Direct Solution of Compensated Radial Distribution Networks with Constant Impedance/Current Loads

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Abstract: - In this paper, a methodology to solve radial distribution networks, with constant current and/or impedance loads as well as shunt capacitor banks, is proposed. The techniques currently available to solve such systems are based either on iterative methods or on the bus impedance matrix. In both cases, the elaboration times are quite high since these are related, for iterative methods, to the overall number of iterations, and, for those methods based on the impedance matrix, to the inversion of the admittance matrix. The method developed here is the extension of a technique that is valid to solve networks made of impedances with one supply point. The methodology can also be applied in the solution, with the backward/forward methodology, of meshed systems having voltage-dependent loads and PV nodes.
After having described the two cited networks' analysis methodologies, the new methodology is developed and its implementation is described. The comparison of the computational performances, in terms of CPU times, between the iterative method, the method using the bus impedance matrix and the one developed in this paper provide evidence regarding the efficiency of the latter in strongly reducing the calculation times.

Key-Words: - Distribution networks, Backward/forward method, Ladder networks.

1 Introduction

The recent growing attention focused on distribution systems, caused by the electrical energy market liberalization and by a greater sensibility towards energy saving and quality problems, has brought about large innovations both in the design and automated operation of these systems. In particular, new functions have been implemented to increase efficiency, service continuity and quality in these systems {compensation, reconfiguration, service restoration, loss minimization ([1÷20])}.

For the solution of the relevant problems, heuristic techniques requiring the analysis of a number of configurations are often employed. It is thus necessary, especially during the solution of online operation problems for these systems, to be able to access a precise and fast solution method.

In general, the networks solution methods depend on their structure, on the nature of the loads and on the adopted schematization for the lines. Since eighties specific methodologies have been set up to analyze distribution systems (radial and weakly meshed). They can be classified into two categories: Newton methods ([21÷22]) and backward/forward methods.

If the network is radial, supplied at one point and loads are with constant power or even mixed (with a component with constant power), the most used solution method is the iterative backward/forward (b/f) method which requires limited calculation resources and allows the attainment of a solution with a few iterations ([23÷34]).

For constant current loads (*I*), constant impedance loads (*Z*) or even mixed (*ZI*), the b/f method is still efficient even if the network can be solved directly; obviously, this is possible because all the system components are modeled by means of linear models such as constant impedance and constant current components.

As far as the new automation functions are concerned, the more frequently made assumptions are those to neglect the lines' capacitances and to consider constant current loads. In this case, associating a constant capacitive current with the compensation banks, the solution of these systems is straightforward since the currents' distribution in all the network branches is immediately known. If instead, lines' capacitances cannot be neglected and capacitor banks are schematized with impedances, then the methods to solve the network are of two types: the iterative b/f method and the direct method based on the bus impedance matrix attained through the inversion of the bus admittance matrix. Both methods are computationally expensive, as an effect of the number of iterations for the first, and because of the necessity to invert the bus admittance matrix for the second; the iterative method is normally faster than the direct method based on the bus impedance matrix.

In this paper a direct method is set up. It does not use the above-mentioned matrices and allows a strong reduction of calculation times also as compared to the iterative methods.

The solution methodology is based on a technique used for networks only made of impedances, suitably modified to take into account the presence of constant current loads. After having illustrated the two methods that allow the solution of the network, the proposed solution method and its implementation are detailed. Finally, the results of the applications executed on some networks showing the efficiency of the proposed methodology in the calculation times' reduction are reported.

2 Solution of the network by the iterative B/F method or by the bus impedance matrix

2.1 Iterative method

The radial system, supplied at one point, and made up of series and shunt impedances, with load currents derived by the nodes can easily be solved through the backward/forward method whose main steps are as follows:

1) the voltages are fixed at all the nodes;
2) the currents derived on the shunt impedances of every node are calculated;
3) starting from the terminal nodes, the branch currents are calculated;
4) starting from the branch connected to the source node, the ending bus voltages of each branch are calculated;
5) the just calculated bus voltage values are compared to the bus voltage values at the beginning of the iteration; if the error is smaller than a given margin, the process stops, otherwise another iteration is begun starting from step 2.

2.2 Bus impedance matrix

Indicating with 0 the source node and with 1 the ending bus of the branch connected to the supply node, the balance of the currents at node 1 gives:

$$\boldsymbol{U_1} \cdot \sum_{J=0,N+1;\, j\neq 1} \boldsymbol{Y_{1,J}} - \sum_{J=2,N} \boldsymbol{Y_{1,J}} \cdot \boldsymbol{U_J} = -\left(-\boldsymbol{Y_{0,1}} \cdot \boldsymbol{U_{source}} + \boldsymbol{I_{L,1}}\right) \qquad (1)$$

where:
$\boldsymbol{I_{L,1}}$ is the constant current load at node *1*;
$\boldsymbol{U_1}$ and $\boldsymbol{U_J}$ are the bus voltages at nodes *1* and *J*;
$\boldsymbol{U_{source}}$ is the source bus voltage;
$\boldsymbol{Y_{1,J}}$ is the admittance of the branch connecting nodes *1* and *J*;
$\boldsymbol{Y_{0,1}}$ is the admittance of the branch connecting nodes *0* and *1*, and
N is the number of nodes of the network not counting the reference node.
The first summation is extended to *N+1* nodes because in it the branch admittance between node *1* and the reference node is included; in the second summation, the impedances are those of the branches connecting node *1* with the other nodes of the network. For another generic node of the network, *K*, the currents balance (with the obvious meaning of the symbols) is given by:

$$\boldsymbol{U_K} \sum_{J=2,N+1;\, j\neq K} \boldsymbol{Y_{K,J}} - \sum_{J=1,N} \boldsymbol{Y_{K,J}} \cdot \boldsymbol{U_J} = -\boldsymbol{I_{L,K}}$$
$$K=2,\ldots,N \qquad (2)$$

Eqns. (1) and (2) can be written in matrix form:

$$\begin{bmatrix} \boldsymbol{Y_{1,1}} & \ldots & \boldsymbol{Y_{1,N}} \\ \ldots & \ldots & \ldots \\ \boldsymbol{Y_{N,1}} & \ldots & \boldsymbol{Y_{N,N}} \end{bmatrix} \begin{bmatrix} \boldsymbol{U_1} \\ \ldots \\ \boldsymbol{U_N} \end{bmatrix} = -\begin{bmatrix} -\boldsymbol{U_{source}} \cdot \boldsymbol{Y_{0,1}} + \boldsymbol{I_{L,1}} \\ \ldots \\ \boldsymbol{I_{L,N}} \end{bmatrix} \qquad (3)$$

where the self-admittance coefficient $\boldsymbol{Y_{i,i}}$ is given, as it is known, by the summation of all the admittances connected to node ***i***, and the mutual admittance coefficient $\boldsymbol{Y_{j,k}}$ is given by the summation, with opposite sign, of all the admittances connecting nodes ***j*** and ***k***.

Eqn. (3) can be written more synthetically:

$$\boldsymbol{Y} \cdot \boldsymbol{U} = -\boldsymbol{I} \qquad (4)$$

being ***Y*** the *NxN* bus admittance matrix, ***U*** the array of bus voltages (unknowns) and ***I*** the array of the *N* known terms; this array is made of the upstream constant current loads except for the first term in which the source bus voltage[1] is considered. The

1 As an alternative, the array of the known terms ***I*** can be considered as the difference between a vector, having as the first term $\boldsymbol{Y_{0,1}} \cdot \boldsymbol{U_{source}}$ and the other terms zero values, and the array of loads currents $\boldsymbol{I_{L,J}}$ (*j=1,...;N*).

bus voltages can be immediately deduced by eqn. (4),

$$\boldsymbol{U} = -\boldsymbol{Y}^{-1} \cdot \boldsymbol{I} = -\boldsymbol{Z} \cdot \boldsymbol{I} \tag{5}$$

where $\boldsymbol{Z}$ is the bus impedance matrix.

3 Solution methodology for networks composed by impedances

Consider the network in Fig. 1, made of a cascade of N two-port networks with Γ structure, each having a series impedance $\boldsymbol{Z}_{ser,i}$ and a shunt impedance $\boldsymbol{Z}_{sh,i}$ ($i=1,2,\ldots,N$) and supplied by a voltage generator $\boldsymbol{U}_{source}$.

For load-flow calculations, this network can be assumed as the circuital model of a distribution line supplied at one end (with voltage $\boldsymbol{U}_{source}$) and with N loads modelled as constant impedance. Concentrating the shunt admittances of each line at the two ends and summing them up with the loads admittances, the impedances $\boldsymbol{Z}_{sh,i}$ represent the shunt elements associated at each node (line capacitance and load).

Hypothesizing that an arbitrary current $\boldsymbol{I}$ flows through the impedance $\boldsymbol{Z}_{sh,N}$:

$$\boldsymbol{I}_N = \boldsymbol{I} \tag{6}$$

all the branch currents, the load currents and the bus voltages can be expressed as:

$$\boldsymbol{I}_{b,i} = \boldsymbol{H}_{Ib,i} \cdot \boldsymbol{I} \tag{7}$$

$$\boldsymbol{I}_i = \boldsymbol{H}_{I,i} \cdot \boldsymbol{I} \tag{8}$$

$$\boldsymbol{U}_i = \boldsymbol{H}_{U,i} \cdot \boldsymbol{I} \tag{9}$$

For the linearity of the circuit model $\boldsymbol{H}_{Ib,i}$, $\boldsymbol{H}_{I,i}$ and $\boldsymbol{H}_{U,i}$ are *transfer functions* of the network depending only on the impedances $\boldsymbol{Z}_{ser,j}$ and $\boldsymbol{Z}_{sh,j}$. As an example, the voltage at the ends of the impedance $\boldsymbol{Z}_{sh,N-1}$ in Fig. 1 and the relevant current can be expressed as:

$$\boldsymbol{U}_{N-1} = (\boldsymbol{Z}_{sh,N} + \boldsymbol{Z}_{ser,N}) \cdot \boldsymbol{I} = \boldsymbol{H}_{U,N-1} \cdot \boldsymbol{I} \tag{10}$$

$$\boldsymbol{I}_{N-1} = \frac{\boldsymbol{U}_{N-1}}{\boldsymbol{Z}_{sh,N-1}} = \frac{\boldsymbol{Z}_{sh,N} + \boldsymbol{Z}_{ser,N}}{\boldsymbol{Z}_{sh,N-1}} \boldsymbol{I} = \boldsymbol{H}_{I,N-1} \cdot \boldsymbol{I} \tag{11}$$

Also the source node voltage can be expressed as a function of $\boldsymbol{I}$:

$$\boldsymbol{U}_o = \boldsymbol{H}_{U,o} \cdot \boldsymbol{I} \tag{12}$$

This voltage, calculated following eqn. (12), differs in general from the corrected value $\boldsymbol{U}_{source}$ due to the arbitrary choice of the current $\boldsymbol{I}$. Considering that the transfer function $\boldsymbol{H}_{U,o}$ does not depend on this choice, the voltage $\boldsymbol{U}_{source}$ can be expressed as:

$$\boldsymbol{U}_{source} = \boldsymbol{H}_{U,o} \cdot \boldsymbol{I}_{true} \tag{13}$$

Thus, by the ratio between eqns. (13) and (12) it is possible to determine a *correction factor* $\boldsymbol{K}$:

$$\boldsymbol{K} = \frac{\boldsymbol{U}_{source}}{\boldsymbol{V}_o} = \frac{\boldsymbol{I}_{true}}{\boldsymbol{I}} \tag{14}$$

This factor, multiplied by any value of voltage and current (already calculated starting from $\boldsymbol{I}$) allows the corrected values to be determined. In particular, for the bus voltages:

$$\boldsymbol{U}_{i,true} = \boldsymbol{H}_{U,i} \cdot \boldsymbol{I}_{true} = \boldsymbol{H}_{U,i} \cdot \boldsymbol{K} \cdot \boldsymbol{I} = \boldsymbol{K} \cdot \boldsymbol{U}_i \tag{15}$$

Consider now the network in Fig. 2 in which, in correspondence of the node D there is a branching point; the branch OD is made up of N two-port networks and the branches DT_1 and DT_2 with N_1 and N_2 two-port networks; the structure of the single two-port networks is identical to that of the two-port networks considered for the passive network in Fig. 1.

Injecting the current $\boldsymbol{I}_{N1}$ (arbitrary) in the last shunt impedance of the branch DT_1 and proceeding upstream following the steps indicated for the network in Fig. 1, the value of the voltage at node D, depending on the current $\boldsymbol{I}_{N1}$, can be determined:

$$\boldsymbol{U}_D = \boldsymbol{H}_{V,D} \cdot \boldsymbol{I}_{N1} \tag{16}$$

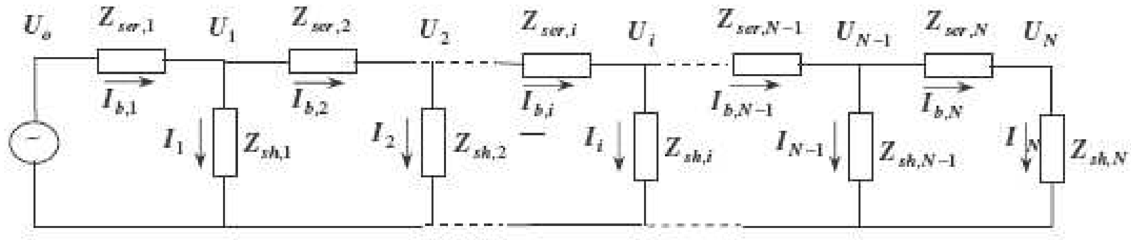

Fig. 1 - Cascade of Γ two-port networks made of series and shunt impedances.

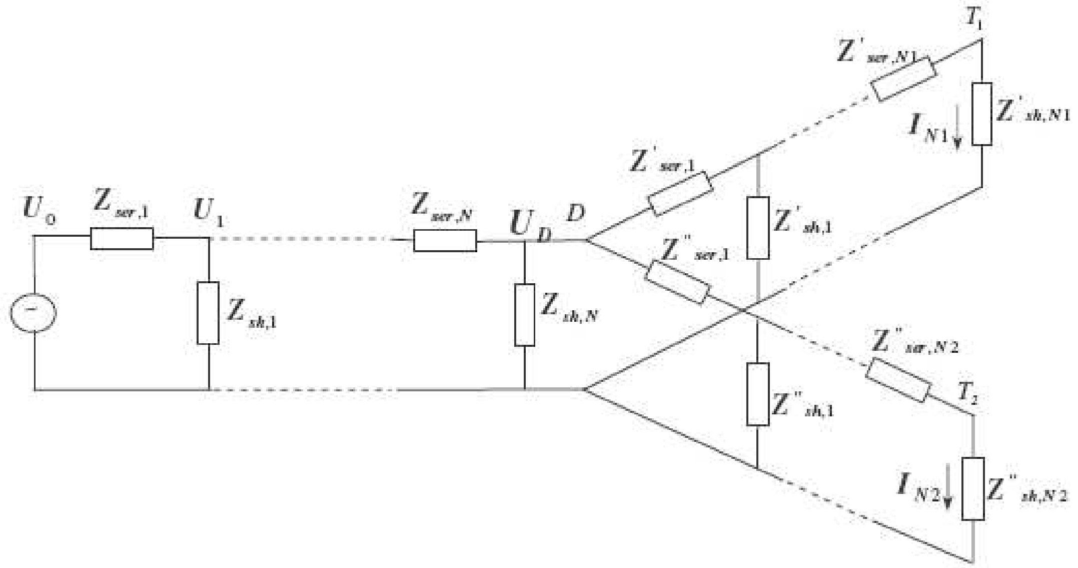

Fig. 2 - Cascade of Γ two-port networks with a branching section in node D.

In the same way, injecting an arbitrary current $\boldsymbol{I}_{N2}$ (normally different from $\boldsymbol{I}_{N1}$) in the last shunt impedance of the branch DT_2 and proceeding upstream in the same way as has just been done, the branch currents and the bus voltages in all the sections of the branch can be calculated.

For the branching node D a voltage value which is generally different from the one calculated before is found:

$$\boldsymbol{U}_D^{'} = \boldsymbol{H}_{V,D}^{'} \cdot \boldsymbol{I}_{N2} \neq \boldsymbol{U}_D \qquad (17)$$

In order to reach that the two values of voltage in node D are the same (as is physically required), then one of the two currents $\boldsymbol{I}_{N1}$ or $\boldsymbol{I}_{N2}$ must be modified.

Suppose, as an example, that we want to modify the current $\boldsymbol{I}_{N2}$. The modified value $\boldsymbol{I}_{N2,mod}$, which determines the equality of the voltage value $\boldsymbol{U}_D^{'}$ with that calculated starting from $\boldsymbol{I}_{N1}$ ($\boldsymbol{U}_D$), can be attained following the same criterion adopted, in the case of the network in figure 1, for the source node:

$$\boldsymbol{I}_{N2,mod} = \frac{\boldsymbol{U}_D}{\boldsymbol{U}_D^{'}} \boldsymbol{I}_{N2} = \boldsymbol{k}_D \cdot \boldsymbol{I}_{N2} \qquad (18)$$

From eqn. (18) it is possible to determine a *local correction factor* $\boldsymbol{k}_D$ that, multiplied by all the electrical quantities already calculated for branch DT_2, will allow the voltage and current values to be found in agreement with those imposed by the arbitrary choice of $\boldsymbol{I}_{N1}$ for the other lateral.

With the values corrected it is then possible to proceed with the balance of the currents at node *D* and evaluate the current circulating in the section upstream of the same node.

As an alternative, with the same criterion, the current $\boldsymbol{I}_{N1}$ can be modified:

$$\boldsymbol{I}_{N1,mod} = \frac{\boldsymbol{U}'_D}{\boldsymbol{U}_D}\boldsymbol{I}_{N1} = \boldsymbol{k}^{**}_D \cdot \boldsymbol{I}_{N1} \tag{19}$$

Multiplying by the *local correction factor* $\boldsymbol{k}^{**}_D$, all the electrical quantities of the branch *DT1* can be adjusted in agreement with the choice of $\boldsymbol{I}_{N2}$.

Once the correction has been executed, the process of calculating the various quantities upstream of the branching node shows no differences as compared to the first case studied. Indeed, for each of the sections of the branch *OD*, the sending bus voltage, the shunt admittance current and the upstream branch current can be calculated. In particular, for the section connected to the source node, the source node voltage is calculated and this is different from the imposed value.

On the basis of the calculated and the imposed values, a *global correction factor* $\boldsymbol{K}$ is deduced.

With this factor the values of voltages and currents can be modified at all buses and in all the branches of the network (branches *OD*, DT_1 and DT_2). The values corrected in this way are the solution of the passive network.

4 Methodology set up for the solution of networks with *ZI* loads

Consider the network in Fig. 3 attained starting from the network in Fig. 1 in which, at each node, a constant current load is injected. Following the same solution methodology set up in the preceding paragraph and under the hypothesis that in the impedance $\boldsymbol{Z}_{sh,N}$ an arbitrary current $\boldsymbol{I}$ flows: $\boldsymbol{I}_N = \boldsymbol{I}$, the current that flows in the impedance $\boldsymbol{Z}_{ser,N}$ is given by:

$$\boldsymbol{I}_{b,N} = \boldsymbol{I}_{L,N} + \boldsymbol{I} \tag{20}$$

where $\boldsymbol{I}_{L,N}$ is the constant load current derived at node *N*. The voltage at node *N-1* is given by:

$$\boldsymbol{U}_{N-1} = \left(\boldsymbol{Z}_{sh,N} + \boldsymbol{Z}_{ser,N}\right)\cdot \boldsymbol{I} + \boldsymbol{Z}_{ser,N}\cdot \boldsymbol{I}_{L,N} = \\ = \boldsymbol{H}_{U,N-1}\cdot \boldsymbol{I} + \boldsymbol{H}^{N}_{U,N-1}\cdot \boldsymbol{I}_{L,N} \tag{21}$$

The current derived from the impedance $\boldsymbol{Z}_{sh,N-1}$ is given by:

$$\boldsymbol{I}_{N-1} = \frac{\boldsymbol{U}_{N-1}}{\boldsymbol{Z}_{sh,N-1}} = \frac{\boldsymbol{Z}_{sh,N} + \boldsymbol{Z}_{ser,N}}{\boldsymbol{Z}_{sh,N-1}}\boldsymbol{I} + \frac{\boldsymbol{Z}_{ser,N}}{\boldsymbol{Z}_{sh,N-1}}\boldsymbol{I}_{L,N} = \\ = \boldsymbol{H}_{I,N-1}\cdot \boldsymbol{I} + \boldsymbol{H}^{N}_{I,N-1}\cdot \boldsymbol{I}_{L,N} \tag{22}$$

The current in the branch *N-1* is given by:

$$\boldsymbol{I}_{b,N-1} = \boldsymbol{I}_{L,N-1} + \boldsymbol{I}_{L,N} + \boldsymbol{I}_{N-1} + \boldsymbol{I} = \\ = \boldsymbol{H}_{Ib,N-1}\cdot \boldsymbol{I} + \boldsymbol{H}^{N}_{Ib,N-1}\cdot \boldsymbol{I}_{L,N} + \boldsymbol{H}^{N-1}_{Ib,N-1}\cdot \boldsymbol{I}_{L,N-1} \tag{23}$$

In general, the current flowing in the *i-th* branch and the voltage at the *i-th* bus can be expressed as:

$$\boldsymbol{I}_{b,i} = \boldsymbol{H}_{Ib,i}\cdot \boldsymbol{I} + \sum_{J=i,N}\boldsymbol{H}^{J}_{Ib,i}\cdot \boldsymbol{I}_{L,J} \tag{24}$$

$$\boldsymbol{U}_{i} = \boldsymbol{H}_{U,i}\cdot \boldsymbol{I} + \sum_{J=i+1,N}\boldsymbol{H}^{J}_{U,i}\cdot \boldsymbol{I}_{L,J} \tag{25}$$

Owing to the linearity of the model $\boldsymbol{H}_{U,i}$, $\boldsymbol{H}_{Ib,i}$, $\boldsymbol{H}^{J}_{U,i}$ and $\boldsymbol{H}^{J}_{Ib,i}$ are *transfer functions* of the network that depend only on the impedances $\boldsymbol{Z}_{ser,j}$ and $\boldsymbol{Z}_{sh,j}$; in particular, the first two are those defined in the case of a network made only with impedances (eqns. 7 and 9).

Therefore, for the considered case, the bus voltages and the branch currents can be attained as the summation of quantities with the same name, but evaluated separately on the network of Fig. 1 and on the network of Fig. 4.

$$\boldsymbol{U}_{i} = \boldsymbol{U}^{net\ Z}_{i} + \boldsymbol{U}^{net\ ZI}_{i} \tag{26}$$

$$\boldsymbol{I}_{b,i} = \boldsymbol{I}^{net\ Z}_{b,i} + \boldsymbol{I}^{net\ ZI}_{b,i} \tag{27}$$

The first network (*net Z*) is composed of the source node and of the shunt and series admittances, the second (*net ZI*) is composed only of the shunt and series admittances and of the currents due to the constant current loads.

It must be noted that the component $\boldsymbol{U}^{net\ ZI}_{EB_i}$ of the voltage at the ending buses, EB_i, of the terminal branches is null; indeed, for the *ZI* network it turns out that $\boldsymbol{I} = 0$ and this implies, from eqn. (21), that, in the sending bus SB_i of the terminal branches, the component $\boldsymbol{U}^{net\ ZI}_{SB_i}$ equals the voltage drop on the impedance $\boldsymbol{Z}_{ser,N}$. Therefore, in the schematization of figure 4, concerning the *ZI* network, the voltage at the ends of the current generator $I_{L,N}$ must be considered null.

The voltage at the source node calculated on the basis of eqn. (26) does not equate to the imposed value, U_{source}, due to the arbitrary choice made on current I;

$$U_0 = H_{U,0} \cdot I + \sum_{J=1,N} H_{U,0}^{J} \cdot I_{L,J} \qquad (28)$$

Considering that transfer functions $H_{U,o}$ and $H_{U,0}^{J}$ ($J=1,N$) do not depend on this choice, voltage U_{source} can be expressed as:

$$U_{source} = H_{U,o} \cdot I_{true} + \sum_{J=1,N} H_{U,0}^{J} \cdot I_{L,J} \qquad (29)$$

Therefore, by means of eqns. (28) and (29) it is possible to determine a *correction factor* K:

$$K = \frac{U_{source} - \sum_{J=1,N} H_{U,0}^{J} \cdot I_{L,J}}{U_o - \sum_{J=1,N} H_{U,0}^{J} \cdot I_{L,J}} = \frac{I_{true}}{I} \qquad (30)$$

that must be applied only to the components $U_i^{net\,Z}$ and $I_{b,i}^{net\,Z}$ to attain, through eqns. (26) and (27), the exact values of bus voltages and branch currents.

For radial systems with branching nodes, the way to proceed is identical to the one described in the preceding paragraph with the sole difference that local correction factors have a similar expression as in eqn. (30).

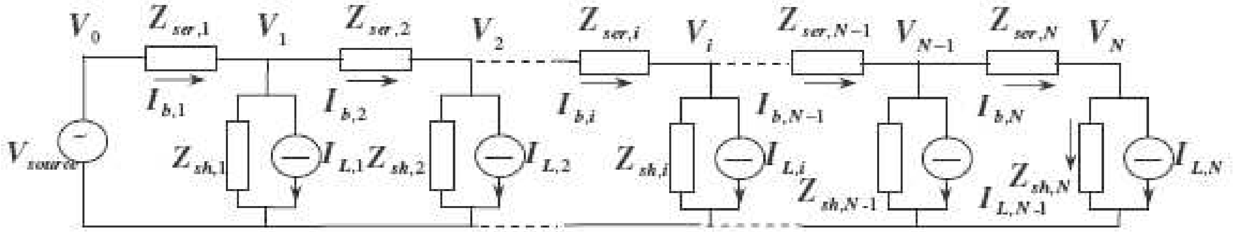

Fig. 3 - Circuit scheme of a feeder with *N* constant impedance/current loads.

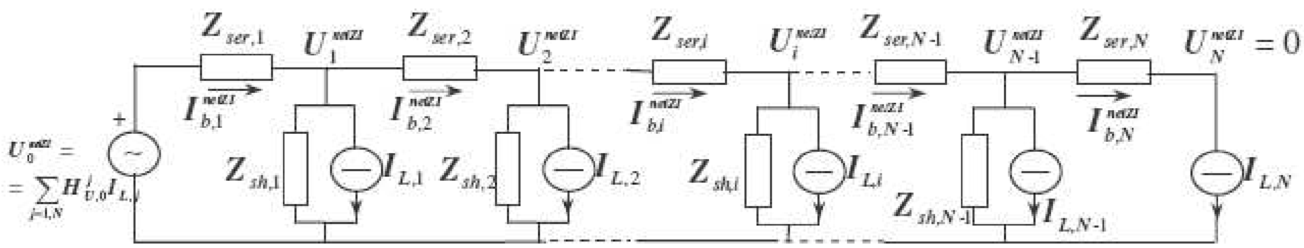

Fig. 4 - *Net ZI* for the calculation of the components $U_i^{net\,ZI}$ and $I_{b,i}^{net\,ZI}$ of the bus voltages and of the branch currents.

5 Implementation of the new methodology

The main solution steps are as follows:

1) arbitrary value attribution for the current on the shunt impedances of the ending nodes of the network;
2) calculation of the component $\boldsymbol{U}^{net\,Z}$ of the voltage in the ending nodes (the component $\boldsymbol{U}_i^{net\,ZI}$ is null in these nodes);
3) starting from the terminal branches, calculation of the two components $\boldsymbol{I}_{b,i}^{net\,Z}$ and $\boldsymbol{I}_{b,i}^{net\,ZI}$ of the current circulating in the generic *i-th* branch;
4) calculation of the two components $\boldsymbol{U}_i^{net\,Z}$ and $\boldsymbol{U}_i^{net\,ZI}$ of the sending bus voltage of the *i-th* branch;
5) calculation of the shunt impedance current at the sending bus of the *i-th* branch;
6) if the starting bus of the *i-th* branch is a branching node, calculation of the local correction factor and modification of bus voltages and of branch currents downstream;
7) if the starting bus of the *i-th* branch is the source node, calculation of the global correction factor and modification of the bus voltages and of the branch currents in the whole network.

6 Applications

The main aim of the applications is to evaluate the calculation times' requirement of the proposed radial networks solution methodology.

The networks considered have constant impedance and constant current loads as well as capacitor banks.

Comparison has been carried out by solving some networks with the three above-described methods:

– the iterative backward/forward method (b/f),
– the method based on the use of the impedance bus matrix (Z) and
– the method proposed here (A).

The relevant algorithms have been implemented in FORTRAN 90 language and the programs run on a mainframe IBM S/390-2003/225; for the inversion of the bus impedance matrix, a subroutine pertaining to the IMSL library has been used. In this manner, the comparisons between the CPU times are meaningful.

For the b/f method, the convergence factor has been fixed so that the values of bus voltages, expressed in p.u., found at the end of the iterative process are equal, up to the fifth decimal digit, to those found with the two direct methods (for the cases examined the factor has been fixed to 10^{-5}).

In Table 1, for the three methods, the number of nodes of the analysed networks and the CPU time to attain the solution are reported; for the iterative method (b/f) the number of iterations is also reported.

From the analysis of the table, it appears that the CPU time required by the method based on the bus impedance matrix is quite high; as figure 5 shows, such consumption grows almost exponentially with the system's size. The CPU time of the iterative method shows a lesser dependency on the number of nodes and is quite low as compared to the Z method (of about one order of magnitude for the 85 bus system).

The method developed here is the fastest, with a reduction of the calculation times, as compared to the b/f method, for the largest network, of about 65 %. In this case, the correlation between CPU times and network size is less evident, since the network configuration and, in particular, the number of branching nodes, influences the number of calculations to be executed.

Indeed, for each branching node a local correction factor must be evaluated and it must be applied to the branch currents and to the bus voltages of the part of the system downstream of the branching node; therefore, for two systems with the same number of nodes, the network showing the larger number of branching nodes requires a larger CPU time.

However, as compared to the other solution methods considered, for the same network, the method developed here is the fastest.

Table 1 - CPU times in the solution of the some networks.

	b/f		**Z**	**A**
Bus number	*Iterations*	*CPU [s]*	*CPU [s]*	*CPU [s]*
12	4	0.00237	0.00652	0.00019
15	4	0.00256	0.00870	0.00031
23	5	0.00373	0.01329	0.00044
28	6	0.00427	0.02113	0.00062
33	4	0.00362	0.02734	0.00069
69	4	0.00557	0.10445	0.00168
85	5	0.00785	0.14571	0.00275

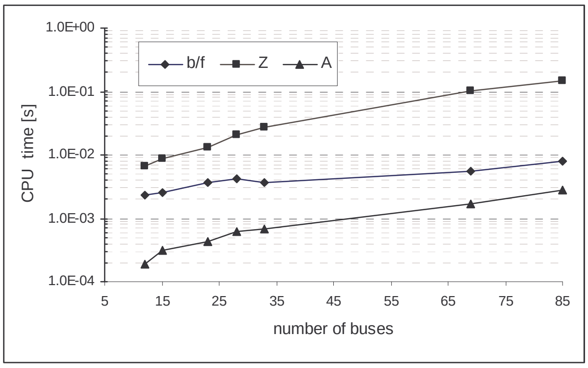

Fig. 5 - CPU time (logarithmic scale) *vs.* node number for the three analysis methods.

7 Conclusions

The solution of radial distribution systems supplying constant current/impedance loads with capacitor banks and with not negligible shunt capacitances can be carried out both with a direct method and with an iterative method.

The main drawback of the direct method, based on the bus impedance matrix, resides in the high consumption of CPU time; the iterative method, although allowing large savings of CPU time, is not completely satisfactory as it solves, in several different attempts, a problem that has a direct solution. In this paper, a methodology with the merits of both procedures is proposed; it solves directly the load flow problem with limited calculation times.

The applications carried out show that the CPU time-saving using the proposed method is large as compared to both methods.

The proposed methodology is not exclusively dedicated to the analysis of the above-mentioned typology of electrical systems (radial structure, constant impedance/current loads, not negligible line capacities, presence of shunt capacitor banks); indeed, the method can also be applied in the solution, with the backward/forward methodology, of meshed systems having voltage-dependent loads and PV nodes; in this case, at each iteration, the simulation of loads and of known capacitor banks at PV nodes with impedances and of meshes and of unknown quantities at PV nodes with breakpoint currents allows the methodology to be used both to solve the radialized system and correct the bus voltage values.

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