Chaos-low periodic orbits transition in a synchronous switched circuit

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Abstract: In this paper we consider a modified model of an Alpazur oscillator with a periodically switching operation. Complex oscillatory phenomena are observed due to both switching operation and the appearance of an additional nonlinear component in circuit model. Indeed, subharmonic oscillations and chaotic behavior are illustrated. In addition, time series, phase trajectories and power spectrum was examined in the analysis of the system dynamics. The transition to chaotic states through doubling period bifurcation cascade and the direct transition from fundamental harmonic or odd subharmonic orbits and transition from a special bifurcation paths to chaotic states are discussed in the state space. The role of higher harmonic spectral lines in period-1 responses under parameter variation have seemingly similarities with the results obtained in previous works for non switched dynamical system. Chaotic states resulting from different transition modes are also presented in this paper. For the numerical simulations, techniques for deriving time and composite Poincaré method are applied.

Key–Words: Switching operation,Transition modes, higher harmonic predominance, doubling period cascade.

1 Introduction

The electrical circuits including switching actions are discussed as a recent interesting subject of search in much of previous works [1],[10],[12],[13],[16],[21]. Much of such circuits are encountered in Power Electronics field which is a discipline spawned by real life applications in industrial, commercial, residential and aerospace environments [2],[3],[14],[19]. Several chaotic behaviors in nonlinear dynamic systems were investigated and classified in many former studies. In a ferroresonant circuit, seemingly random oscillations were observed in steady state, the impact of source voltage and initial conditions on the so called chaotic ferroresonance was highlighted in [17]. There are two types of chaotic systems, autonomous and non autonomous [18]. In the last case the systems are submitted to an external time varying source. A well known example studied in [6] dealt with the Duffing type equation describing an electrical circuit, the amplitude and the frequency of the sinusoidal signal both contribute to the chaotic dynamics of such circuit. The Duffing-Van der Pol oscillator of [4] shows a broad spectrum of dynamic behaviors, both chaotic as well as periodic.

Studies of routes to chaos exhibit several configurations of bifurcation paths leading to chaotic orbits.The period doubling succession plays an important role in the occurrence of chaotic attractors, it generally constitutes a veritable route to chaos. Other bifurcation paths leading to chaotic states are available in literature. Among theses cases one can itemize the transition from period 3 orbits to chaos wherein the oscillator's responses enter abruptly chaotic regions under a parameter variation. In certain cases, pursuing to vary the same parameter, the system dynamics exit chaotic regions and turn again to period 3 orbit. In fundamental harmonic regime taken as the standard situation, the system responses are synchronous with period one orbit. Nevertheless, the nature, the period and the stability of the responses can undergo an unexpected change under a parameter variation. Theses sudden changes are called bifurcation phenomena. In non linear autonomous systems some bifurcation scenarios involve transition to chaotic states via stable equilibrium, limit cycles and quasi-periodic orbits [20]. Traditional bifurcation appear when one of the eigenvalues reaches critical values (eigenvalues of the Jacobian Matrix) leading to a change of stability or order or nature. Some analysis tools issued from bifurcation theory will be applied to a two dimensional switching dynamical circuit.

Circuits with one or more switches, also called on-off circuits, are generally described by dynamical differential equations switched in a certain manner typically synchronous or asynchronous modes. In synchronous mode , the switching is done by a periodic external independent state excitation [5],[8]. Whereas in asynchronous mode, toggling is controlled by a depending state excitation [9],[10]. Thus a switching circuit can be described as a piecewise switched circuit which assumes different topologies at different times. It is worth noting that in recent few years , it has been gradually recognized that the switched dynamical systems exhibit many interesting phenomena such as periodic, quasi-periodic, subharmonic and chaotic behaviors, border collision bifurcation and so forth [15]. As an illustrated example, we investigated the behavior of a modified Alpazur oscillator which includes two nonlinearities: the nonlinear conductor having a cubic current-voltage characteristic and a nonlinear inductor represented similarly by a third order function. The operational mode of the circuit is changed by a switch turned on and off periodically resulting in a sequence of nonlinear circuits being toggled in a supposedly orderly manner. Thus, the considered switched circuit involves two differential equations one in each region so the corresponding map is continuous across and its derivatives become discontinuous. Applying the general methodology, the Poincaré mapping was taken as a composite discrete mapping. Three main transition modes will be outlined in this paper such as doubling period bifurcations succession, direct transition from fundamental harmonic or odd subharmonic responses to chaos and a special transition through period-n orbits succession, $n = 4, 5, 6, 7..$.

In what follows we will outline the plethora of different dynamical behaviors exhibited by the considered oscillator. Section 2 is mainly intended to describe the modified Alpazur oscillator including an external force depending switch. Section 3 is devoted to present three different ways leading to chaotic behaviors. In section 4, we have attempted to characterize intermittent phenomenon by subharmonic predominance via spectral analysis accordingly to former studies [6]. Section 5 is reserved to present chaotic regions resulting from two different transition modes.

2 System description and general reminders

2.1 Modified Alpazur oscillator

Firstly, we should note that the modification done on the classical Alpazur oscillator discussed in [8] consists in replacing the linear inductor by a nonlinear one so that the circuit includes two nonlinear elements the conductor G and the inductor L. Such circuit is a combination of a Rayleigh-type oscillator unit and two supplied dc power interchanged by a switch S_p which operates at switching edges P_1 and P_2 as shown in Figure 1. For $0 < t \leq \tau_{p_1}; \tau_{p_1} = \rho.T$, S_p is

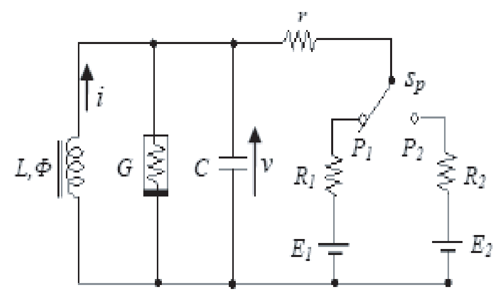

Figure 1: Modified Alpazur Oscillator circuit

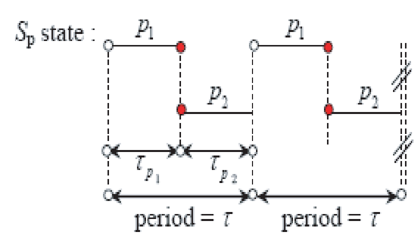

Figure 2: A chronogram of the switch excitation

thrown into contact with position P_1 so the circuit dynamics is governed by two dimensional autonomous system:

$$\begin{cases} \frac{d\phi}{dt} = -v \\ C\frac{dv}{dt} = i - G(v) + \frac{E_1 - v}{r + R_1} \end{cases}$$

$$0 < t \leq \rho.T \tag{1}$$

Then it is switched to position P_2 for so the governing equations are: $\begin{cases} \frac{d\phi}{dt} = -v \\ C\frac{dv}{dt} = i - G(v) + \frac{E_2 - v}{r + R_2} \end{cases}$

$$\rho.T < t \leq T \tag{2}$$

Where the capacitor voltage $v(t)$ and the magnetic flux $\phi(t)$ in the inductor are state variables of the system. The nonlinear current-voltage characteristic of the conductor G is a smooth cubic function $G(v) = -v + \frac{1}{3}v^3$ and the nonlinear current-flux characteristic of the inductor L is supposed to have the form $i = \alpha\phi + \beta\phi^3, \alpha > 0, \beta > 0$. After normalization of the state variables and the parameters, the differential systems (1)and (2) are respectively rewritten as:

$$\begin{cases} \frac{dx_1}{dt} = -x_2 \\ \frac{dx_2}{dt} = \alpha x_1 + \beta x_1^3 + (1 - \varepsilon_1)x_2 - \frac{1}{3}x_2^3 + H_1 \end{cases} \quad 0 < t \leq \rho.T \tag{3}$$

$$\begin{cases} \frac{dx_1}{dt} = -x_2 \\ \frac{dx_2}{dt} = \alpha x_1 + \beta x_1^3 + (1 - \varepsilon_2)x_2 - \frac{1}{3}x_2^3 + H_2 \end{cases} \quad \rho.T < t \leq T \tag{4}$$

where $\varepsilon_1 = \frac{1}{r+R_1}$, $\varepsilon_2 = \frac{1}{r+R_2}$, $\mathrm{H}_1 = \varepsilon_1 E_1$, $\mathrm{H}_2 = \varepsilon_2 E_2$, $C = 1.0$, $\tau_{p_1} = \rho.T$ and $\tau_{p_2} = (1-\rho).T$ (i.e. $\tau_{p_1} + \tau_{p_2} = T$), and by considering the following transformation of the state variables $\phi = x_1$ and $v = x_2$, The switching sequence is given in Figure 2.

2.2 Some properties of Poincaré map

The differential systems (3) and (4) can be written in a compact form:

$$\begin{aligned} \frac{dx}{dt} &= f_{p_1}(x, \lambda, \lambda_1), \quad 0 < t \leq \rho.T \\ \frac{dx}{dt} &= f_{p_2}(x, \lambda, \lambda_2), \quad \rho.T < t \leq T \end{aligned} \tag{5}$$

The general solution of such kind of differential equation can be expressed as :

$$x(t) = \varphi(t, x_0, \lambda, \lambda_1, \lambda_2) \tag{6}$$

Where $t \in IR$ is the time, $x_0 \in IR^2$ is the state variable, $(\lambda_c, \lambda_1, \lambda_2) \in IR^3$. λ_c is a common parameter for the functions f_{p_1} and f_{p_2}, whereas λ_1 and λ_2 are specific parameters for each of them respectively. These functions are assumed to be smooth and differentiable as many times as necessary in regard to the variables and parameters. Referring to the illustrative trajectory example shown in Figure 3, and taking into account the border continuity conditions, it is obvious to see that the switching system possesses a composite solution, we then define the following two maps:

$$\begin{aligned} M_{p_1} &: IR^2 \to IR^2 \\ x_0 &\mapsto x_1 = \varphi_{p_1}(\rho.T, x_0, \lambda_c, \lambda_1) \end{aligned} \tag{7}$$

$$\begin{aligned} M_{p_2} &: IR^2 \to IR^2 \\ x_1 &\mapsto x_2 = \varphi_{p_2}((1-\rho).T, x_1, \lambda_c, \lambda_2) \end{aligned} \tag{8}$$

Where
$\varphi_{p_1}(\rho.T, x_0, \lambda_c, \lambda_1) = \varphi(\rho.T, x_0, \lambda_c, \lambda_1, \lambda_2)$
$\varphi_{p_2}((1-\rho).T, x_0, \lambda_c, \lambda_2) = \varphi((1-\rho).T, x_0, \lambda_c, \lambda_1, \lambda_2)$
Poincaré section is given by choosing transversal plan defined $t = k.T$, the resulting composite mapping M_c expressed as:

$$\begin{aligned} M_c &: IR^2 \to IR^2 \\ x_0 &\mapsto M_c(x_0) = M_{p_2} \circ M_{p_1}(x_0) \end{aligned} \tag{9}$$

For fixed points investigation we merely solve $M_c(x_0) = x_0$. Additional condition about the derivative is required for solving such equation:

$$\left.\frac{\partial M_c}{\partial x_0}\right|_{t=T} = \left.\frac{\partial M_{p_2}}{\partial x_1}\right|_{t=(1-\rho).T} . \left.\frac{\partial M_{p_1}}{\partial x_0}\right|_{t=\rho.T} \tag{10}$$

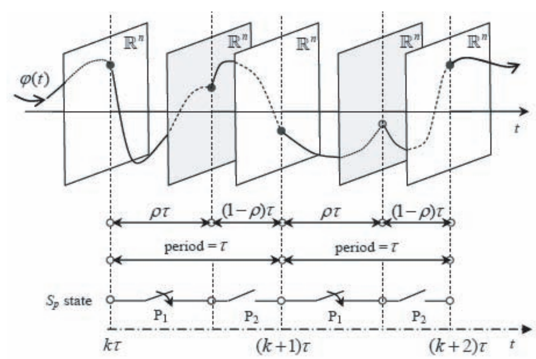

Figure 3: Composite trajectory

2.3 Basic methods of analysis

In this section, we will briefly introduce the method previously discussed in [12] to investigate the governing equations of system under study, which typically consist of two nonlinear autonomous systems. More generally, we consider the following n-dimensional differential equations: The solution of these equations can be expressed as: For $0 < t \leq \rho.T$, S_p is at p_1:

$$x(t) = \varphi_{p_1}(t,\, x_0,\, \lambda_c,\, \lambda_1) \tag{11}$$

For $\rho.T < t \leq T$, S_p is at p_2:

$$x(t) = \varphi_{p_2}(t - \rho.T,\, \varphi_{p_1}(t,\, x_0,\, \lambda_c,\, \lambda_1),\, \lambda_c,\, \lambda_2) \tag{12}$$

based on the equations, we can easily define the following two maps: In practice the Poincaré section can be generated by choosing the transversal plan defined at $t = \rho.T$. Thus, By using the property of the switching actions, Poincaré mapping was constructed as a composite discrete mapping M_c of M_{p_1} and M_{p_2}:

$$\begin{aligned} M_c : IR^n &\rightarrow IR^n \\ x_0 &\mapsto M_c(x_0) = M_{p_2} \circ M_{p_1}(x_0) \end{aligned} \tag{13}$$

It is obvious to verify that a fixed point x_0 in the chosen surface section, is a point of the map M_c, where $M_c(x_0) = x_0$. The knowledge of the singularities of M_c enables to study the dynamical behaviors of the initial system. Indeed a a fixed point of M_c is associated to a periodical solution having the period of the fundamental harmonic (or a period-1 orbit) whereas a k-order cycle of M_c is associated to a periodical solution having the period of a k-order subharmonic (or a period-k orbit).

To numerically solve the equation mentioned above we can apply the Newton's method. In fact, we must calculate the derivative term presented in this method witch is defined as:

We note that the eigenvalues of the Jacobian matrix $\frac{\partial M_c}{\partial x_0}$ at a fixed point are called the characteristic multipliers, and they will be noted here by $(s_i)_{i=1,\,2,\,\ldots\,n}$, which are the roots of the characteristic equation defined as:

$$\chi(\mu) = \left| \frac{\partial M_c}{\partial x_0} - sI_n \right| = 0 \tag{14}$$

Generally, there are three critical points associated to the value of (s_i), called bifurcation points: Fold and Pitchfork bifurcation ($s = 1$), Flip or period-doubling bifurcation ($s = -1$) and Neimark bifurcation (complex conjugate multipliers with modulus 1). Consider a period-1 solution of (4) and (5), its Fourier series expansion and the corresponding frequency spectrum (made of of lines). Let r be place occupied by an order p higher harmonic spectral line from an ordering based on the amplitudes of spectral lines in descending order [6].

A given order p higher harmonic is said predominant if it occupies the second place $(r = 2)$ in the above classification. It is said fully predominant if it occupies the first place $(r = 1)$.

Choosing an arbitrary parameter plane, it can be considered as made up of sheets (foliated representation), each one being associated with a well defined response. For two dimensional nonlinear dynamical system, every period k orbit (associated with order k cycle) possesses two multipliers s_1 and s_2 determined from the Jacobian matrix, so a fold bifurcation curve is the junction of two sheets of same period, one is related to a saddle fixed point $(s_1 < 1, 0 < s_2 < 1)$ the other to a fixed point with $|s_i| < 1, i = 1, 2$, (stable node, or stable focus). A flip bifurcation curve is the junction of three sheets. one is associated to a stable period k cycle with $|s_i| < 1, i = 1, 2$, another with a saddle type period k cycle with $s_1 < -1, |s_2| < 1$, the third sheet is associated to a period 2k cycle having $|s_i| < 1, i = 1, 2$ '(stable node, or stable focus). A pitchfork bifurcation curve is the junction of four sheets. Three are related to node cycles having the same order and the fourth is linked to a saddle cycle having the same order too.

3 Transition modes to chaotic orbits

The main property of a chaotic response is that it is not asymptotically stable and closely correlated initial conditions have trajectories which quickly become unrelated. Among the characteristics of chaos, that can be quantified we distinguish apparent randomness in the time variation, the broad band components to the power spectrum and sensitive dependence on initial conditions.

The Lyapunov characteristic exponent gives the rate of exponential divergence from perturbed initial conditions, The dominant Lyapunov exponent is one of the most widely used indicators to describe the qualitative behavior in a dynamical system . A system of order n acquire n Lyapunov exponents. In a chaotic behavior, the Lyapunov exponents can measure how two different trajectories starting from different initial conditions converge to or diverge from each other.

Under a parameter change, transition to chaotic states can be gradually or abruptly as it will be seen in next sections.

Varying system parameters, one can observe three different cases leading to chaotic orbits, the classical doubling period bifurcation, direct transition to chaotic states from fundamental harmonic or odd subharmonic solutions and a special case raising the succession of subharmonic orbits of orders 4, 5,6 and 7 ended by chaotic orbit.

Many interesting features investigated in phase plan known as strange attractors are shown in illustrative figures below.

3.1 Transition to chaos via doubling period succession

The period doubling bifurcation plays an important role in the occurrence of chaotic attractors. In a Duffing type equation [6], the bifurcation structure shows flip bifurcation closed curves surrounding chaotic regions. Thus when a system parameter is varied, the responses gradually enter into chaotic regions then its exit these domains in order to turn to lower order cycles. This sequence is repeatedly reproduced as many times as the flip bifurcation curves number is. Aiming to maintain an acceptable stable operating point one has to investigate in the system parameters in order to narrow the chaotic zone or make it entirely disappear. It is worth noting, that coming close to the chaotic regions, the flip bifurcations become increasingly closer to each other. Crossing a small range of T values, one can meet four different solutions schemed in phase plane as shown in Figure 4, it is obvious to remark that when T increases from $T = 0.98$ to $T = 10.016$ the system responses undergo a doubling period cascade of bifurcations. The red points plotted in all phase portraits belong to the discrete trajectory generated by the composite Poincaré map.

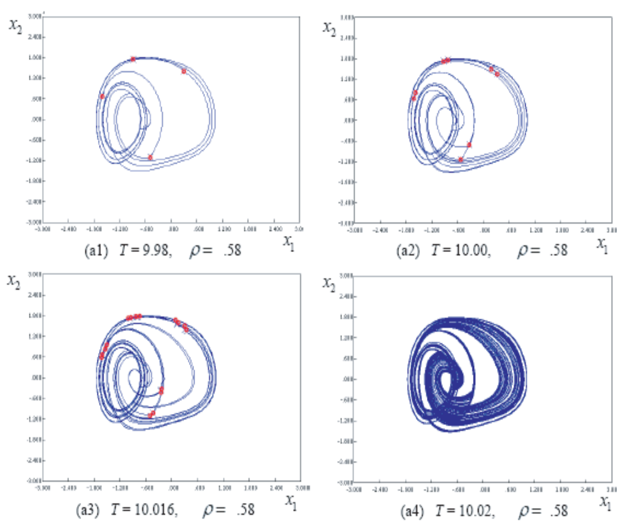

Figure 4: phase portraits for different values of T and $\rho = 0.58$,(a1) period-4 orbit, (a2) period-8 orbit, (a3) period-16 orbit, (a4) chaotic state

Figure 5 exhibits a doubling period sequence of a period-3 orbit under variation of T. chaotic state is obtained here for decreasing values of T from 5.2620 to 5.2224. In a doubling period cascade of bifurcation, the bifurcation points become closer and closer to each other, so that at a certain order n it is difficult to distinguish period-2^n orbits.

In Figure 6, we plot phase portraits of the state

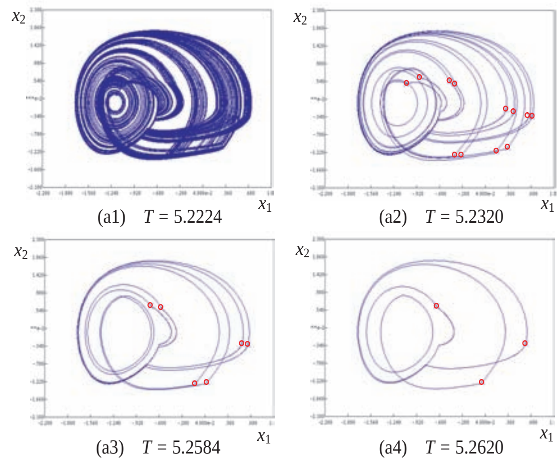

Figure 5: phase portraits for different values of T and $\rho = 0.58$,(a1) chaotic orbit, (a2) period-12 orbit, (a3) period-6 orbit, (a4) period-3 orbit

variables (x_2, x_1) for different values of the parameter T in order to give an illustrative example of intermittent chaotic states resulting from doubling period cascades.It is shown that the system intermittently bifurcates from an initial regular chaotic state for $T = 5.0604$ to lower subharmonic orbits for increasing values of T and then turns again to another chaotic state for $T = 5.1926$ through the same bifurcation type but in the reverse path.

3.2 Direct transition from harmonic or subharmonic orbits to chaos

From [11] it was stated that it is sufficient for one dimensional case to have a period 3 orbit to deduce the existence of a chaotic behavior. But later, it was proved that actually many other bifurcations types can lead to chaotic orbits such as the period doubling bifurcation succession of either a fixed point giving rise to 2,4,8,16,...-order cycles or of k-order cycle leading to 2k,4k,8k,16k,...order cycles. From Figure 7, the graph clearly shows the abrupt transition from subharmonic response of order 3 to chaotic state, under a parameter variation from $\rho = 0.898$ to $\rho = 0.90$. Another remarkable observation is that the direct transition happens from odd subharmonic orbit to a chaotic state, this can be illustrated by a second example of transition from period-5 orbit to chaos for a parameter change from $\rho = 0.9309$ $\rho = 0.9310$ see Figure 9. In Figure 11, it is shown the transition from fundamental harmonic orbit to chaotic state by varying H_2 from 0.8353 to 0.836.

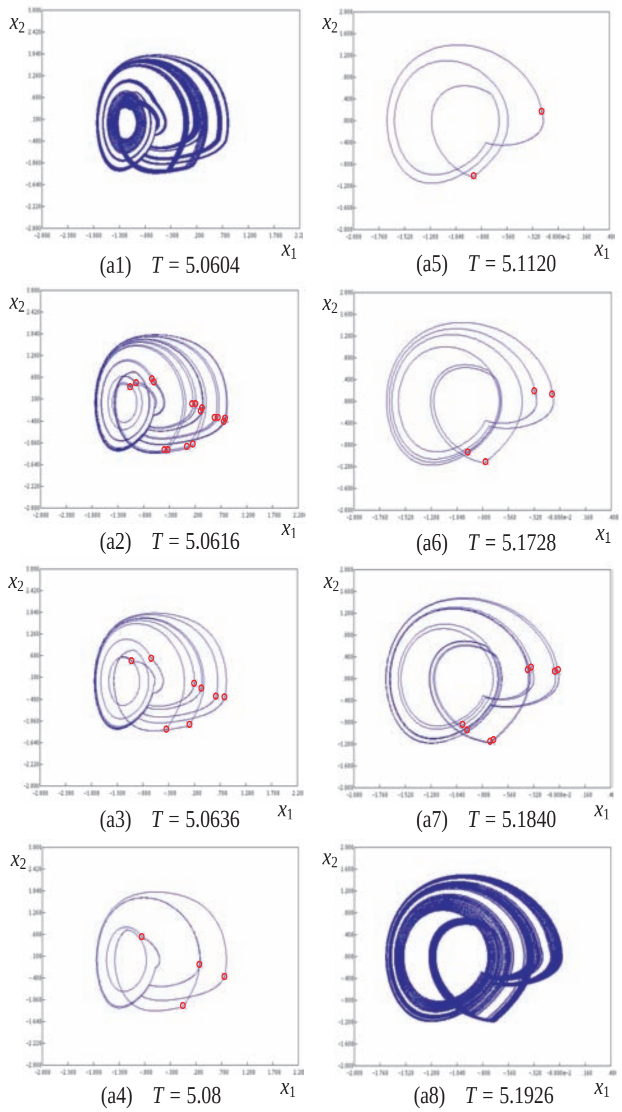

Figure 6: intermittency of chaotic states

Figures 8 and 10 show the direct transition from period-3 orbits to chaos for relatively small and large values of parameter T. Note that the obtained results are derived from observed examples. Hence the statement that there is a direct transition from an odd subharmonic to chaotic state is a subject for further investigations and development.

3.3 Transition to chaos via a special period-n orbit succession

Varying the parameter ε_2 over a small range of magnitudes, our investigation indicates that the traces in Figure 12 change following a certain order of merging of period-n, $n = 4, 5, 6, 7$ orbits ended by a chaotic state

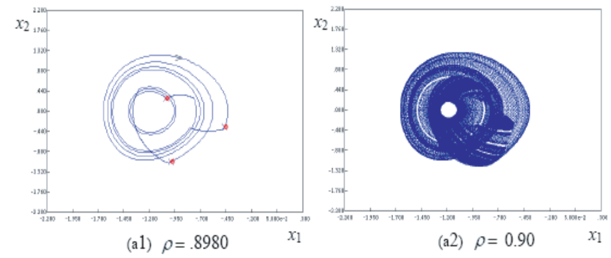

Figure 7: (a) period-3 orbit, (b) chaotic orbit

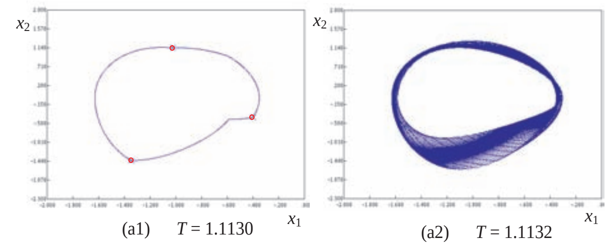

Figure 8: (a) period-3 orbit, (b) chaotic orbit

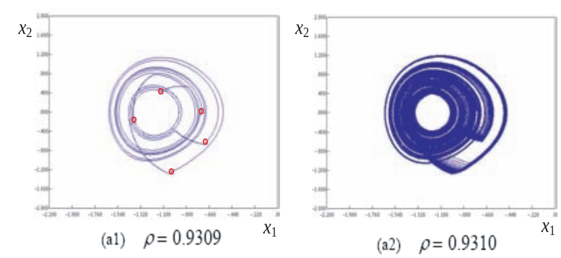

Figure 9: (a) period-5 orbit, (b) chaotic orbit

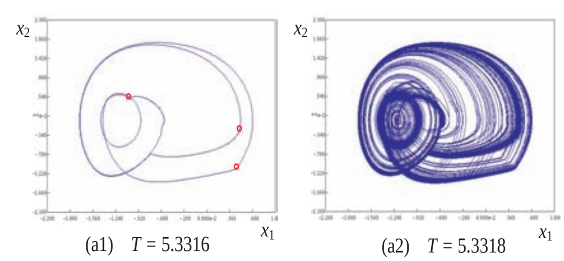

Figure 10: (a) period-3 orbit, (b) chaotic orbit

for $\varepsilon_2 = 0.4342$. To our knowledge this succession of appearance of subharmonics with 'incremental' order leading to a chaotic behavior, different from doubling period cascade and from direct transition from odd period-n orbit to chaotic orbit, is a special interesting case. Thus further investigation is necessary to characterize this behavior in dynamical switching systems.

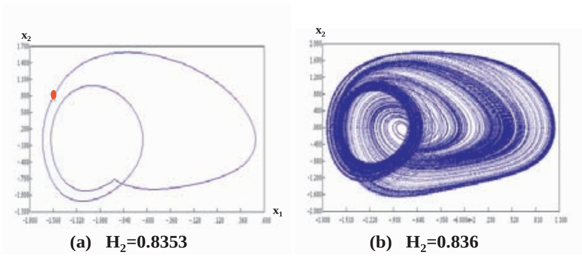

Figure 11: (a) period-1 orbit, (b) chaotic orbit

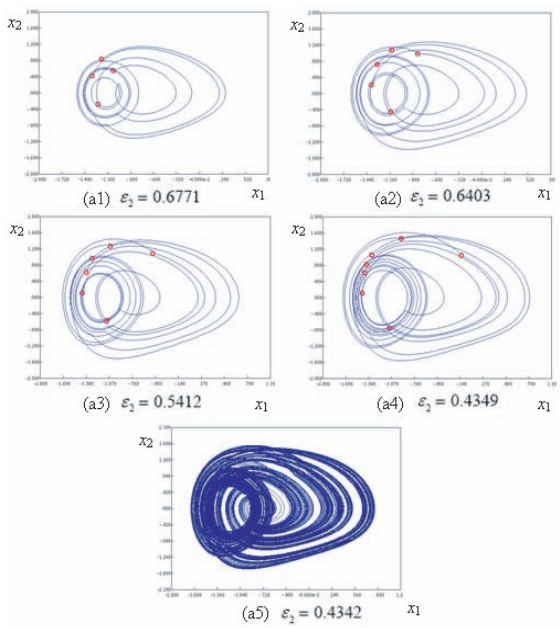

Figure 12: phase portraits for different values of ε_2 ,(a1) period-4 orbit, (a2) period-5 orbit, (a3) period-6 orbit, (a4) period-7 orbit, (a5) chaotic state

The transition from period-4 orbit to period-5 orbit, then from period-5 orbit to period-6 orbit and so on may be related to a certain type of bifurcation that is not revealed here. But one can say that it is not a fold or pitchfork bifurcation, because the order of periodic orbits which undergo such type of bifurcation does not change. Furthermore, the hypothetical bifurcation is not a flip a period doubling bifurcation. Border collision which plays an important role in switched dynamical systems was said in recent studied that it can be connected to saddle-node bifurcation. The bifurcation succession identified can have a subtle link with border collision bifurcation.

4 Higher harmonic predominance

4.1 Spectral analysis of period-1 orbits

Generally, in series resonance circuits periodic responses, terms of harmonics higher than the third are certain to be present but are ignored to this order. Nevertheless in certain cases namely in nonlinear systems described by a Duffing type equation, the periodic responses spectra can include higher harmonics with amplitudes greater than the third and even the fundamental harmonic. Spectral analysis of periodic responses in such systems led to find the role linking between higher harmonics amplitudes and a particular fold bifurcation structure namely isoordinal cascade of lips[6]. This structure contains a finite set of pairs of Fold bifurcation curves (lips) associated to well defined domains for which the amplitude of a rank-m higher harmonic line, $m = 1, 2, 3, ...$, has the place r ($r = 1, 2$) from an ordering based on higher harmonics amplitudes in descending order. Such domains defined for fixed points bifurcation structure are called *predominance domains* of the rank-m harmonics $r = 2$, and *full predominance domains* when $r = 1$.

For a period-1 response, the corresponding spectral ordering contains a supplementary useful information about the response that should be considered among other characteristics such as order, stability and the nature of the period-1 orbit.

4.2 Simple predominance case

From figure 13, $(a_i), i = 1, 6$ are the phase trajectories of fixed points corresponding to different values of H_2, (b_i) are the corresponding x_2-spectra. For $H_2 = 0.01$, the higher harmonic classification shown in Figure 13 (b_1) presents the fundamental harmonic in the first rank, and the order-2 higher harmonic in the second rank so this case is regarded as a simple predominance of second higher harmonic. The higher harmonic spectral lines classification is then $\{1, 2, 3, 5, 4, 8, 7, ...\}$. For the same fixed parameters a slight variation of H_2 around $H_2 = 0.01$ make the spectral lines amplitudes vary smoothly so that it exists a predominance domain of order-2 higher harmonic containing at least such point. Computation of predominance domains corresponding to the different higher harmonic orders in a given parameter plane provide supplementary characterization of the periodic orbits identified in these domains. Increasing the value of H_2 to $H_2 = 0.24$ we have also a fixed point having the phase portrait of Figure 13 (a_2) but seemingly with simple predominance of third higher harmonic see Figure 13 (b_2). The spectral lines classification is deduced from Figure 13(b2) as

$\{1, 3, 5, 4, 2, 8, 7, ...\}$. A first glimpse to the reorganization of the spectral lines classification under variation of H_2 from 0.01 to 0.24 leads to point out several changes. Firstly the order-2 higher harmonic moves from the 2nd to the 5th rank and we have the order-3 higher harmonic instead of it (i.e in 2nd rank).
Secondly the obtained spectrum for $H_2 = 0.24$ reveals that the amplitudes of order-3 and order-5 harmonics are apparently equal. This kind of situation was called in [6] a permutation point of the third and the fifth higher harmonic. Subsequently for $H_2 = 0.71$ the obtained period-1 orbit spectra is characterized by a fourth order higher harmonic simple predominance see Figure 13 (a_3) and (b_3). Thus, The spectral lines classification $\{1, 4, 2, 5, 3, 8, 7, ...\}$ corresponds to a periodic solution which belongs to a fourth order higher harmonic predominance domain and it is close to a permutation point of the second and the fifth higher harmonics being in the third rank of the classification. The studied switched circuit, under a gradual increasing of H_2 values, exhibits an intermittent appearance of order m higher harmonics simple predominance: m=2,3,4.., therefore an isoordinal cascade of lips may exist in the neighborhood of the H_2 values range given above. The identification of such bifurcation structure in a given parameter plane is let to further researches.

4.3 Full predominance case

Accordingly to their power spectra, the remaining fixed points of Figure 13 ((a_4),(a_5),(a_6)) obtained for $H_2 = 5.185$,$H_2 = 8.76$ and $H_2 = 11.26$ are associated to full predominance of higher harmonics of orders 2,3 and 4 respectively. For $H_2 = 5.185$, the corresponding spectral lines classification is $\{2, 3, 1, 4, 5, 6, 11, ...\}$, see Figure 13(b4); it is obvious to remark that the fundamental harmonic occupies the third rank and the second order higher harmonic has the greatest amplitude in the called spectral lines classification. For $H_2 = 8.76$, the spectral lines classification of a period -1 solution is $\{3, 4, 1, 2, 6, 5, 10, ...\}$,see Figure 13(b5) and the fundamental harmonic is similarly in the third rank but the 2nd order higher harmonic moves to the fourth rank of the decreasing amplitudes classification. A particular case in which the fundamental harmonic moves to the fourth rank for $H_2 = 11.26$ and the classification becomes $\{4, 2, 3, 1, 6, 4, 10, 7, ...\}$ see Figure 13(b6). For such values range of H_2 or in other words for large amplitudes of the second DC generator E_2, the higher harmonics play an important role regarding to their increasing amplitudes and become more predominant than the low order ones in the Fourier spectrum of period-1 solutions. The trajectories shown in phase plane present as more undulations as H_2 increases.

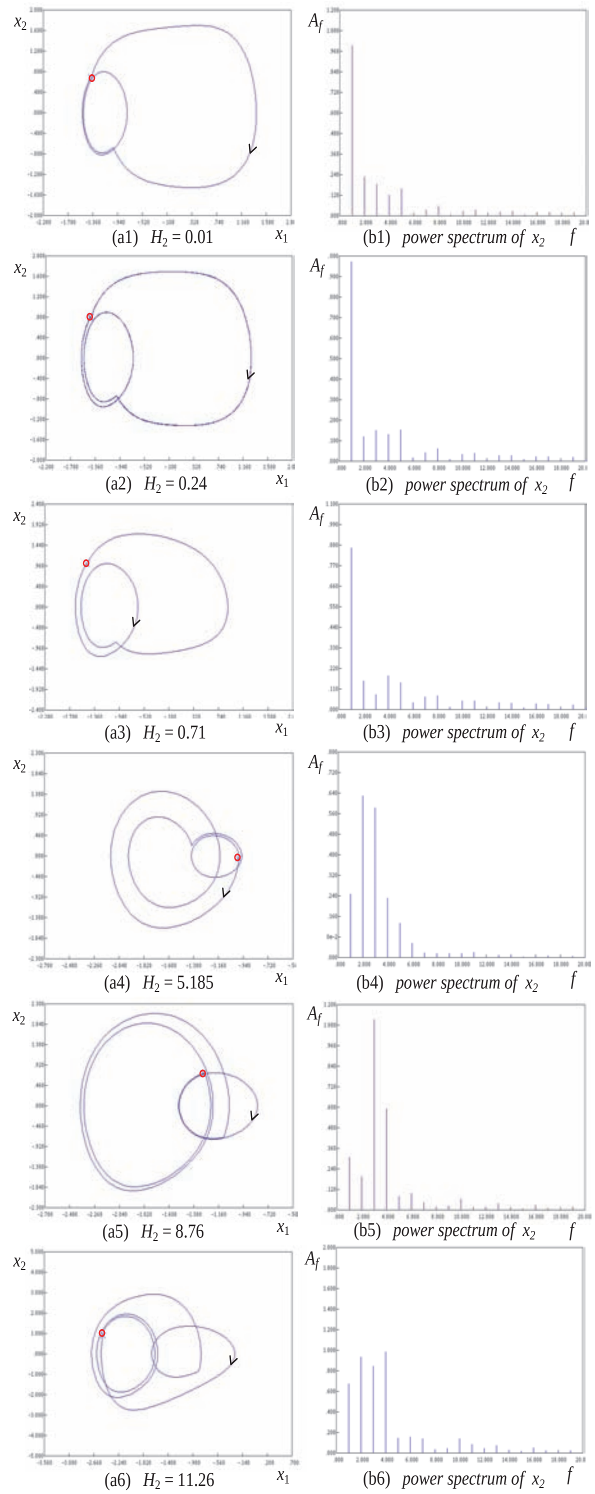

Figure 13: Higher harmonic predominance

This raises the following question: do the higher harmonic predominance change under parameter variation shown in the fixed points spectra is connected to an isoordinal cascade of fold bifurcation similarly to the case of Duffing type equation as in [6]? To settle this issue further investigation to characterize certain bifurcation structure in switching dynamical systems through higher harmonic predominance is needed.

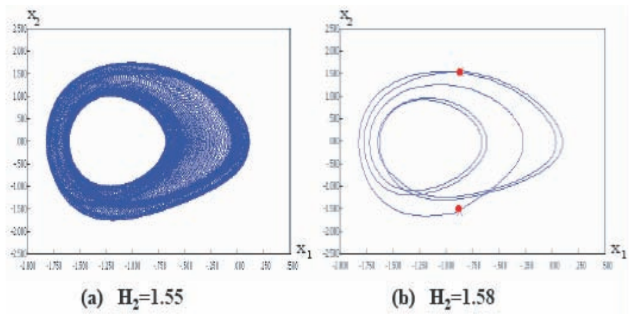

Figure 14: (a) chaotic orbit, (b)period-2 orbit

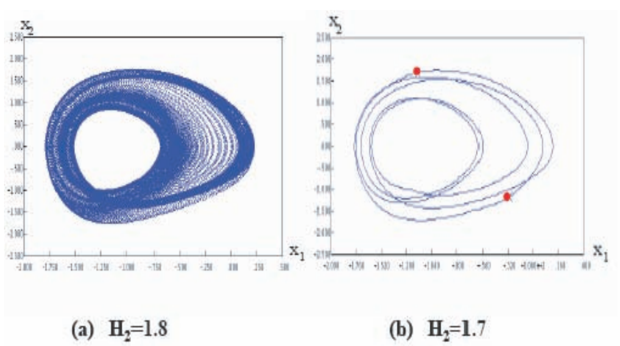

Figure 15: (a) chaotic orbit, (b)period-2 orbit

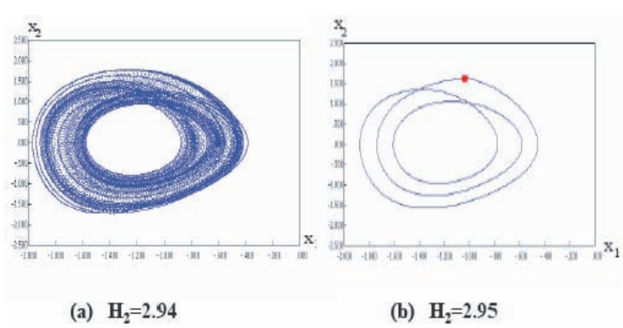

Figure 16: (a) chaotic orbit, (b)period-1 orbit

5 Chaotic states from different transition modes

5.1 Diagram of a reverse bubble

The doubling period bifurcation succession leads generally to a chaotic state it is also called Myrberg cascade of bifurcations. Two illustrative patterns of such type of bifurcation were observed in Chua's circuit namely diagram of a bubble and diagram of a reverse bubble [7]. The reverse bubble reveals the unavoidable reversals of doubling period which is a typical phenomenon of nonlinear circuits. Increasing the values of H_2 the Alpazur circuit responses cross the regions D_1, D_2, D_3, D_4, D_5 displayed in figure 17. Each region is associated to harmonic,subharmonic or chaotic behavior.

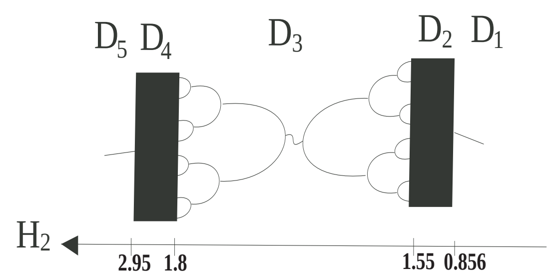

Figure 17: reverse bubble amid two direct transition modes

5.2 Transition modes between the five regions

The region D_1 corresponds to period 1 orbit, changing the value of H_2 from 0.8353 to 0.836, the responses suddenly go into chaotic region D_2. The fundamental harmonic orbit and the chaotic orbit obtained from transition from D_1 to D_2 or vice versa are given in figure 11. The chaotic domain D_2 is located in parameter interval $H_2 \in [0.836; 1.55]$.
For $H_2 = 1.58$ a transition from chaotic region D_2 to low order orbits region D_3 happens resulting in a period 2 orbit see figure 14. Varying H_2 from $H_2 = 1.7$ to $H_2 = 1.8$, the responses go out the region D_3 and turn again to a chaotic region D_4, the transition orbits are given in figure 15. Then, for $H_2 = 2.95$ the responses exit the region D_4 and enter abruptly the region D_5 giving rise to a period-1 orbit see figure 16.The chaotic domain D_4 is enclosed in parameter interval $H_2 \in [1.8; 2.95]$.

5.3 Miscellaneous chaotic regions

It is worth noting that the chaotic regions D_2 and D_4 do not outcome from only one transition mode. Either transition from fundamental harmonic orbit or from doubling period succession can lead to a chaotic state inside such regions. In a parameter plane a closed flip bifurcation curve includes period doubling sequences associated with a corresponding reverse period doubling cascade,chaotic regions generally appear amid these two cascades. The chaotic states belonging to such regions which purely result from doubling period succession are dissimilar from those of the regions D_2 and D_4.

6 Conclusion

In this paper, we have considered a modified Alpazur oscillator and through numerical investigation a variety of observations over time, Poincaré maps, phase trajectories, and power spectrum is established. Three main remarks are deduced from this work, even though rigorous proof is needed. The first point deals with bifurcations leading to chaotic states, three different ways are described: doubling period cascade of bifurcation, direct transition from fundamental harmonic or odd subharmonic orbits to chaos and a special bifurcation succession following a certain order $(4 - 5 - 6 - 7)$ of subharmonic responses merging ended by a chaotic orbit. The second point is the higher harmonic behavior under a parameter variation which encourages to prospect the existence of isoordinal cascade of lips in a proper parameter plane. The third point concerns the existence of chaotic regions resulting from either doubling period cascade or direct transition from a fundamental harmonic orbit. The results reported in this paper pave the way to further investigations for proper understanding of the bifurcation paths leading to chaotic states in switched dynamical systems. These results also call for further developments of the theory of higher harmonic oscillations in regard to fold bifurcation cascade (or isoordinal cascade of lips).

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