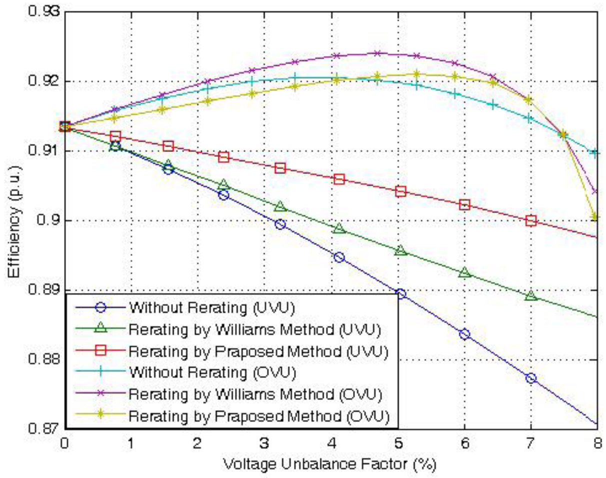


Fig. 6 Variation of efficiency with VUF

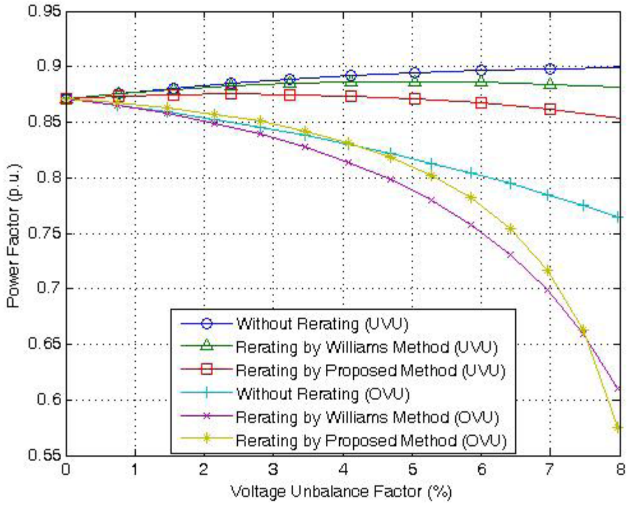


Fig. 7 Variation of power factor with VUF

7. Conclusions

Induction motor is an important type of electric machines which is widely used and acceptable in industrial, commercial and domestic applications. These machines are meant to operate under balanced and rated supply system. But it is found that absence of voltage unbalance on a distribution system is almost impossible due to randomness of the connections and disconnection of single phase loads, uneven distributions of single-phase loads on the three phases and inherent asymmetry of power system. Sometimes this results in to the compulsion to operate such machines under unbalanced supply system.

Therefore, in this paper an attempt has been made to analyze and control the operation of three-phase induction motor operating with unbalanced supply system. For the first time MATLAB based programming, MATLAB SIMULINK & PSIM environments are simultaneously used to estimate the steady state performance of motor. Comparison of results indicates that any methodology can be adopted for the analysis of machine with unbalanced supply system. Both 'under voltage unbalance' as well as 'over voltage unbalance' have been considered for analysis purpose. A new approach has been proposed and adopted for rerating of the motor and is found to be effective to control the excessive heating of the motor operating with unbalanced supply system.

In future this research work may be extended for the analysis of induction machine operating under other types of unbalanced supply systems as described in section 2.

Appendix-1

Specifications of the induction motor used for simulation results are;

Three phase, 4 pole, 10 HP,

400/230 volt, 50 Hz, Y- connected, 1440 rpm

Rs=0.7384ohm,

Rr=0.7422 ohm,

Xs=Xr=0.9566 ohm,

Xm=38.9872 ohm,

Moment of Inertia=0.0343 kg-m^2,

Friction factor =0.000503

All parameters are on per phase basis.

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