# REI Equivalent Design for Electric Power Systems with Genetic Algorithms

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*Abstract:* - Present day power systems are often large or very large systems, with a high degree of interconnectivity. Their analysis can be simplified using network equivalents, which decrease the size of the system by replacing a significant part of it with only a few nodes. The chosen equivalent is the REI equivalent. This paper describes a new approach to the problem of the REI equivalent design optimization, based on the sensitivity of the complex bus voltage from the internal unmodified section of the power system to a set of simulated representative contingencies. The optimal equivalents are determined using artificial intelligence techniques, namely genetic algorithms. The optimum design of the REI equivalent aimed to determine the number of REI buses to be used and the aggregation of external buses into the REI buses. The method was tested on a slightly modified version of the IEEE 57 bus test system. The obtained results prove the efficiency of the proposed method.

*Key-Words:* - Static network equivalents, REI equivalent, Load flow analysis, Contingency, Genetic algorithms, Sensitivity analysis.

## 1 Introduction

The actual development of power systems and the ever growing of power exchanges between systems, under more and more complex operating conditions, related to system control and mutual support, have determined an increasing interconnection degree between power systems. At present, this trend is strengthened by the globalization of electricity markets, where electricity is traded at national or regional level. This fact entails the need for the scheduling and control of important power exchanges between systems through existing or new interconnection power lines.

Present day power systems are basically large or very large systems, known also as wide area power systems (WAPS), whose on-line or off-line analysis and control often implies a serious computational burden. The analysis of such power systems aims particularly at two types of problems: development (off-line) and security assessment and control (on-line). To efficiently approach this analysis simplifying assumptions must be taken. Basically, such assumptions refer to the manner in which different parts of the power system areas are represented and interact.

The wide-spread approach uses static network or system equivalents. Such equivalents are widely used especially when the main interest lies in the analysis of a local power system interconnected with other neighboring systems, case in which the remote system is replaced by a simpler, equivalent network, while the local system is represented by more accurate models.

Basically, there are two main types of applications that use static network equivalents: system development studies and on-line monitoring and control of the power system, using contingency analysis.

In on-line applications, equivalencing techniques can considerably improve computational performances due to the decrease in the size of the system replaced by the equivalent. With this aim in view, practically all equivalencing methods used in present on-line applications divide the original power system into three subsystems, namely: (i) the internal power system (IPS), i.e. the part of the power system under analysis; this is the best known part of the whole system and its operating state is to be determined (ii) the external power system (EPS), i.e. the part of the system to be replaced by the equivalent, and (iii) the boundary power system or, simply, boundary nodes (BNs), i. e. the set of nodes which separate IPS from EPS and to which the equivalent will be connected.

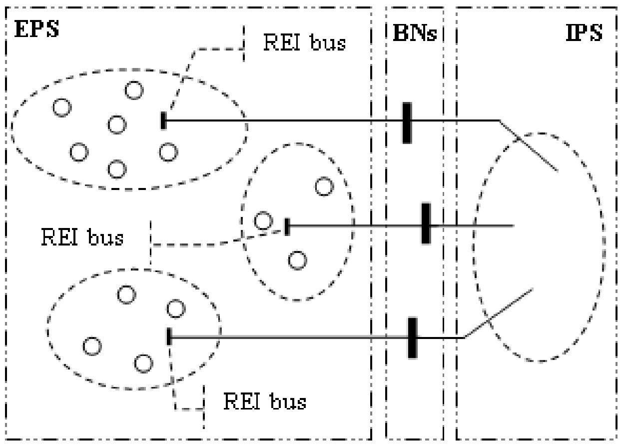



Fig. 1 Grouping buses from the EPS to create REI equivalent buses.

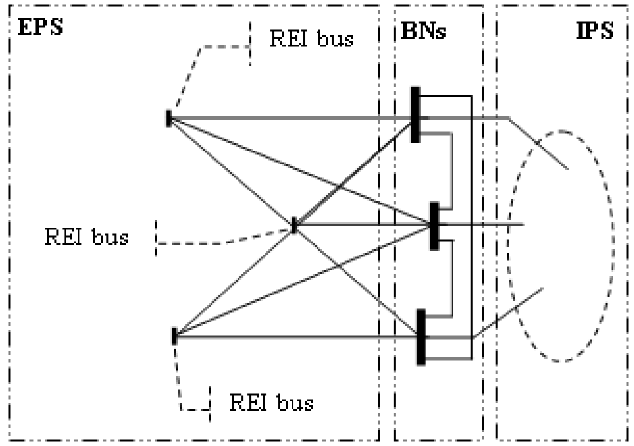



Fig. 2 The model of the REI equivalencing technique.

The first system equivalent studies were initiated in the mid of the 19-th century, when J. B. Ward has proposed the static equivalent named after him [10]. The Ward equivalent is widely employed even today in simple system studies, for its simplicity and ease of use.

Later, in the 1970s, P. Dimo has defined the REI equivalent and has proven the possibility of using this paradigm in power system studies [2]. The next type of network equivalent, the so called Ideal Transformers Equivalent, was introduced in 1977 by a group of researchers from EPRI, as an alternative for the REI equivalent concept, using ideal transformers instead of impedances for the equivalent network [6].

The REI equivalent represents one of the most efficient equivalencing techniques. Such an equivalent is based on external system bus reduction, preserving in a certain extent the effect of the external generator or load buses on the operating conditions in the IPS.

The solution proposed by P. Dimo [2] replaces the EPS by one or more fictitious buses, designated as *REI buses*, that group together different external buses. The basic REI model either groups all buses from the EPS into a single REI bus, or uses 2 REI buses, one for the load (PQ) buses, and the other for the generator (PV) buses.

However, a general model can be imagined that uses multiple REI buses, and bus-grouping procedures. Moreover, the resulting equivalent network can contain more generator REI buses, more load REI buses or even mixed REI buses, which group together load and generator buses. The grouping procedure is a question which should take into consideration its influence over the accuracy of the IPS operating conditions computed using the REI equivalent(s) for different contingencies in the IPS.

For instance, paper [9] presents an approach to this problem based on the analysis of the sensitivity of power flows on the branches of the IPS to finite changes of the power generated or consumed in the buses of the EPS. These sensitivities are computed for pairs of buses from the EPS, which change their load or generation in steps of 1 MW / MVAr. Other rules for grouping real buses into a REI bus are presented in [3, 8, 9, 16]

This paper presents a new approach to the problem of the REI equivalent design optimization based on the sensitivity of the complex bus voltage to a set of simulated contingencies. The optimal solution of this problem, for different values of the number of the REI buses used by the equivalent and a given set of branch and bus contingencies, is determined using a genetic algorithm, applied to a modified form of the standard IEEE 57 bus test system as a case study.

## 2 REI Equivalents

REI equivalents were introduced in the 1970’s by P. Dimo [2]. This type of static network equivalents is still widely used in various applications of power system analysis [11].

### 2.1 Basics on REI Equivalents

For any power system from which the IPS and the EPS have previously been separated, linked through boundary nodes / buses, the computation of the REI equivalent needs to define one or more sets of real buses to be replaced by fictitious REI buses (Fig. 1).

Each group of real buses and adjacent branches is replaced by a so called REI network. While the IPS can maintain a close/loop structure, the REI network has a radial structure. Each REI bus is connected directly, not through other nodes or transformers, to all boundary nodes (Fig. 2). This explains the REI acronym: Radial – Equivalent – Independent.

The power injections from the EPS nodes are linearized by being replaced with transversal admittances or equivalent currents, which are subsequently grouped in order to define non-linear power injections at the equivalent REI buses connected to the IPS.

The REI equivalents are built taking into consideration some specific requirements such as: (a) seen from the boundary nodes, the equivalent should represent accurately the structure and the behavior of the EPS; (b) the equivalent should describe as accurately as possible the reaction of the EPS to changes in the IPS with respect to the reference operating conditions, and (c) the REI equivalent should contain a minimum number of REI buses.

## 2.2 The Zero Power Balance Network

One important characteristic of the REI equivalent compared with other equivalencing techniques is that the first one preserves power losses in the initial and equivalent networks. This behavior is possible due to the special procedure used to build the equivalent, starting from the so called Zero Power Balance Network (ZPBN). This is a fictitious, temporary network which links the buses from the EPS that are to be eliminated to the fictitious REI buses. The ZPBN is a linear lossless network, which eventually is eliminated by applying a simple Gauss reduction technique.

The ZPBN for a single REI node is built using the following procedure (see also Fig. 3):

1. For each EPS bus associated to the REI bus, the load is linearized by replacing the power injection with transversal bus admittances $\underline{y}_{i,0}$, $\underline{y}_{j,0}$,and $\underline{y}_{k,0}$,, which are defined between the initial nodes and the fictitious ground bus, denoted by *0*, which has also zero voltage. The admittances are computed using equation:

$$\underline{y}_{p,0} = \frac{\underline{S}_p^*}{U_p^2}, \quad (p = i, j, k) \tag{1}$$

2. The current injections between the three nodes and the fictitious ground node are computed using the next formula:

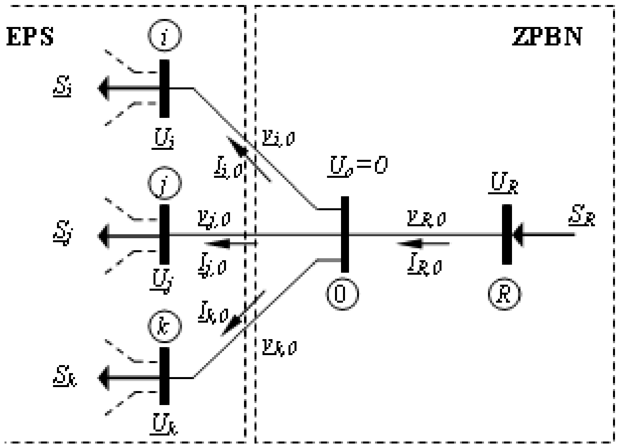



Fig. 3 Building the ZPBN network

$$\underline{I}_{p,0} = \frac{\underline{S}_p^*}{\underline{U}_p}, \quad (p = i, j, k) \tag{2}$$

3. The existing network, including the ground node, is extended by adding the fictitious REI node, denoted with *R*, which will replace the real nodes *i, j* and *k*.

4. The apparent power injection in the *R* bus is computed using equation (3) as the sum of the apparent power injections from the real buses associated to it (the ZPBN network will have thus zero losses and the mathematical model is deliniarized again).

$$\underline{S}_R = \sum_p \underline{S}_p = \underline{S}_i + \underline{S}_j + \underline{S}_k \tag{3}$$

5. The current flow between the fictitious nodes *0* and *R* is determined using the first Kirchoff theorem:

$$\underline{I}_{R,0} = \sum_p \underline{I}_{p,0} = \underline{I}_{i,0} + \underline{I}_{j,0} + \underline{I}_{k,0} \tag{4}$$

6. Based on apparent power $\underline{S}_R$ and equivalent current injections $\underline{I}_{p,0}$, voltage at the REI bus $\underline{U}_R$ and the complex admittance of the 0-R equivalent branch are computed as:

$$\underline{U}_R = \frac{\underline{S}_R}{I_{R,0}^2}$$
$$\underline{y}_{R,0} = -\frac{\underline{S}_R^*}{U_R^2} \tag{5}$$

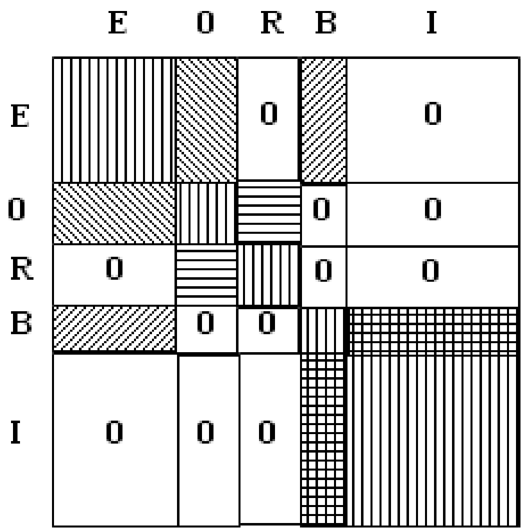
The number of equivalent REI buses and the way in which the real buses from EPS are associated to each of them can influence in a great extent the accuracy of the REI equivalent. The study case presented in this paper aims to address specifically this problem by finding the optimal number of REI buses and the sets of EPS buses association to each of them.

### 2.3 Building REI Equivalents

After building all ZPBNs associated to the REI buses, using the algorithm described above, the network is reduced applying a traditional Gauss reduction technique, which aims to bring the nodal equation:

$$[\underline{Y}]\cdot[\underline{U}]=[\underline{I}] \qquad (6)$$

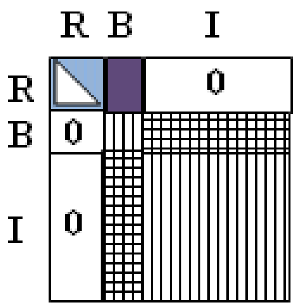
to a partially triangular form. Matrix [ $\underline{Y}$ ] from the above equation has the general form:




(7)

where 0 denotes a vector or a matrix filled with zeros. Other notations are: **E** – external buses (buses from the EPS); **0** – fictitious ground buses; **R** – fictitious REI buses; **B** – boundary buses; **I** – internal buses (buses from the IPS).

The application of Gauss reduction procedure to the matrix from equation (7) ends when all the lower diagonal elements from the **E, 0** and **R** columns have been zeroed. At this stage, the right-lower submatrix is extracted to represent the admittance matrix in the equivalent network (IPS and ZPBN):




(8)

With the exception of the lower-right corner of the matrix from equation (8), the rest of non-zero blocks change their values as compared to the structure in equation (7).

## 3 Problem formulation

Starting from the ZPBN building procedure described in section 2.2, the problem aims to determine the REI equivalent that best fit the operating conditions of the IPS for a given set of contingencies that occur in this system.

The accuracy of the equivalent network is assessed using the mean absolute percentage error of the complex voltage in the IPS buses, for all contingencies in the input data set:

$$FO=\frac{1}{NC\cdot NI}\cdot\sum_{i=1}^{NI}\sum_{j=1}^{NC}\frac{\left|\underline{U}_{i,j}^{ref}-\underline{U}_{i,j}^{eq}\right|}{\left|\underline{U}_{i,j}^{ref}\right|}\cdot 100 \qquad (9)$$

where: $\underline{U}_{i,j}^{ref}$ - the complex voltage from bus *i*, for contingency *j* and the reference operating condition (which represent in details the whole power system); $\underline{U}_{i,j}^{eq}$ - the complex voltage from bus *i*, for contingency *j* when the EPS is replaced with the REI equivalent; *NI* – number of buses in the IPS; *NC* – number of contingencies considered in the input data set.

For each contingency $j = 1, \dots, NC$ the values of the complex voltages in the buses of the IPS will depend on the features of the REI equivalent. The basic two features of a REI equivalent that influence these values are:

- The number of REI buses used by the equivalent, denoted by $N_{REI}$ and
- The set of buses from the EPS associated to each REI bus

A solution to this problem must describe exactly these two features.

The model corresponding to equation (5) and to the above two features describes in fact a combinatorial analysis problem, which can efficiently be addressed using evolutionary computation techniques, namely the so-called genetic algorithms.

## 4 Genetic Algorithms

Genetic algorithms (GAs) are adaptive techniques that determine an optimal or near-optimal solution for an optimization problem using mechanisms specific to genetics and natural selection [1, 4]. Given the advances in terms of computing power in the last years, they are used today with notable results in a wide range of applications in the power system field, including, but not limited to complex systems' control [12], system operation and structure optimization [13, 14] and operation forecast [15].

GAs represent the admissible solutions as strings or chromosomes, which change their structure from the actual to the next generation, directing themselves towards the optimal solution. The initial population has a random composition and generations give birth one to another following the "fittest survive" principle. GAs use five major components:

1) *Admissible solutions* of a given problem are represented as strings or *chromosomes* of a fixed length, which inherently is imposed by the problem itself. The elements of the strings (*genes*) frequently use a binary representation (e.g. 0-1; on-off; present-absent), but real number representation is also possible.

2) *The fitness function* determines how well the solution described by a chromosome fits to the problem (how close is it to the optimal solution). By their intimate mechanisms GAs tend to maximize the fitness function. Hence, if the optimization problem aims to minimizing the objective function F, the fitness function f must be computed as the reciprocal of F (f=1/F).

3) *Reproduction or selection*. Using the values of the fitness functions computed in the previous step, a crossover pool is created using chromosomes from the current generation. Those chromosomes with higher values of the fitness function (the better adapted ones) have higher probabilities to produce more offsprings to the next generation. Therefore, more copies of these chromosomes will reach the crossover pool.

4) *Crossover*. Each chromosome in the actual generation contains a part of the information that forms the optimal solution. One way to assemble this information into a single string is to recombine the chromosomes in the crossover pool. The most efficient implementations of the GAs apply a stochastic crossover operator using a probabilistic crossover rate. At the end of this step the population is completely renewed. The parent chromosomes are replaced by their offsprings, which form a new generation.

5) *Mutation*. It is highly probable to start the algorithm from an initial population which contains poor information with respect to the optimal solution. In this case there will be no possible crossover operation between the strings in the actual or the ever next generation to drive the GA towards the optimal solution. The population diversification from one generation to another may be encouraged using a mutation operator. Mutations modify at random one or more elements in the string with a probabilistic mutation rate.

To simulate more accurately the process of natural selection and as a tool to fine-tune the algorithm's performance, the crossover and mutation operators are applied with a certain probability modeled through crossover and a mutation rates.

A way to improve the performances of the GA is to use the *elitism*, by perpetuating the best adapted chromosome from each generation to the next. This method was used in the case study.

The basic form of the GA is:

- Set the initial population ***P(Gen), Gen=1*** ;
- Compute fitness functions for the initial population

***repeat***

  - Apply the *selection* operator to send parent strings in the crossover pool.
  - Recombination: apply the crossover operator to form a new population ***P(Gen+1)*** ;
  - Mutation: apply mutations to change the structure of the new population ***P(Gen+1)*** ;
  - Compute fitness function for the new population ***P(Gen+1)*** ;
  - Next generation: ***Gen=Gen+1*** ;

***until*** {ending condition}

- The optimal solution corresponds to the string with the highest value of the fitness function.

## 5 Case study

The implementation of the GA to solve the problem of the REI equivalent design optimization was studied on a modified form of the IEEE 57 bus test system.

The inner part of the IEEE 57 bus test system was considered as the IPS, while the marginal buses and branches was considered to form the EPS. The type of buses from the test system is shown in Table 1. Buses with bold face in Table 1 are generators (PU), the one with bold and underline face is the slack bus, while the rest, with normal face are load buses (PQ).

The slack bus was changed from bus #1, which becomes an external bus, to bus #15. In fact, bus #1 was renamed as bus #58 and a new bus named bus

Table 1 The type of buses from the IEEE 57 bus test system.

| External Power System (EPS) | Internal Power System (IPS) | Boundary nodes (BNs) |
|---|---|---|
| **2**, **3**, 4, 5, **6**, **8**, **9**, 10, 11, **12**, 13, 14, 16, 17, 58 | <u>**1**</u>, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39,40, 42, 43, 44, 45, 47, 48, 50, 52, 53, 54, 56, 57 | 7, 15, 18, 41, 46, 49, 51, 55 |

Table 2 Contingencies used in the GA implementation for the REI equivalent design optimization problem.

| Location and type of contingency | EPS | IPS |
|---|---|---|
| Branch | 12-13, 7-8 | 38-48, 29-52, 28-29, 22-23 |
| Bus | 8 (-20%), 12 (-50%) | 38 (-100%), 47 (-100%), 50 (-100%), 53 (-50%) |

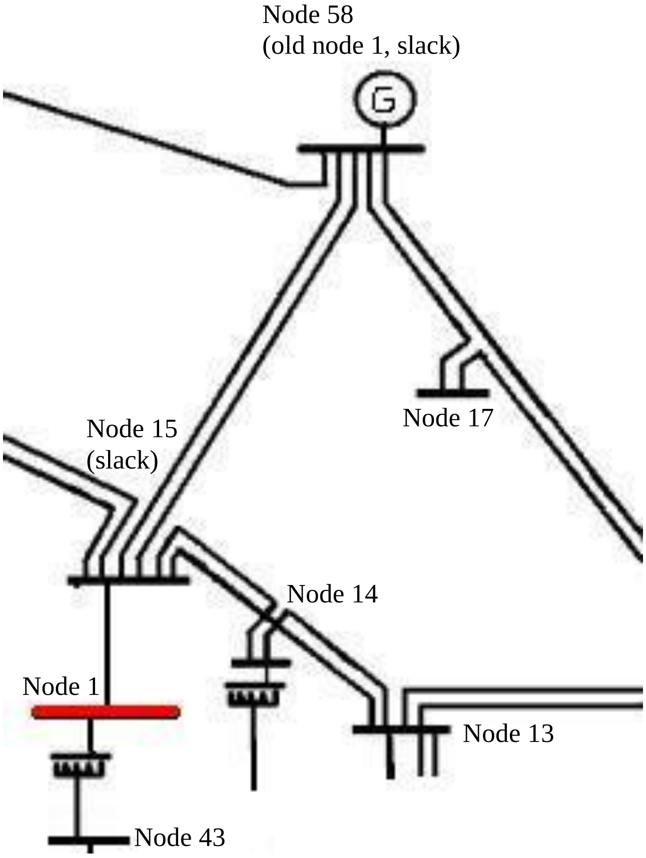



Fig. 4 Changes made to the IEEE 57 bus system.

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The slack bus was changed from bus #1, which becomes an external bus, to bus #15. In fact, bus #1 was renamed as bus #58 and a new bus named bus #1 was added between buses #15 and #43, through a fictitious line of zero-admittance connected to bus #15, which becomes a boundary node (see Table 1 and Fig. 4). The complete one-line diagram of the original IEEE 57 bus test system can be consulted in reference [5].

For this system, the problem consists in selecting the best number of REI buses and the aggregation of the external buses from Table 1 that produce the minimal value of the objective function from equation (9) for a given set of contingencies (see Table 2).

The model has considered two types of contingencies (branch and bus contingencies) and two locations (the EPS and the IPS). Although the network equivalent analysis is centered on the IPS, and hence contingencies in this system are primarily envisaged, events in the EPS can also influence the behavior of the IPS. Therefore, contingencies in the EPS have been considered too. However, for a contingency in the EPS, this event is considered unknown for the operator in the IPS and no corrective action is considered. Practically, this means that for any contingency in the EPS, the operating conditions of the IPS will result the same, i.e. those computed with the IPS, the BNs and the REI equivalent for the reference conditions.

The set of contingencies used by the GA implementation is the one from Table 2.

This is the general framework of the REI equivalent design optimization problem for the case of the IEEE 57 bus test system. To solve this problem using the GA approach from section 4, the structure of the string that describes a possible

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Genes | 2 | 6 | 1 | 6 | 2 | 2 | 2 | 6 | 1 | 2 | 4 | 1 | 4 | 2 | 6 |
| Buses in Table 1 | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 16 | 17 | 58 |

Fig. 5 The representation of a possible solution in a string used by the GA.

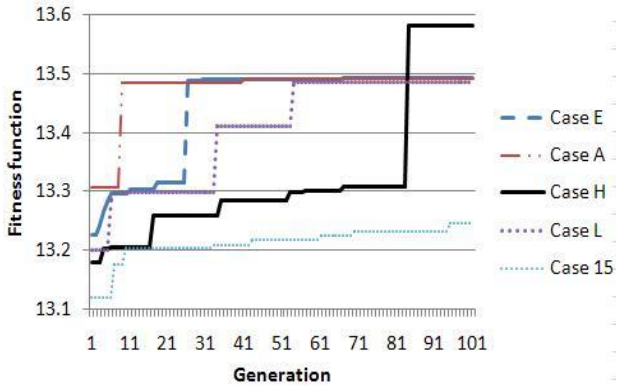


Fig. 6 The evolution of the fitness function during the run of the GA for 4 cases from Table 3 and for the case of $N_{REI}$ = 15 ('Case 15").

Table 3 Possible solutions for different values of parameter $N_{REI}$.

| | 2 | 3 | 4 | 5 | 6 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 16 | 17 | 58 | **Buses** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $N_{REI}$ = 2 | | | | | | | | | | | | | | | Fitness |
| **A** | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 2 | 1 | 1 | 1 | 2 | 2 | 1 | 2 | 13.4929 |
| **B** | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 2 | 1 | 1 | 2 | 2 | 1 | 2 | 2 | 13.4862 |
| | $N_{REI}$ = 3 | | | | | | | | | | | | | | | |
| **C** | 1 | 2 | 1 | 2 | 1 | 1 | 1 | 3 | 1 | 1 | 2 | 2 | 3 | 1 | 2 | 13.5779 |
| **D** | 1 | 3 | 1 | 1 | 1 | 1 | 1 | 3 | 3 | 1 | 3 | 2 | 1 | 2 | 2 | 13.4952 |
| **E** | 1 | 2 | 3 | 2 | 1 | 1 | 1 | 3 | 3 | 1 | 1 | 3 | 2 | 1 | 3 | 13.4929 |
| **F** | 3 | 2 | 2 | 3 | 3 | 3 | 3 | 2 | 2 | 3 | 1 | 2 | 3 | 2 | 2 | 13.4869 |
| **G** | 1 | 1 | 2 | 1 | 1 | 3 | 1 | 2 | 2 | 3 | 3 | 1 | 2 | 2 | 1 | 13.3262 |
| | $N_{REI}$ = 4 | | | | | | | | | | | | | | | |
| **H** | **3** | **4** | **1** | **4** | **3** | **3** | **3** | **4** | **1** | **3** | **2** | **1** | **2** | **3** | **4** | **13.5821** |
| **I** | 4 | 1 | 3 | 3 | 4 | 4 | 4 | 2 | 4 | 4 | 1 | 3 | 1 | 4 | 1 | 13.5791 |
| **J** | 3 | 4 | 3 | 3 | 3 | 3 | 3 | 1 | 4 | 3 | 4 | 4 | 3 | 1 | 1 | 13.4963 |
| **K** | 1 | 1 | 4 | 1 | 1 | 4 | 1 | 1 | 4 | 4 | 3 | 1 | 3 | 3 | 1 | 13.3395 |
| | $N_{REI}$ = 5 | | | | | | | | | | | | | | | |
| **L** | 5 | 3 | 5 | 5 | 5 | 5 | 5 | 1 | 5 | 5 | 4 | 1 | 5 | 2 | 1 | 13.4875 |
| **M** | 2 | 4 | 4 | 2 | 2 | 4 | 2 | 2 | 1 | 4 | 5 | 5 | 1 | 4 | 2 | 13.2993 |

solution of the problem must be derived. One of the simplest structures that can be easily manipulated by the GA is the one presented in Fig. 5. Here the string has 15 genes, a number equal to the number of buses in the external system (see Table 1). The maximum number of REI buses to be used by the GA model ($N_{REI}$) is given as a parameter of the GA. The value stored by a gene from the string is a number between 1 and $N_{REI}$. This value shows to which fictitious REI bus is allocated the external bus corresponding to that gene.

For instance, the string from Fig. 5 describes a solution that can use a maximum number of 6 REI buses, but actually uses only 4 buses, denoted by 1, 2, 4 and 6. Buses 4, 11 and 14 are associated to the first REI bus, buses 2, 6, 8, 9, 12 and 17 – to the 2nd REI bus, buses 13 and 16 – to the 3rd REI bus, and buses 3, 5, 10 and 58 – to the 4th REI bus.

All tests with the GA have used a population of 100 strings, during a lifelong of 100 generations. In a first stage of the experiment the authors have considered various conditions concerning the maximum number of REI buses. Tests using large values for parameter $N_{REI}$ were driven (e.g. $N_{REI}$ = 15 or 10). During these tests, the fitness function (computed as the inverse of the objective function from equation (9)) did not go beyond 13.3028. On the other hand, these tests have shown that the usage of the maximum number of REI buses is discouraged by the algorithm, since only 7 and 5 REI buses were actually used for $N_{REI}$ = 15 and 10, respectively. Even when using a maximum number of 6 REI buses, only 5 buses were actually used. Of course, due to the course of dimensionality the use of large values for $N_{REI}$ is far away of being efficient since the best solution could require a large number of generations of the GA. Hence modest values for parameter $N_{REI}$ have been considered for continuing the experiments.

In the next stage of the experiment the GA was run using for parameter $N_{REI}$ values between 2 and 5. The results of these tests, describing the aggregation form and the value of the fitness function, are presented in Table 3. For comparison reasons, Fig. 6 shows the evolution of the fitness function for 4 cases from Table 3 and for the case $N_{REI}$ = 15, denoted by "Case 15" in this figure.

As one can see from the values in this table, the fitness function takes values in a relative narrow interval around 13.5. However, a clear optimum point can be identified for $N_{REI}$ = 4 and a fitness function of 13.5821 (case H in table 3). This solution aggregates the buses as follows: The next stage of the experiment consists in analyzing the performances of the optimum solution identified in the previous step (case H from Table 3). With this aim in view, the authors have considered two sides of the analysis, namely the bus voltage profile, and the branch power flow profile. Both sides were analyzed from the viewpoint of the set of

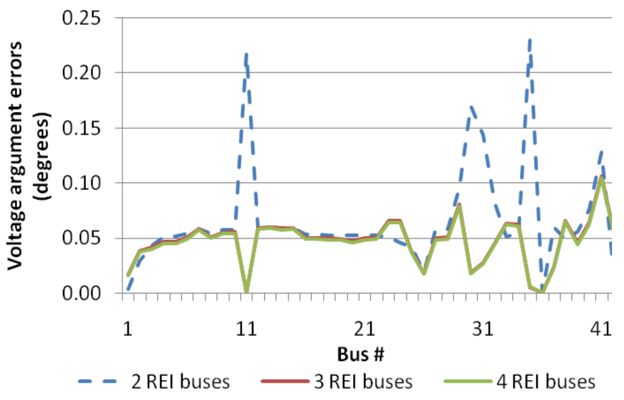


Fig. 7 Voltage argument errors for branch contingency #3 from Table 2.

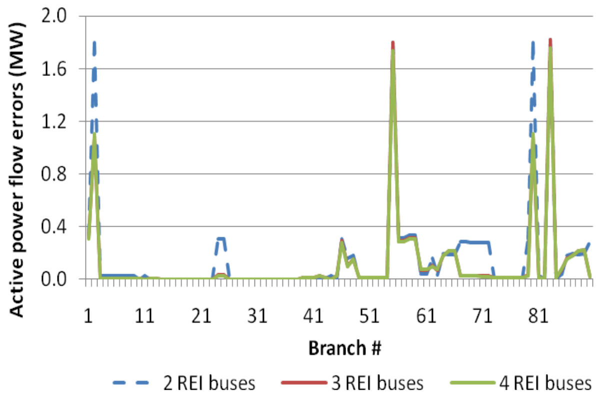


Fig. 8 Active power flow errors for on the IPS branches.

Table 4 Mean absolute errors for active and reactive power flows on the IPS branches (MW and MVAr)

| Contingency | Active power flows | | | Reactive power flows | | |
|---|---|---|---|---|---|---|
| | 2 REI | 3 REI | 4 REI | 2 REI | 3 REI | 4 REI |
| Branch 28-29 | 0.160 | 0.114 | 0.112 | 0.218 | 0.172 | 0.170 |
| Branch 22-23 | 0.120 | 0.089 | 0.087 | 0.254 | 0.240 | 0.239 |
| Bus 50 (-100%) | 0.093 | 0.108 | 0.106 | 0.581 | 0.576 | 0.576 |
| Bus 53 (-50%) | 0.151 | 0.101 | 0.099 | 0.431 | 0.415 | 0.414 |

contingencies considered to compute the fitness function. To illustrate the results of the analysis, only four contingencies among the 12 from Table 2 were considered.

On the bus voltage side, the voltage variations on the buses of the IPS have a low sensitivity with respect to the number of REI buses considered in the analysis. Voltage amplitudes are almost constant, but significant changes can be observed in the arguments of the bus voltages. For instance, for branch contingency # 3 from Table 2, as one can observe from Fig. 7, voltage argument errors for 3 and 4 REI buses are practically the same (the solution with 4 REI buses is however slightly superior), and the errors are less than 0.1 degree. However, for the solution with 2 REI buses there are buses in the IPS where voltage argument errors change its value in comparison with the 3 and 4 REI buses. Such buses are buses #: 7, 29, 51, 52, 53, 54. This behavior is strengthened for the second, the third and the fourth contingencies, when branch between buses 22 and 23 is lost or loads from buses 50, and 53 diminishes with 100% or 50%, respectively. Buses #7 and 29 are the most sensitive with respect to voltage angle variation for different contingency hypothesis.

On the branch power flow side, active and reactive power flows through the branches of the IPS vary in a greater extent, up to several tens of percentage. However, the most significant variations were recorded on low loaded branches. Generally, the optimal solution for 3 and 4 REI buses produce similar results from the view point of errors between real power flows and simulated power flows based on the REI equivalent network. On the contrary, solutions for 2 REI buses produce higher errors for all contingencies from Table 2.

The most sensitive branches with respect to power flow variations for different designs of the REI equivalent are branches # 41-43 and # 1-15. Both branches are sensitive to active power flows. On the reactive power flow side, only branch #1-15 manifests a constant sensitivity for all contingencies.

For instance, if a contingency is considered on branch 28-29, the reference value of the active power flow on branch # 41-43 is about 12.67 MW, while the estimation errors based on REI equivalents with 2 to 4 buses vary between 1.75 and 1.82 MW, or a percentage error between 13.8 % and 14.4%. For the same contingency, but for branch # 1-15, the reference value of the active power flow is greater (about 41.0 MW) and the estimation errors vary between 1.09 and 1.8 MW, or between 2.65% and 4.39%. For the same branch, the changes in the reactive power flows are larger, variating between 50 % and 65 %, for a reference value of 10.13 MVAr.

Table 5 Active and reactive power flows in the IPS branches for the reference conditions (REF) and three REI equivalent designs (2 REI buses, 3 REI buses and 4 REI buses).

| N1 | N2 | REF | 2 REI | 3 REI | 4 REI | N1 | N2 | REF | 2 REI | 3 REI | 4 REI | N1 | N2 | REF | 2 REI | 3 REI | 4 REI |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 45 | 44.63 | 44.30 | 44.32 | 44.33 | 32 | 33 | 3.81 | 3.81 | 3.81 | 3.81 | 47 | 46 | -55.44 | -55.40 | -55.36 | -55.36 |
| 1 | 15 | -41.02 | -42.82 | -42.11 | -42.13 | 32 | 34 | -9.87 | -9.87 | -9.87 | -9.87 | 48 | 38 | 28.90 | 29.06 | 29.01 | 29.00 |
| 19 | 20 | 2.72 | 2.69 | 2.71 | 2.71 | 33 | 32 | -3.80 | -3.80 | -3.80 | -3.80 | 48 | 47 | -25.58 | -25.54 | -25.51 | -25.50 |
| 19 | 18 | -6.02 | -5.99 | -6.01 | -6.01 | 34 | 32 | 9.87 | 9.87 | 9.87 | 9.87 | 48 | 49 | -3.32 | -3.52 | -3.50 | -3.50 |
| 20 | 19 | -2.69 | -2.66 | -2.69 | -2.69 | 34 | 35 | -9.87 | -9.87 | -9.87 | -9.87 | 50 | 49 | -6.48 | -6.29 | -6.27 | -6.27 |
| 20 | 21 | 0.39 | 0.36 | 0.39 | 0.39 | 35 | 34 | 9.95 | 9.95 | 9.95 | 9.95 | 50 | 51 | -14.52 | -14.71 | -14.73 | -14.73 |
| 21 | 20 | -0.39 | -0.36 | -0.39 | -0.39 | 35 | 36 | -15.95 | -15.95 | -15.95 | -15.95 | 52 | 29 | -20.44 | -20.15 | -20.41 | -20.41 |
| 21 | 22 | 0.39 | 0.36 | 0.39 | 0.39 | 36 | 35 | 16.10 | 16.10 | 16.10 | 16.10 | 52 | 53 | 15.54 | 15.25 | 15.51 | 15.51 |
| 22 | 21 | -0.39 | -0.36 | -0.38 | -0.39 | 36 | 37 | -18.56 | -18.56 | -18.54 | -18.54 | 53 | 52 | -15.35 | -15.07 | -15.32 | -15.33 |
| 22 | 23 | 33.69 | 33.69 | 33.69 | 33.69 | 36 | 40 | 2.46 | 2.46 | 2.44 | 2.44 | 53 | 54 | -4.65 | -4.93 | -4.68 | -4.67 |
| 22 | 38 | -33.30 | -33.33 | -33.30 | -33.30 | 37 | 36 | 18.71 | 18.72 | 18.70 | 18.70 | 54 | 53 | 4.75 | 5.03 | 4.77 | 4.77 |
| 23 | 22 | -33.57 | -33.56 | -33.56 | -33.56 | 37 | 38 | -21.81 | -21.82 | -21.79 | -21.79 | 54 | 55 | -8.85 | -9.13 | -8.87 | -8.87 |
| 23 | 24 | 27.27 | 27.26 | 27.26 | 27.26 | 37 | 39 | 3.10 | 3.10 | 3.09 | 3.09 | 56 | 40 | -2.45 | -2.45 | -2.43 | -2.43 |
| 24 | 23 | -25.86 | -25.86 | -25.86 | -25.86 | 38 | 22 | 33.54 | 33.57 | 33.54 | 33.54 | 56 | 42 | -2.39 | -2.39 | -2.40 | -2.40 |
| 24 | 25 | 11.46 | 11.46 | 11.46 | 11.46 | 38 | 37 | 22.29 | 22.30 | 22.27 | 22.27 | 56 | 57 | 3.63 | 3.63 | 3.64 | 3.64 |
| 24 | 26 | 14.40 | 14.40 | 14.40 | 14.40 | 38 | 44 | -31.18 | -30.87 | -30.89 | -30.89 | 56 | 41 | -6.39 | -6.39 | -6.40 | -6.40 |
| 25 | 24 | -11.46 | -11.46 | -11.46 | -11.46 | 38 | 48 | -28.48 | -28.64 | -28.59 | -28.58 | 57 | 39 | -3.10 | -3.10 | -3.09 | -3.09 |
| 25 | 30 | 5.16 | 5.16 | 5.16 | 5.16 | 38 | 49 | -10.18 | -10.36 | -10.33 | -10.33 | 57 | 56 | -3.60 | -3.60 | -3.61 | -3.61 |
| 26 | 24 | -14.40 | -14.40 | -14.40 | -14.40 | 39 | 37 | -3.10 | -3.10 | -3.09 | -3.09 | 7 | 29 | 38.06 | 37.75 | 38.03 | 38.03 |
| 26 | 27 | 14.40 | 14.40 | 14.40 | 14.40 | 39 | 57 | 3.10 | 3.10 | 3.09 | 3.09 | 15 | 1 | 41.02 | 42.82 | 42.11 | 42.13 |
| 27 | 26 | -13.92 | -13.92 | -13.92 | -13.92 | 40 | 36 | -2.45 | -2.45 | -2.43 | -2.43 | 18 | 19 | 6.22 | 6.19 | 6.21 | 6.21 |
| 27 | 28 | 4.62 | 4.62 | 4.62 | 4.62 | 40 | 56 | 2.45 | 2.45 | 2.43 | 2.43 | 41 | 42 | 9.74 | 9.74 | 9.75 | 9.75 |
| 28 | 27 | -4.60 | -4.60 | -4.60 | -4.60 | 42 | 56 | 2.41 | 2.40 | 2.42 | 2.42 | 41 | 43 | -12.67 | -14.49 | -14.49 | -14.43 |
| 29 | 52 | 21.06 | 20.75 | 21.03 | 21.03 | 42 | 41 | -9.51 | -9.50 | -9.52 | -9.52 | 41 | 56 | 6.64 | 6.63 | 6.65 | 6.65 |
| 29 | 7 | -38.06 | -37.75 | -38.03 | -38.03 | 43 | 41 | 12.67 | 14.47 | 14.47 | 14.41 | 46 | 47 | 56.27 | 56.23 | 56.19 | 56.18 |
| 30 | 25 | -5.10 | -5.10 | -5.10 | -5.10 | 44 | 38 | 31.46 | 31.15 | 31.16 | 31.17 | 49 | 38 | 10.46 | 10.65 | 10.62 | 10.62 |
| 30 | 31 | 1.50 | 1.50 | 1.50 | 1.50 | 44 | 45 | -43.46 | -43.15 | -43.16 | -43.17 | 49 | 48 | 3.38 | 3.58 | 3.56 | 3.56 |
| 31 | 30 | -1.48 | -1.48 | -1.48 | -1.48 | 45 | 1 | -44.63 | -44.30 | -44.32 | -44.33 | 49 | 50 | 6.53 | 6.34 | 6.31 | 6.31 |
| 31 | 32 | -4.32 | -4.32 | -4.32 | -4.32 | 45 | 44 | 44.63 | 44.30 | 44.32 | 44.33 | 51 | 50 | 14.85 | 15.05 | 15.08 | 15.08 |
| 32 | 31 | 4.46 | 4.46 | 4.46 | 4.46 | 47 | 48 | 25.74 | 25.70 | 25.66 | 25.66 | 55 | 54 | 9.07 | 9.35 | 9.09 | 9.09 |

The sensitivity of the power flows on branches # 41-43 and #1-15 is illustrated for the case of active power flows in Fig. 8. In this figure, the first and the last peaks corresponds to branches 1-15 and 15-1, while the second and the third peaks correspond to braches 41-43 and 43-41.

The mean absolute errors for the active and reactive power flows on the IPS branches, for different contingencies and REI equivalent designs, are presented in Table 4. As one can see from this table, the values of the power flow errors are better for the 3 and 4 REI buses solutions, with the exception of the active power flows for contingency in bus 50, when the error is slightly superior for the 2 REI buses solution.

To illustrate the above comments, Table 5 present the values of the active and reactive power flows in the IPS branches for one contingency (branch 28-29 is lost).

As concluding remarks, we can state that for the IEEE 57 bus test system considered in our analysis, the best REI equivalent design uses 4 REI buses and an aggregation of the buses from the EPS as shown in Case H from Table 3. This design of the REI equivalent determines the best results in terms of the estimation accuracy for both bus voltage phasors and branch power flows, for the whole set of contingencies.

# 6 Conclusion

The development of power systems and the growing of power exchanges between systems, have determined an increasing interconnection degree between power systems. The analysis of such large scale networks, with complex operation conditions, requires a heavy computational effort, which can be simplified using system equivalents.

The work described in this paper presents a new approach to the problem of the REI equivalent design optimization. The optimization process is based on the sensitivity of the complex bus voltage from the internal power system to a set of simulated contingencies and was conducted using the optimization model of genetic algorithms.

The optimum design of the REI equivalent aimed to determine the number of REI buses to be used and the aggregation of external buses into the REI buses. Test results show that the optimal solutions generated by multiple runs of the genetic algorithm tend to use a moderate number of REI buses. For the IEEE 57 bus test system, the optimal solution consists in using 4 REI buses, each of them aggregating 2, 3, 4 or 6 buses from the external system.

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