Current Decomposition Methods Based on p-q and CPC Theories for Active Filtering Reasons

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Abstract: - This paper presents a comparative analysis of three current decomposition methods in three-phase systems without neutral wire on the basis of some case studies. The analysis takes into consideration the Currents' Physical Components (CPC) introduced by Professor Czarnecki in 1988 [1], the active and the reactive components of the p-q theory of the instantaneous power introduced by Professor Akagi and his coauthors in 1983 [2] and a modified variant of the Akagi's components proposed by the authors for the operation under unbalanced or nonsinusoidal conditions [3]. We propose four components of the current space-vector in terms of DC and AC components of the instantaneous active and reactive powers. The term of supplementary useless current vector is also pointed out. The analysis shows that the current decomposition which respects the definition of the instantaneous apparent power vector is useful for compensation reasons only if the supply voltages are sinusoidal. A modified definition of the components of the current is proposed for the operation under nonsinusoidal voltage conditions. This case studies based-analysis allows finding again some deficiencies that Professor Czarnecki has pointed out relating to Akagi's current decomposition [4], [5]. It is shown that the proposed active and reactive current components are identical to those of Czarnecki's CPC theory.

Key-Words: - Active current, Reactive current, Unless current, Unbalanced load, Nonsinusoidal conditions

1 Introduction

A wide variety of approaches have been proposed to decompose the current waveform into various components in the general case of nonsinusoidal conditions [4]–[7]. Especially, the decomposition into necessary and useless components is needed for the control of compensators such as active filters.
Making evident the components of the current, especially the active one, is an old concern of researchers in straight connection with the necessity of substantiation of performant methods for improving power factor. Thus, when total compensation is expected, the active filter has to provide the current vector

$$\underline{i}_F = \underline{i}_l - \underline{i}_a , \qquad (1)$$

where and ia are the load current vector and its active component.
In 1983, Akagi and his coauthors introduced the so-called "p–q theory" in three-phase, three-wire systems which was expected to be valid for any instantaneous variation of voltage and current [3]. This theory uses the complex space vector theory and introduces the concepts of instantaneous active power (p) and instantaneous reactive power (q). Then, the definitions of d and q-axis instantaneous active and reactive currents use only the instantaneous powers and the voltages in d-q coordinates [5]. Many extensions of the original p-q theory have been developed [8]–[10].
However, some conceptual limitations of this theory were pointed out by Willems in [11], [12]. Moreover, Professor L.S. Czarnecki from Louisiana State University has investigated how power phenomena and properties of three-phase systems are described and interpreted by the instantaneous p–q theory [13], [14]. The argumentation through which Czarnecki disagrees with the p-q theory is principally based on the relativity of the active and reactive character of the currents defined by Akagi and his followers. Czarnecki introduced his own current decomposition in 1988 [15]. These components are referred to as Current's Physical Components (CPC) and used as a tool for study [16]. Still, there are discussions about p–q and CPC theories [17], [18].
The decomposition of the currents proposed by the

authors avoids both terminology and interpretation ambiguities by practical examples.
Under nonsinusoidal voltage conditions, the proposed components of the current do not respect the definition of the complex apparent power introduced by Akagi.

2 p-q Theory and Currents' Components

The instantaneous apparent complex power for three-phased system is defined by voltage space vector and the complex conjugate current space vector [19]:

$$\underline{s} = p + jq = \frac{3}{2}\underline{u} \cdot \underline{i}^{*} \qquad (2)$$

The expression (1) allows expressing the current space-vector by

$$\underline{i} = i_d + ji_q = \frac{2}{3}\frac{1}{u_d^2 + u_q^2}\underline{u} \cdot \underline{s}^{*}. \qquad (3)$$

H.Akagi proposed to compensate the AC components of the real and imaginary parts (p and q) of $\underline{s}$, that is to calculate the reference compensation currents on the basis of expression (3) [2]. By expressing the scalar product in (3), the following expression is obtained:

$$\underline{i} = \frac{2}{3}\frac{1}{|\underline{u}|^2}\left[pu_d + qu_q + j\left(-qu_d + pu_q\right)\right]. \qquad (4)$$

On this basis, Akagi, Nabae and their co-authors (1993) [5] defined the following components of the current:

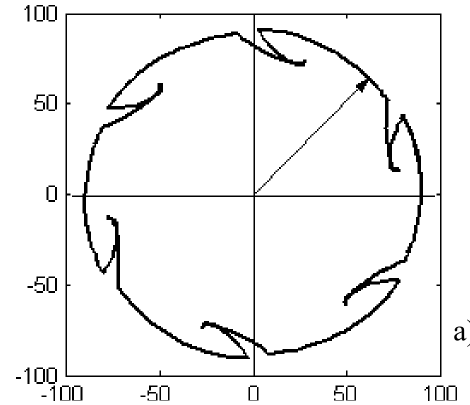
1. The active current vector (ia), whose components are:

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}p; i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}p; \qquad (5)$$

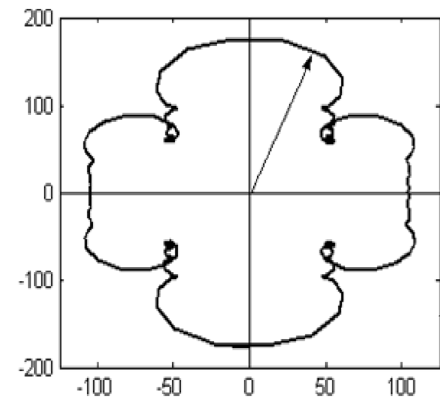
2. The reactive current vector (ir), whose components are:

$$i_{rd} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}q; i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}q. \qquad (6)$$

In the above three expressions, $|\underline{u}|^2$ is the square of voltage space-vector modulus. It should be noticed that the voltage space-vector modulus is not time-dependent only if the supply voltages are sinusoidal. This aspect is pointed out by the space-vector trajectories of the voltages for two practical cases: the secondary of a transformer which supplies a DC motor via a three-phase full controlled rectifier (Fig. 1a) and the output of a three-phase voltage source inverter (Fig. 1b).



a)



b)

Fig. 1 Geometric place of the voltage space vector modulus: to input of a three-phase bridge controlled rectifier - a) and to output of a voltage inverter - b)

3 Currents' Physical Components (CPC) Power Theory for Three-Phase Circuits

Unlike the generalized theory of the instantaneous power, the main concern in developing the Currents' Physical Components theory was the connection with power phenomena. Power properties in three-phase circuits under sinusoidal conditions are determined by the load features and the following independent phenomena [1]:
1. the permanent energy transmission between the supply network and load is associated with the active power;
2. the reactive components of the load generate the reactive power;
3. the unbalanced loads generate an asymmetrical supply current.
The main goal of CPC's theory is to decompose the load current into orthogonal components associated with different power phenomena. As the circuits' complexity is increasing, the number of the different phenomena is also increasing at the same time.

Consequently, the complexity of CPC's theory is increasing. A three-phase linear time-invariant load feed by a sinusoidal voltages system of positive sequence can be equivalent to a load of equivalent conductance Ge with respect to the active power P at the same voltage. So, the equivalent conductance of the three-phase load can be expressed as [1]

$$G_e = \frac{P}{\|\mathbf{u}_R\|^2 + \|\mathbf{u}_S\|^2 + \|\mathbf{u}_T\|^2} = \frac{P}{\|\mathbf{u}\|^2} \tag{7}$$

On the other hand, each three-phase load feed by a three-phase voltage supply can be equivalent to a delta structure with respect to the line current. The active power of such a load is [1]

$$P = \mathrm{Re}\{\mathbf{Y}_{RS} + \mathbf{Y}_{ST} + \mathbf{Y}_{Tr}\}\|\mathbf{u}\|^2 . \tag{8}$$

The term $\mathbf{Y}_{RS} + \mathbf{Y}_{ST} + \mathbf{Y}_{TR} = \mathbf{Y}_e = G_e + jB_e$ is named the equivalent admittance of the three-phase load. Its real part is the equivalent conductance Ge and the imaginary part is the equivalent susceptance Be.

The line current equivalent to the resistive load is the active current given by the expression [1]

$$\mathbf{i}_a = \begin{bmatrix} i_{Ra} \\ i_{Sa} \\ i_{Ta} \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\begin{bmatrix} G_e\mathbf{U}_R \\ G_e\mathbf{U}_S \\ G_e\mathbf{U}_T \end{bmatrix} = \sqrt{2}\,\mathrm{Re}\{G_e\mathbf{U}e^{j\omega t}\} \tag{9}$$

It is the minimum current of the load needed to provide permanent energy transmission. As the remaining current component (i-ia) does not contribute to permanent energy transmission, it is the useless current component which contributes to increasing the rms value of the supply current. Two different components of the current can be distinguished in the useless current. The former (ir) exists when the equivalent susceptance of the load (Be) is not null and can be expressed as [1]

$$\mathbf{i}_r = \sqrt{2}\,\mathrm{Re}\{jB_e\mathbf{U}e^{j\omega_1 t}\} . \tag{10}$$

A reactive power [1],

$$Q = -\mathrm{Im}\{\mathbf{Y}_{RS}\|\mathbf{u}_{RS}\|^2 + \mathbf{Y}_{ST}\|\mathbf{u}_{ST}\|^2 + \mathbf{Y}_{TR}\|\mathbf{u}_{TR}\|^2\} \tag{11}$$

is associated with such a situation, where YRS, YST and YTR are the line-to-line admittances of the load. The latter component of the useless current (i_u) [1],

$$\mathbf{i}_u = \sqrt{2}\,\mathrm{Re}\{\mathbf{A}\mathbf{U}^{\#}e^{j\omega_1 t}\} \tag{12}$$

is associated with the load imbalance and consequently it is called the unbalanced current.

This component occurs in the load current only if the negative sequence phasor associated with line admittances (A),

$$\mathbf{A} = -\left(\mathbf{Y}_{ST} + \alpha\mathbf{Y}_{TR} + \alpha^*\mathbf{Y}_{RS}\right) \tag{13}$$

is not null. The vector $\mathbf{U}^{\#}$ corresponds to the negative sequence voltages, respectively

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_R \\ \mathbf{U}_S \\ \mathbf{U}_T \end{bmatrix}, \quad \mathbf{U}^{\#} = \begin{bmatrix} \mathbf{U}_R \\ \mathbf{U}_T \\ \mathbf{U}_S \end{bmatrix}. \tag{14}$$

Consequently, the supply current in three-phase systems with sinusoidal voltage can be decomposed into three physical components, the active, reactive and unbalanced currents,

$$\mathbf{i} = \mathbf{i}_a + \mathbf{i}_r + \mathbf{i}_u . \tag{15}$$

4 Correct Interpretation of p-q Theory

In order to obviate the ambiguities generated by Akagi's current components, a possible decomposition of the current space-vector takes into account the DC components and the AC components of the instantaneous powers p and q. Thus, expression (4) becomes

$$\underline{i} = \frac{2}{3}\frac{1}{|\underline{u}|^2}\begin{bmatrix} (P + p_\sim)u_d + (Q + q_\sim)u_q + \\ + j\left(-(Q + q_\sim)u_d + (P + p_\sim)u_q\right) \end{bmatrix}. \tag{16}$$

Starting from this expression, the following current space-vectors are defined.

1. The active current vector (ia), whose components are

$$i_{ad} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}P; \quad i_{aq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}P . \tag{17}$$

2. The reactive current vector (ia), whose components are

$$i_{rd} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}Q; \; i_{rq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}Q . \tag{18}$$

3. The supplementary useless current vector on account of p~ (isp), whose components are

$$i_{spd} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}p_\sim ; \; i_{sqp} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}p_\sim . \tag{19}$$

4. The supplementary useless current vector on account of q~ (isq), whose components are

$$i_{sdq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}q_\sim ; \; i_{sqq} = -\frac{2}{3}\frac{u_d}{|\underline{u}|^2}q_\sim . \tag{20}$$

It is also possible to define the total supplementary useless current vector (is) as a sum of the two supplementary useless current vectors. Thus, its components are

$$i_{sd} = \frac{2}{3}\frac{u_d p_\sim + u_q q_\sim}{|\underline{u}|^2}; \; i_{sq} = \frac{2}{3}\frac{u_q p_\sim - u_d q_\sim}{|\underline{u}|^2} \tag{21}$$

It is easy to see that the moduli of above vectors comply with the next orthogonality condition

$$\left|\underline{i_a} + \underline{i_{sp}}\right|^2 + \left|\underline{i_r} + \underline{i_{sq}}\right|^2 = \left|\underline{i}\right|^2 . \tag{22}$$

So,

$$\left|\underline{i_a} + \underline{i_{sp}}\right|^2 + \left|\underline{i_r} + \underline{i_{sq}}\right|^2 = \frac{4}{9}\frac{1}{\left|\underline{u}\right|^2} \times$$
$$\left[\left(P + p_\sim\right)^2 + \left(Q + q_\sim\right)^2\right] = \frac{4}{9}\frac{\left|\underline{s}\right|^2}{\left|\underline{u}\right|^2} = \frac{\left|\underline{u}\right|^2\left|\underline{i}\right|^2}{\left|\underline{u}\right|^2} = \left|\underline{i}\right|^2 \tag{23}$$

As far as sum of ia, ir and is moduli are concerned, we have found that

$$\left|\underline{i_a}\right|^2 + \left|\underline{i_r}\right|^2 + \left|\underline{i_s}\right|^2 = \frac{4}{9}\frac{1}{\left|\underline{u}\right|^2}\left(P^2 + Q^2 + p_\sim^2 + q_\sim^2\right). \tag{24}$$

By integrating (22), we get

$$\frac{1}{2\pi}\int_0^{2\pi}\left[\left|\underline{i_a}\right|^2 + \left|\underline{i_r}\right|^2 + \left|\underline{i_s}\right|^2\right]d(\omega t) = $$
$$= \frac{4}{9}\frac{1}{2\pi}\int_0^{2\pi}\frac{1}{\left|\underline{u}\right|^2}\left(P^2 + Q^2 + p_\sim^2 + q_\sim^2\right)d(\omega t) . \tag{25}$$

Taking into account that

$$\frac{1}{2\pi}\int_0^{2\pi}\left(P^2 + Q^2 + p_\sim^2 + q_\sim^2\right)d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi}\left[P^2 + \right.$$
$$\left. + Q^2 + p_\sim^2 + q_\sim^2 + 2Pp_\sim + 2Qq\right]d(\omega t) = \frac{9}{4}\left|\underline{s}\right|^2 ,$$

we found that the rms values of the current components moduli are mutually ortogonal only under sinusoidal voltage operation, i.e.

$\left|\underline{u}\right| = \text{Constant}.$

If the voltages waveform is not sinusoidal, then |u| is not constant and, consequently,

$$I_a^2 + I_r^2 + I_s^2 = I^2 - \frac{4}{9}\frac{1}{2\pi}\int_0^{2\pi}\frac{2\left(Pp_\sim + Qq_\sim\right)}{\left|\underline{u}\right|^2}d(\omega t)$$

So, $I_a^2 + I_r^2 + I_s^2 \neq I^2$ because of

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{2\left(Pp_\sim + Qq_\sim\right)}{\left|\underline{u}\right|^2}d(\omega t) \neq 0 .$$

5 Case Studies

5.1 Three-phase sinusoidal voltage system with single-phase purely resistive load

Let us consider the first illustration given by Czarnecki in [13]. It is about an ideal D/Y transformer of ratio 1:1 which supplies a single-phase purely resistive load, connected as shown in Fig. 2. The R-phase instantaneous voltage of the positive sequence voltage system is $u_R = \sqrt{2}U\cos\omega_1 t, U = 120V$ and the load resistance is of 2Ω..

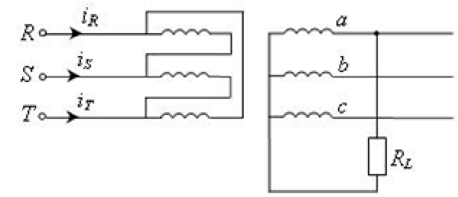


Fig. 2 - Three-phase circuit with single-phase purely resistive load

Professor Czarnecki expressed:

- the line currents in the primary of the transformer, $i_R = \sqrt{2}I\cos(\omega_1 t + 30^0) = -i_S$; $i_T = 0$; I=103,9A;
- the supply current in the α and β coordinates,

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} \sqrt{3}I\cos\left(\omega_1 t + 30^0\right) \\ -I\cos\left(\omega_1 t + 30^0\right) \end{bmatrix}; \tag{26}$$

- the instantaneous power p and q,

$$p = \sqrt{3}UI\left[1 + \cos 2\left(\omega_1 t + 30^0\right)\right];$$
$$q = -\sqrt{3}UI\sin 2\left(\omega_1 t + 30^0\right) . \tag{27}$$

Then, according to the definition relations (5) and (6) introduced by Akagi, the instantaneous active and reactive components of the current in (α, β) coordinates have been expressed. Finally, the active and reactive currents in the line R (Fig. 3) have been obtained through the inverse Clarke Transform:

$$i_{Ra} = \sqrt{\frac{2}{3}}I\left[1 + \cos 2\left(\omega_1 t + 30^0\right)\right]\cos\omega_1 t ; \tag{28}$$

$$i_{Rr} = \sqrt{\frac{2}{3}}I\sin 2\left(\omega_1 t + 30^0\right)\sin\omega_1 t . \tag{29}$$

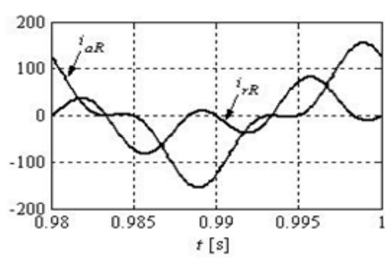


Fig. 3 The Akagi's current components in line R

On this basis, two consequences which disagree with physical phenomena have been pointed out:

1. The active current given by (28) is non sinusoidal and contains the third harmonic even the supply voltage is sinusoidal and the active power transfer is achieved only on the fundamental frequency.
2. Even if the load is purely resistive, there is a

reactive component of the current (29).
Consequently, the current component given by (28) can not be the active one and the reactive current has nothing in common with expression (29).
In order to apply the proposed definitions to this case study, the DC and AC components of the real imaginary parts of the complex apparent power (i.e. P, p~, Q and q~) must be pointed out.
These components are:

$P=\sqrt{3}UI;\quad p_{\sim}=\sqrt{3}UI\cos 2\left(\omega_1 t+30^0\right)$;

$Q=0$; $q_{\sim}=-\sqrt{3}UI\sin 2\left(\omega_1 t+30^0\right)$.

Thus, the proposed components of the current can be expressed from (17)-(21) as follows:
- the active current in (α, β) coordinates,

$$\begin{bmatrix} i_{a\alpha} \\ i_{a\beta} \end{bmatrix}=\begin{bmatrix} I\cos\omega_1 t \\ I\sin\omega_1 t \end{bmatrix}; \tag{30}$$

- the supplementary useless current in (α, β) coordinates,

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}=\begin{bmatrix} I\cos\left(3\omega_1 t+30^0\right) \\ I\sin\left(3\omega_1 t+30^0\right) \end{bmatrix}; \tag{31}$$

- the active current in the phase coordinates,

$$\begin{bmatrix} i_{aR} \\ i_{aS} \\ i_{aT} \end{bmatrix}=\begin{bmatrix} \sqrt{2}\dfrac{I}{\sqrt{3}}\cos\omega_1 t \\ \sqrt{2}\dfrac{I}{\sqrt{3}}\cos\left(\omega_1 t-120^0\right) \\ \sqrt{2}\dfrac{I}{\sqrt{3}}\cos\left(\omega_1 t+120^0\right) \end{bmatrix}. \tag{32}$$

The reactive current is null. Therefore, the proposed expressions for the active and reactive components of the currents lead to the same results as CPC's theory. As regards the Czarnecki's unbalanced current (18), it corresponds to the supplementary useless current introduced by (28). In this way, it results that the supplementary useless currents exist on account of load unbalance. After their compensation, the supply currents are composed of only active components, i.e. there is a symmetrical balanced three-phase current system that corresponds of the optimal energetic regime.

5.2 Three-phase sinusoidal voltage system with single-phase purely inductive load

Let us consider the second illustration given by Czarnecki in [8]. The only difference between this case study and the previous one is the character of the single-phase load. A purely inductive load of $X_L=2\Omega$ is considered this time. Similar to previous example, Professor Czarnecki found the line currents in the primary of the transformer,

$i_R=\sqrt{2}I\cos(\omega_1 t-60^0)=-i_S$; $i_T=0$; I=103,9A;

and the active current in the line R, by Akagi's relations

$$i_{Ra}=\frac{I}{\sqrt{6}}\left[\cos\left(\omega_1 t-30^0\right)+\cos\left(3\omega_1 t-30^0\right)\right]. \tag{33}$$

These results allowed pointing out two consequences of Akagi's p-q theory which disagree with physical phenomena:
1. The active current (33) is non sinusoidal, i.e. it has a different shape related to supply voltage.
2. An active current occurs even in purely reactive circuits.
Certainly, as an active power does not exist, the active component of the current must be zero and consequently, the current given by (33) cannot be an active current. To calculate the proposed components of the current, the DC and AC components of instantaneous powers p and q must be expressed. So, these components are:

$P=0$; $p_{\sim}=\sqrt{3}UI\cos\left(2\omega_1 t-30^0\right)$;

$Q=-\sqrt{3}UI$; $q_{\sim}=-\sqrt{3}UI\sin\left(2\omega_1 t-30^0\right)$.

By using (17)-(21), the following currents are obtained:
- the active current in (α, β) coordinates is null;
- the reactive current in (α, β) coordinates,

$$\begin{bmatrix} i_{r\alpha} \\ i_{r\beta} \end{bmatrix}=\begin{bmatrix} -I\sin\omega_1 t \\ I\cos\omega_1 t \end{bmatrix}; \tag{34}$$

- The supplementary useless current in (α, β) coordinates,

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix}=\begin{bmatrix} I\cos\left(3\omega_1 t-30^0\right) \\ I\sin\left(3\omega_1 t-30^0\right) \end{bmatrix}; \tag{35}$$

- the reactive current in the phase coordinates,

$$\begin{bmatrix} i_{rR} \\ i_{rS} \\ i_{rT} \end{bmatrix}=\begin{bmatrix} -\sqrt{2}\dfrac{I}{\sqrt{3}}\sin\omega_1 t \\ -\sqrt{2}\dfrac{I}{\sqrt{3}}\sin\left(\omega_1 t-120^0\right) \\ -\sqrt{2}\dfrac{I}{\sqrt{3}}\sin\left(\omega_1 t+120^0\right) \end{bmatrix}. \tag{36}$$

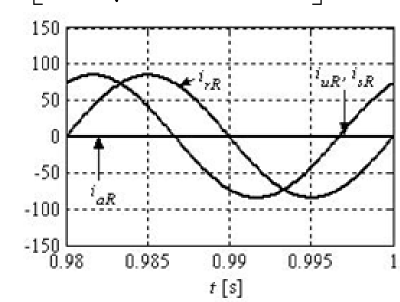


Fig. 4 The identical current components of CPC's and proposed theories in line R for purely inductive load

The above equations show again that the proposed expressions for the current decomposition lead to the same results as CPC's theory, i.e. the reactive currents are sinusoidal, symmetrical and balanced (like the supply voltages) and the active currents are zero because of purely inductive load (Fig. 4).

5.3 Three-phase sinusoidal voltage system with unbalanced load without active and reactive powers

In this case study, the unbalanced load is formed by connecting an inductance and a capacitance of equal impedances $X_L = X_C = 2\Omega$, between R and S phases and N. This time, the line currents are symmetrical. The waveforms of the line current components calculated through the three methods (Fig. 5) allow us to emphasize some aspects.

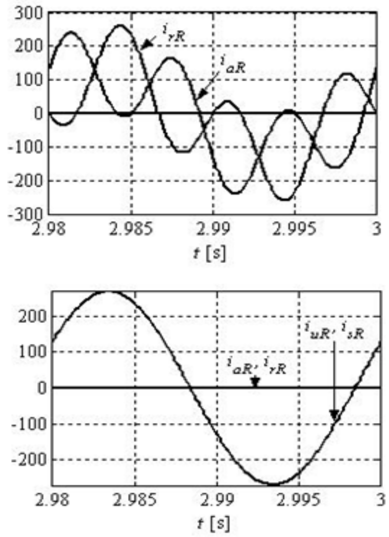


Fig. 5 The Akagi's current components and identical current components of CPC's and proposed theories in line R

- According the CPC's theory, as the equivalent admittance of the load given by (13) is zero, the active and reactive currents do not exist. Therefore, the unbalanced components of the currents coincide with the real supply currents.
- In spite of zero active power (P), in accordance with Akagi's p-q theory there is a nonzero active current (Fig. 9b). It means that there is not a relationship between the active power and Akagi's active current.
- Similarly, Akagi's reactive current occurs in line supply current in spite of zero reactive power.
- In accordance with the proposed decomposition, only the supplementary component of the current occurs in the supply line current. Both active and reactive components are zero.

5.4 Sinusoidal Voltages and Nonsinusoidal Currents

Let us consider the thee-phase system with sinusoidal voltages and nonsinusoidal currents in the primary of a D/Y transformer which supplies a DC motor via a full controlled rectifier. The waveforms of phase voltage and distorted current for a control angle of 30° are shown in Fig. 6.

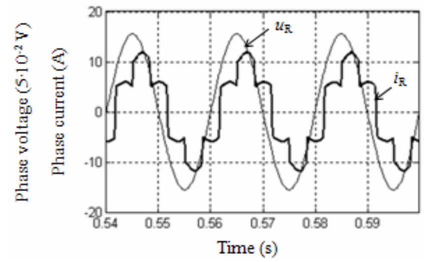


Fig. 6 Phase voltage and current in the primary of the transformer

As it can be seen in Fig. 7, the proposed active current waveform is sinusoidal, unlike the active current defined by Akagi (Fig. 8), although the both currents are in phase with the phase voltage.

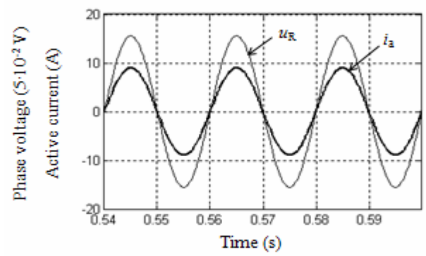


Fig. 7 Phase voltage and proposed active current waveforms

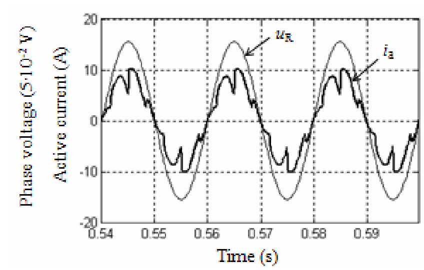


Fig. 8 Phase voltage and active current defined by Akagi

This happens because the Akagi's active current contains both the proper active component and the distortion component.

As regards the proposed reactive component of the current, it has a sinusoidal shape like the active component, but shifted by 90° behind the voltage, as expected (Fig. 9).

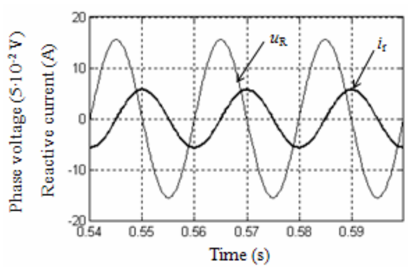


Fig. 9 Phase voltage and proposed reactive current waveforms

Although the Akagi's reactive current expressed by (6) lags the voltage by 90°, it is much distorted owing to its components which characterize the nonsinusoidal conditions (Fig. 10).

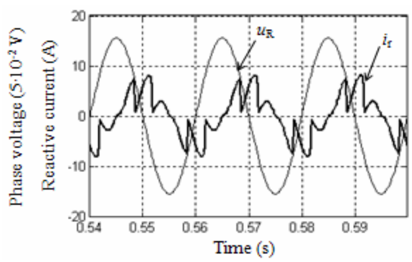


Fig. 10 Phase voltage and reactive current defined by (6)

As the active power transfer is achieved only on the fundamental frequency in the case of sinusoidal voltage conditions, the components of the current introduced by (5) and (6) have nothing in common with the meaning of the active and reactive currents as used in electrical engineering [1], [8], [10], [11]. Obviously, the trajectories of the active and reactive current space-vectors are circles only with the proposed definitions (Fig. 11).

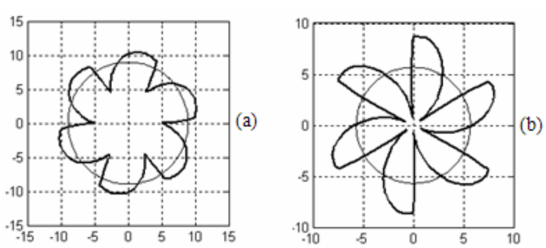


Fig. 11 Space-vector trajectories of the active current (a) and reactive current (b) defined by Akagi (thin line) and proposed (thick line) by (8) and (9)

The total supplementary useless current according to (12) and its vector locus are shown in Fig. 12 and Fig. 13.

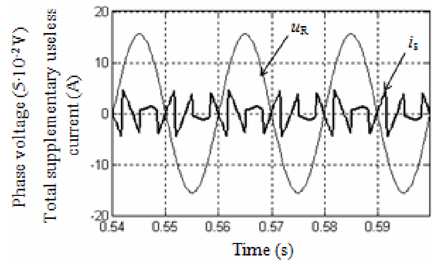


Fig. 12 Phase voltage and total supplementary useless current

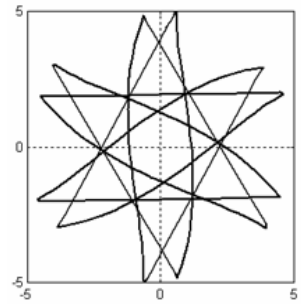


Fig. 13 Space-vector trajectory of the supplementary useless current

5.5 Nonsinusoidal Voltages and Balanced Resistive Load

As voltages in the secondary of the transformer have low distortion level, we have chosen another case study to serve as a model to current decomposition. A three-phase balanced resistive load of R = 2 Ω is supplied by a three-phase nonsinusoidal voltage system as follows:

$$u_R = \sqrt{2}(100 \sin \omega t + 50 \sin 5\omega t);$$

$$u_S = \sqrt{2}(100 \sin(\omega t - 2\pi/3) + 50 \sin 5(\omega t - 2\pi/3));$$

$$u_T = \sqrt{2}(100 \sin(\omega t + 2\pi/3) + 50 \sin 5(\omega t + 2\pi/3)).$$

The waveforms of phase voltage and supply current are both nonsinusoidal but they are in-phase (Fig. 14).

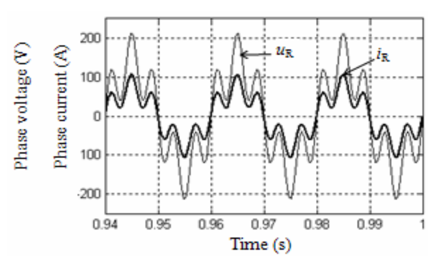


Fig. 14 Nonsinusoidal supply voltage and current waveforms in the case of purely resistive load

As it can be seen, the active current, as defined by (8), is substantially different in shape compared to the supply voltage even in the case of linear load (Fig. 15).

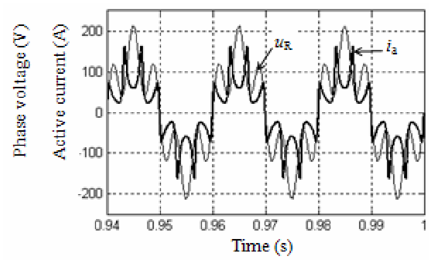


Fig. 15 Nonsinusoidal supply voltage and active current, as defined by (8), in the 5.5 case

This situation is generated by the fact that the square of voltage vector modulus in active current component definition is time-dependent (Fig. 16).

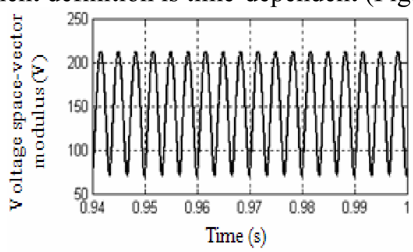


Fig. 16 Evolution of the voltage vector modulus

As expected, the reactive component of the current does not exist and the supplementary useless current is shown in Fig. 17.

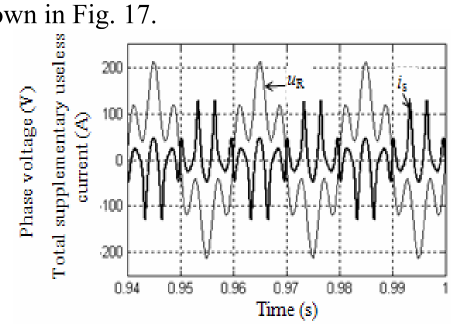


Fig. 17 Total supplementary useless current, as defined by (12), related to supply voltage in the case of linear balanced load

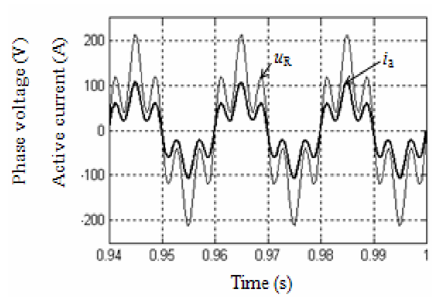


Fig. 18 Akagi's active current related to supply voltage in the case of linear balanced load

On the other hand, the linear character of the balanced load makes the Akagi's current defined by (5) have the same waveform as the supply voltage in this particular situation (Fig. 18).

5.6 Nonsinusoidal Voltages and Balanced Nonlinear Load

In this example, a series RL load of R = XL = 2 □ is supplied by the three-phase nonsinusoidal voltage system specified by (19). This time, the distorted current and voltage have different waveforms. Moreover, a delay of the supply current with respect to the supply voltage occurs in such a circuit (Fig. 19).

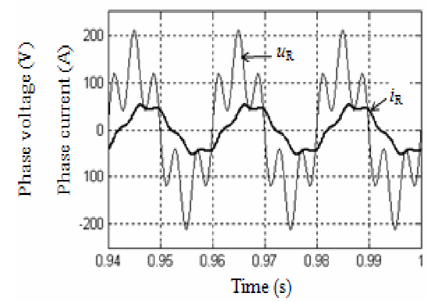


Fig. 19 Nonsinusoidal supply voltage and current waveforms in the case of RL load

The distorted active component of the current (Fig. 20), as defined by (8), has the following properties: its zero-passing coincide with the voltage zero-passing; it leads to an active power of 7.6 kW which is equal to the power consumed by the resistive component of the load; its rms value is of 29.4 A.

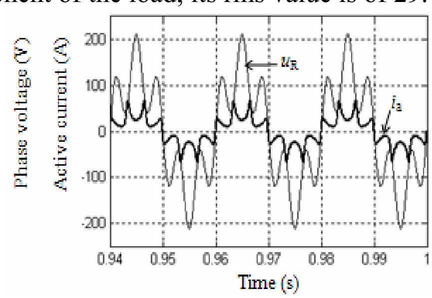


Fig. 20 Nonsinusoidal supply voltage and active current, as defined by (8), in the case of nonlinear balanced load

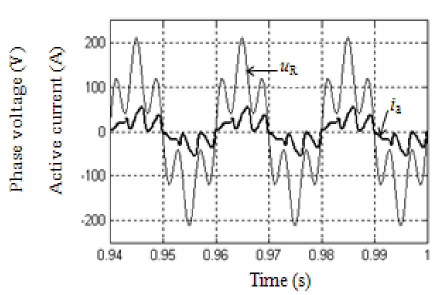


Fig. 21 Akagi's active current related to supply voltage in the case of nonlinear balanced load

The nonlinear character of the load makes the Akagi's active current be much distorted with respect to the supply voltage (Fig. 21), unlike the purely resistive load situation shown in Fig. 18.
As it can be seen in Fig. 22, the reactive current, as proposed by (9), lags the voltage by 90°.

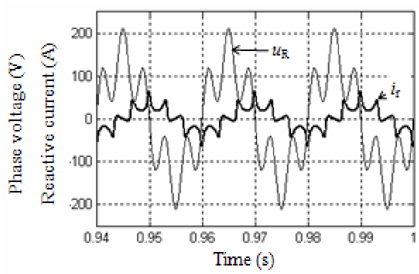


Fig. 22 Nonsinusoidal supply voltage and reactive current, as defined by (9), in the case of nonlinear balanced load

6 Conclusion

Taking into account the results obtained by analyzing the previous typical examples, we can make evident some concluding remarks with reference to decomposition of the nonsinusoidal current in three-phase, three-wire systems.

1. The Akagi's component of the current, as introduced by (5), can be an active one only if the load is linear and balanced.
2. In the case of an unbalanced load, the active and reactive currents defined by Akagi have nothing in common with the active and reactive powers. These limitations of Akagi's current decomposition were pointed out by Czarnecki who introduced the CPC's theory in accordance with the physical phenomena.
3. For a sinusoidal voltage supply, the active and reactive components of the current proposed in this paper through expressions (24) and (25) lead to the same results as CPC's theory irrespective of character of the load.
4. The supplementary component of the current defined by (28) coincides with the unbalanced current introduced by Czarnecki.
5. The proposed manner of interpretation based on p-q theory of instantaneous power removes Czarnecki's critical remarks related to Akagi's definitions and makes similar the two theories at least under sinusoidal voltage conditions.
6. The component of the current, as proposed by (8), can be the active one only under sinusoidal voltage conditions for both linear and nonlinear balanced load.
7. If the supply voltage system is not sinusoidal, the current proposed by (8) cannot be an active component. This result can be explained by the fact that the voltage vector modulus in the denominator has a time variation (Fig. 16). As a result, the harmonics spectrum of this component of the current is not the same with the voltage harmonics spectrum.

The result in these simple case studies allows us to conclude that the current components expressed by (8)–(12) are not useful for reference current calculation in active filtering if the voltages have not a sinusoidal shape.

Indeed, for the fifth case study, if the compensation is achieved by a parallel active filter and its reference current is distorted related to the supply voltage, the RMS value of the supply current is higher than the initial load current even if this new current provides the necessary active power, removes the AC component of the instantaneous active power ($p_{\sim}$) and has the same phase with the voltage. For example, in this case study, the RMS initial load current is exceeded by about 30% after compensation. Consequently, it is not a better solution.

In the last case study, the component of the current defined by (8) contains harmonics whose order is 6k+1. Clearly, such a current generates active power only on fundamental frequency, which explains the rms value of 29.4 A of this current.

8. In order to solve this aspect of the problem, we propose the replacement of $|\underline{u}|$ in (8)–(12) with its RMS value, i.e.

$$U^2 = \frac{1}{T}\int_{t-T}^{t} |\underline{u}|^2 dt \tag{34}$$

After this replacement, the new active, reactive and supplementary useless components of the current are:

$$i_{ad} = \frac{2}{3}\cdot\frac{u_d}{U^2}P; \quad i_{aq} = \frac{2}{3}\cdot\frac{u_q}{U}P; \tag{35}$$

$$i_{rd} = \frac{2}{3}\cdot\frac{u_q}{U^2}Q; \quad i_{rq} = -\frac{2}{3}\cdot\frac{u_d}{U^2}Q.; \tag{36}$$

$$i_{sd} = \frac{2}{3}\frac{u_d p_{\sim} + u_q q_{\sim}}{U^2}; \quad i_{sq} = \frac{2}{3}\frac{u_q p_{\sim} - u_d q_{\sim}}{U^2}.$$

It is obvious that the use of expression (35) for the active current calculation makes this current keep the voltage waveform. In the case of last case study, the active current calculated with (35) provides the required active power with only 22.8 A RMS value of this current (Fig. 23).

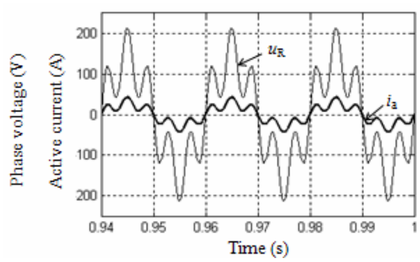


Fig. 23 Nonsinusoidal supply voltage and active current, as defined by (22), in the case of nonlinear balanced load

9. Undoubtedly, the proposed current decomposition based on complex apparent power vector is useful in the calculation of the reference current for active power filters. Thus, when total compensation is expected, the reference current requires only the load current and its active component.

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