Stability of Driving Systems with Induction Motors. A New Method of Analysis

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Abstract: - This paper analyzes the stability of the driving systems with induction motors by two methods.The first one consists in using the geometrical locus of the roots and the second one is an original method developed by authors. The paper details the mathematical models which are used, the results obtained by computer simulation and the conclusions obtained by carrying out the analysis.

Key-Words: - Driving system, stability, static converter, induction motor, simulation.

1 Introduction

The driving systems with induction motors involve, obviously, some dynamic regimes. These ones, at their turn, impose thorough analyses of the stability which confirm the possibility of their adequate operation.

In order to carry out an adequate study in this field, it is imposed to use an adequate mathematical model.

The model is optimum from the drive point of view when it generates an as simple as possible structure of the control system. In this situation, because of the great number of equations and parameters occurring in the case of the induction motors speed adjustment by modifying the supply frequency, some simplifications have been necessary.

But this fact leads to a model which does not reflect correctly the dynamic behaviour of the system.

So, it is imposed to use an as complete as possible model, materialized in a great number of differential equations.

For writing this mathematical model, which is to facilitate the study of the influence of the filtration circuit parameters of a voltage and frequency static converter on the stability of the VFSC-induction motor assembly, the structure of such a system will be taken into account; this structure is depicted in figure 1.

The notations have the following meaning:
- R - rectifier;
- F - filtration circuit;
- I - inverter;
- IR - industrial robot.

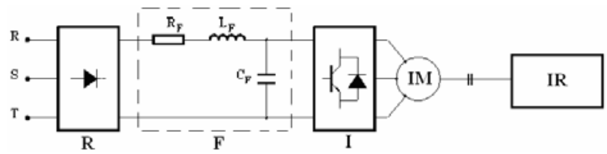


Fig. 1. Structure of driving system.

2 Operation equations of system

The mathematical model of the converter-induction motor assembly will be established further on. Thus, in accordance with references, the two axes voltage equations of a squirrel cage induction machine, written in operational, have the following matrix form:

$$\begin{vmatrix} U_{ds}(s) \\ U_{qs}(s) \\ 0 \\ 0 \end{vmatrix} =$$

$$= \begin{vmatrix} R_s + sL_s & -L_s\omega_s & sL_{sh} & -L_{sh}\omega_s \\ L_s\omega_s & R_s + sL_s & L_{sh}\omega_s & sL_{sh} \\ sL_{sh} & -L_{sh}\omega_r & R_r^{/} + sL_r^{/} & -L_r^{/}\omega_r \\ L_{sh}\omega_r & sL_{sh} & L_r^{/}\omega_r & R_r^{/} + sL_r^{/} \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{dr}^{/}(s) \\ I_{qr}^{/}(s) \end{vmatrix} \qquad (1)$$

where s is the Laplace operator.

The equation (1) gets the following form, by processing it adequately and by considering in addition that the stator voltage phasor is oriented after the real axis ($u_{qs} = 0$):

$$\begin{vmatrix} sI_{ds}(s) \\ sI_{qs}(s) \\ s\left[I_{ds}(s)+\frac{L_{sh}}{L_s}I'_{dr}\right] \\ s\left[I_{qs}(s)+\frac{L_{sh}}{L_s}I'_{qr}\right] \end{vmatrix} =$$

$$= \begin{bmatrix} -\dfrac{\frac{R_s}{L_s}+\frac{R'_r}{L'_r}}{1-\frac{L_{sh}^2}{L_sL'_r}} & \omega_r & \dfrac{\frac{R'_r}{L'_r}}{1-\frac{L_{sh}^2}{L_sL'_r}} & \dfrac{\omega}{1-\frac{L_{sh}^2}{L_sL'_r}} \\ -\omega_r & -\dfrac{\frac{R_s}{L_s}+\frac{R'_r}{L'_r}}{1-\frac{L_{sh}^2}{L_sL'_r}} & -\dfrac{\omega}{1-\frac{L_{sh}^2}{L_sL'_r}} & \dfrac{\frac{R'_r}{L'_r}}{1-\frac{L_{sh}^2}{L_sL'_r}} \\ -\frac{R_s}{L_s} & 0 & 0 & \omega_s \\ 0 & -\frac{R_s}{L_s} & -\omega_s & 0 \end{bmatrix} \cdot$$

$$\cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{ds}(s)+\frac{L_{sh}}{L_s}I'_{dr} \\ I_{qs}(s)+\frac{L_{sh}}{L_s}I'_{qr} \end{vmatrix} + \begin{vmatrix} \dfrac{1}{L_s\left(1-\frac{L_{sh}^2}{L_sL'_r}\right)} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} U_{ds}(s) \qquad (2)$$

The following notations will be used further on:
- time constant of the stator winding;
- time constant of the rotor winding;
- leakage coefficient of the machine windings;

$$I_{df}(s) = I_{ds}(s) + \frac{L_{sh}}{L_s} I'_{dr}(s)$$

$$I_{qf}(s) = I_{qs}(s) + \frac{L_{sh}}{L_s} I'_{qr}(s) \qquad (3)$$

The relation (2) becomes with these notations:

$$s \cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} =$$

$$= \begin{vmatrix} -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \frac{-\omega}{\sigma} & \frac{1}{T_r\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 \end{vmatrix} \cdot$$

$$\cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\sigma} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} \cdot U_{ds}(s) \qquad (4)$$

The motion equation is added to this system:

$$\frac{J}{p} \cdot \frac{d\omega}{dt} = M + M_m \qquad (5)$$

where

$$M = \frac{3}{2} pL_{sh}(i_{qs}i'_{dr} - i_{ds}i'_{qr}) \qquad (6)$$

There result from (3):

$$I'_{dr}(s) = \frac{L_s}{L_{sh}}\left[I_{df}(s) - I_{ds}(s)\right]$$

$$I'_{qr}(s) = \frac{L_s}{L_{sh}}\left[I_{qf}(s) - I_{qs}(s)\right] \qquad (7)$$

The following relation results by applying Laplace transformation to the equation (5) and by replacing then and given by (7):

$$\frac{J}{p} s\omega - M_m = \frac{3}{2} pL_s\left[I_{qs}(s)I_{df}(s) - I_{ds}(s)I_{qf}(s)\right] \qquad (8)$$

The filtration circuit will be simulated further on.

This one is described by the following equations (in operational):

$$U_R(s) = (R_F + sL_F)\cdot I_R(s) + U_F(s)$$

$$I_R(s) = I_F(s) + C_F \cdot sU_F(s) \qquad (9)$$

The previous system can be also written in the form:

$$s\begin{vmatrix} U_F(s) \\ I_R(s) \end{vmatrix} = \begin{vmatrix} 0 & \dfrac{1}{C_F} \\ -\dfrac{1}{L_F} & -\dfrac{R_F}{L_F} \end{vmatrix} \cdot \begin{vmatrix} U_F(s) \\ I_R(s) \end{vmatrix} + $$

$$+\begin{vmatrix} -\dfrac{1}{C_F} \\ 0 \end{vmatrix} \cdot I_F(s) + \begin{vmatrix} 0 \\ \dfrac{1}{L_F} \end{vmatrix} \cdot U_R(s) \qquad (10)$$

It will be also considered that the inverter behaves like a common amplifier without delay time, characterized by the modulation factor k_m :

$$k_m = \frac{U_{ds}(s)}{U_F(s)} \qquad (11)$$

The relation (12) can be written by considering that is always equal to zero and by replacing adequately in accordance with the relation (11):

$$I_R(s) \cong I_F(s) = k_m I_{ds}(s) \qquad (12)$$

In conclusion, the mathematical model of the converter-induction machine assembly has the following form, by taking into account the relations (4), (8) and (10):

$$s\begin{vmatrix} \Delta I_{ds}(s) \\ \Delta I_{qs}(s) \\ \Delta I_{df}(s) \\ \Delta I_{qf}(s) \\ \Delta U_F(s) \\ \Delta I_R(s) \\ \Delta\omega(s) \end{vmatrix} =$$

$$= \begin{vmatrix}
-(\frac{1}{T_s}+\frac{1}{T_r})\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} & \frac{k_m}{L_{s\sigma}} & 0 & \frac{I_{qf0}}{\sigma} \\
-\omega_r & -\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & -\frac{\omega}{\sigma} & \frac{1}{T_r\sigma} & 0 & 0 & -\frac{I_{df0}}{\sigma} \\
-\frac{1}{T_s} & 0 & 0 & \omega_s & \frac{k_m}{L_s} & 0 & 0 \\
0 & -\frac{1}{T_s} & -\omega_s & 0 & 0 & 0 & 0 \\
-\frac{k_m}{C_F} & 0 & 0 & 0 & 0 & \frac{1}{C_F} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{L_F} & -\frac{R_F}{L_F} & 0 \\
k_c I_{qf0} & -k_c I_{df0} & -k_c I_{qs0} & k_c I_{ds0} & 0 & 0 & 0
\end{vmatrix} \cdot$$

$$\cdot\begin{vmatrix} \Delta I_{ds}(s) \\ \Delta I_{qs}(s) \\ \Delta I_{df}(s) \\ \Delta I_{qf}(s) \\ \Delta U_F(s) \\ \Delta I_R(s) \\ \Delta\omega(s) \end{vmatrix} + \begin{vmatrix}
0 & I_{qsD} & \frac{U_{FD}}{L_{s\sigma}} \\
0 & -I_{dsD} & 0 \\
0 & I_{qfD} & \frac{U_{FD}}{L_s} \\
0 & -I_{dfD} & 0 \\
0 & 0 & -\frac{I_{dsD}}{C_F} \\
1 & 0 & 0 \\
0 & 0 & 0
\end{vmatrix} \cdot \begin{vmatrix} \Delta U_R(s) \\ \Delta\omega_r(s) \\ \Delta k_m(s) \end{vmatrix} \qquad (13)$$

where $k_c = -p^2 \cdot \dfrac{L_s}{J}$.

The system is therefore described by seven equations which can be written again in the known compact form:

$$s|Y(s)| = |A| \cdot |Y(s)| + |B| \cdot |U(s)|. \qquad (14)$$

where the input quantities are $\Delta U_R(s)$, $\Delta\omega(s)$ and $\Delta k_m(s)$.

3 Simulations results

A Matlab program has been conceived with the help of the mathematical model detailed before; its results are presented further on.

In order to emphasize the filtration capacitor influence on stability, a numerical analysis of the problem has been performed for several values of the capacitor .

Figure 2 has been obtained by running the computation program.

A few values of the seven roots corresponding to four values of the filtration capacitor are presented further on in order to emphasize the emerging conclusions.

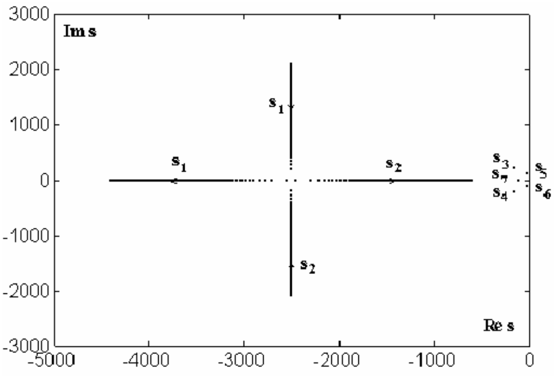


Fig. 2. Geometric locus of roots for C_F= 470 μF - 1,88 mF (step 5 μF).

Table 1. Roots corresponding to the filtration capacitor.

	C_F			
	470 μF	940μF	1,41 μF	1,88 mF
s_1	-2499,2 +2094,7i	-3465,2	-4144,4	-4394,9
s_2	-2499,2 -2094,7i	-1532,9	-853,6	-603,2
s_3	-155,5 +218,4i	-155,5 + 218,5i	-155,6 +218,6i	-155,5 +218,8i
s_4	-155,5 -218,4i	-155,5 -218,5i	-155,6 -218,6i	-155,5 -218,8i
s_5	-18,4 +119,2i	-18,4 +119,2i	-18,4 +119,2i	-18,5 +119,2i
s_6	-18,4 -119,2i	-18,4 -119,2i	-18,4 -119,2i	-18,5 -119,2i
s_7	-103,2	-103,2	-103,2	-103,2

Thus, while the roots s_3, s_4, s_5, s_6 and s_7 are almost "immobile", the roots s1 and s_2 displace (at the same time with the increase of the capacitor value) in accordance with the arrows. If its value does not exceed 850 μF the roots s1 and s_2 have constant real part, the system stability remaining practically unchanged. Over this value, the stability begin to decrease because of the fact that s_2 comes nearer the imaginary axis.

An increased value is imposed for C_F for having an as good as possible filtration of the rectified voltage. However, in the case of the studied converter, although there are reserved on the base board four seats for fixing the filtration capacitors, still only two capacitors of 470 μF have been connected in parallel (C_F=940 μF). One of the reasons for preferring this combination is (in accordance with the ones mentioned before) even the increased stability of the system for that case.

For studying the filtration inductivity influence the following results have been obtained (fig. 3 and table 2).

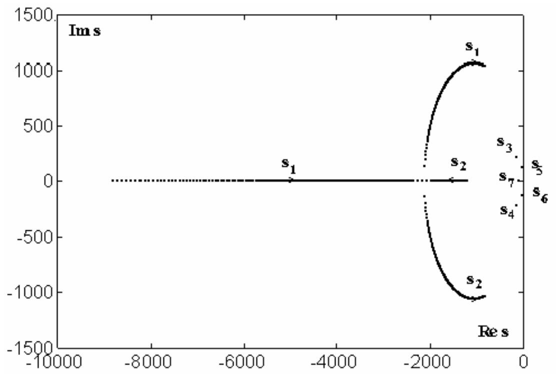


Fig. 3. Geometric locus of roots for L_F=0,1-0,6 mH (step 1 mH).

The studied system is always stable because all roots have negative real part.

In the previous representation the arrows show the sense for the filtration inductivity increase. In order to justify their senses further on we present the roots values for four particular values of the inductivity L_F.

Table 2. Roots corresponding to the inductivity L_F.

	L_F			
	0,1 mH	0,2 mH	0,3 mH	0,4 mH
s_1	-8789,7	-3465,2	-1665,8 + 876,0i	-1249,1 +1047,3i
s_2	-1208,4	-1532,9	-1665,8 - 876,0i	1249,1 -1047,3i
s_3	-155,5 + 218,5i	-155,5 + 218,5i	-155,5 + 218,5i	-155,5 + 218,5i
s_4	-155,5 - 218,5i	-155,5 - 218,5i	-155,5 - 218,5i	-155,5 - 218,5i
s_5	-18,4 + 119,2i	-18,4 + 119,2i	-18,4 + 119,2i	-18,4 + 119,2i
s_6	-18,4 - 119,2i	-18,4 - 119,2i	-18,4 - 119,2i	-18,4 - 119,2i
s_7	-103,2	-103,2	-103,2	-103,2

As one can notice from the previous table, the characteristic equation have five immobile roots in the complex plan (s_3, s_4, s_5, s_6 and s_7). The other two, depending on the filtration inductivity value, displace in accordance with the arrows indication (they come nearer the imaginary axis at the same time with its increase).

The emerging conclusion is the same as that one deduced in the case C_F=0, therefore when the filter inductivity value increases the system stability decreases.

The modulation factor influence is analyzed in the same way.

The following results have been obtained by doing as in the previous cases (fig. 4 and table 3).

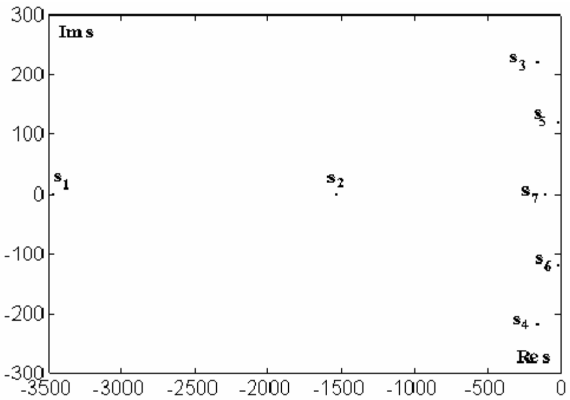


Fig. 4. Geometric locus of roots for k_m= 0-0,5 (step 0,01).

Table 3. Roots corresponding to k_m.

	k_m			
	0	0,2	0,4	0,5
s_1	-3464,8	-3465,0	-3465,5	-3465,9
s_2	-1535,2	-1534,3	-1531,4	-1529,3
s_3	-154,6 +218,4i	-155,0 +218,4i	-156,1 +218,5i	-157,0 +218,6i
s_4	-154,6 -218,4i	-155,0 -218,4i	-156,1 - 218,5i	-157,0 - 218,6i
s_5	-18,5 +119,5i	-18,5 +119,4i	-18,4 + 119,1i	-18,4 +118,9i
s_6	-18,5 -119,5i	-18,5 -119,4i	-18,4 -119,1i	-18,4 -118,9i
s_7	-103,1	-103,1	-103,2	-103,3

As it can be noticed, the modulation factor influence on stability is very small.

However, owing to the fact that at the same time with its increase the root s_2 comes nearer the imaginary axis (while the other roots are almost “fixed”), we can say that the studied system stability, in the mentioned situation, tends to decrease.

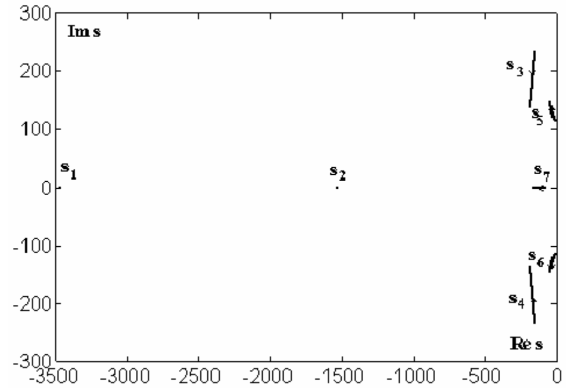


Fig. 5. Geometric locus of roots for $R_r^/$=4-10 Ω.

The arrows show the sense of the roots displacement in the case of the rotor winding increase, fact confirmed by data from table 4, too.

Table 4. Roots corresponding to $R_r^/$.

	$R_r^/$			
	3,5 Ω	5,5 Ω	7,5 Ω	8,5 Ω
s_1	-3464,8	-3464,8	-3464,8	-3464,8
s_2	-1535,2	-1535,2	-1535,2	-1535,2
s_3	-145,9 + 238,2i	-154,6 + 218,4i	-164,2 + 191,2i	-179,9 + 151,2i
s_4	-145,9 -238,2i	-154,6 -218,4i	-164,2 -191,2i	-179,9 -151,2i
s_5	-9,3 +114,9i	-18,5 +119,5i	-29,0 +127,5i	-38,7 +141,4i
s_6	-9,3 -114,9i	-18,5 -119,5i	-29,0 -127,5i	-38,7 -141,4i
s_7	-66,5	-103,1	-135,1	-156,4

The induction machine parameters influences on stability when considering the converter parameters too, can also be studied with the help of the relations (13). The conclusions are identical with the ones obtained by using the method shown in references.

The roots geometric locus when modifying the rotor winding resistance is depicted further on for exemplification.

The conclusion emerging from the analysis of these results is identical with the one from references, therefore at the same time with the increase of the rotor winding resistance the studied system stability also increases.

The characteristics depicted in the previous figures have been obtained for the case when: C_F=940 μF, L_F=0,3mH, f=50 Hz, U_{F0}=522 V, I_{ds0}=1,84 A, I_{qs0}=0 A (values measured on the converter CEGELEC VNTV 4004).

It has also been considered that the machine operates without load. This means that it can be considered that (by neglecting the rotor current components over the stator current components) I_{df0}=1,84 A, I_{qf0}=0 A.

4 A new analysis method of stability

The equations system that is used have the following form [5]:

$$\omega_s^* = s_{ks}(\underline{\Psi}_s^* - k\underline{\Psi}_r^{/*}) + \frac{d\underline{\Psi}_s^*}{dt^*} + j\omega_s^*\underline{\Psi}_s^* \tag{15}$$

$$0 = s_{kr}(\underline{\Psi}_r^{/*} - k\underline{\Psi}_s^*) + \frac{d\underline{\Psi}_r^{/*}}{dt^*} + j(\omega_s^* - \omega^*)\underline{\Psi}_r^{/*}$$

$$h \cdot \frac{d\omega^*}{dt^*} = -\frac{k}{x_{rt}^{/*}} \operatorname{Im}\left[(\underline{\Psi}_s^*)^* \underline{\Psi}_r^{/*}\right] - m_r^*$$

These equations are linearized further on.

In order to do this thing it is considered that the pulsation modifies in saltus with a very low value. This variation will lead implicitly to a voltage modification, in saltus too, with the same value, so that the two quantities ratio to remain constant.

In this hypothesis the system (1) will modify as follows.

$$\omega_s^* + \Delta\omega_s^* = s_{ks}\left[\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^* - k(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right] + \frac{d(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)}{dt^*} + j(\omega_s^* + \Delta\omega_s^*)(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*) \tag{16}$$

$$0 = s_{kr}\left[\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} - k(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)\right] + \frac{d(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})}{dt^*} + j(\omega_s^* + \Delta\omega_s^* - \omega^* - \Delta\omega^*)(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})$$

$$h \cdot \frac{d(\omega^* + \Delta\omega^*)}{dt^*} = -\frac{k}{x_{rt}^{/*}} \cdot \operatorname{Im}\left\{\left[(\underline{\Psi}_s^*)^* + \Delta(\underline{\Psi}_s^*)\right] \cdot (\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\right\} - m_r^*$$

By applying Laplace transformation to the first two equations of the systems (1) and (2), by subtracting member by member and by neglecting the products of the form $\Delta \cdot \Delta$, one obtains:

$$\Delta\omega_s^* = (s_{ks} + j\omega_s^* + s) \cdot \Delta\underline{\Psi}_s^* - s_{ks} \cdot k \cdot \Delta\underline{\Psi}_r^{/*} + j \cdot \underline{\Psi}_s^* \cdot \Delta\omega_s^* \tag{17}$$

$$0 = -s_{kr} \cdot k \cdot \Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} + j(\Delta\omega_s^* - \Delta\omega)\underline{\Psi}_r^{/*}$$

$$h\frac{d(\Delta\omega^*)}{dt} = -\frac{k}{x_{st}^*}\operatorname{Im}\left[\left(\underline{\Psi}_s^*\right) \cdot \Delta\underline{\Psi}_r^{/*} + \underline{\Psi}_r^{/*} \cdot \Delta\left(\underline{\Psi}_s^*\right)^*\right]$$

where the operational variable is noted with s.

It must also be noticed that for simplifying the writing and for not producing confusions, both in the previous relation and in the following ones, it has been given up both to indicate the quantities depending on s ($\Delta\omega_s^*(s)$, $\Delta\omega^*(s)$ etc.) and to note them with capitals.

If it is considered that $\Delta\omega^*$ is not less than 0,1 in the previous relations the following approximations may be made:

$$j\underline{\Psi}_s^* = 1 \quad \text{and} \quad j\underline{\Psi}_r^{/*} = k\,. \tag{18}$$

This way, the first two relations from (3) become:

$$0 = (s_{ks} + j\omega_s^* + s)\Delta\underline{\Psi}_s^* - s_{ks} \cdot k \cdot \Delta\underline{\Psi}_r^{/*}$$

$$k(\left(\Delta\omega^* - \Delta\omega_s^*\right) = -s_{kr} \cdot k \cdot \Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} \tag{19}$$

The analysis of these relations can be simplified if it is considered $R_s \cong 0$. But this simplifying hypothesis leads to satisfactory results only in the field $\omega_s^* \in (0{,}5 \div 1)$.

So it is imposed to analyze the situation when $R_s \neq 0$, but considering that the studied phenomenon is linearized.

For this, it is considered that the motor operated no-load before modifying the frequency. In this situation, owing to the low frequency of the rotor current, its active component may be neglected.

Thus, one can write:

$$\Delta\underline{i}_r^{/*} = \Delta i_{dr}^{/*} + j\Delta i_{qr}^{/*} \cong \Delta i_{dr}^{/*} = \frac{\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*}{dx_s^*}\,. \tag{20}$$

By solving the system (5) relatively to $\Delta\Psi_s^*$ and $\Delta\Psi_r^{/*}$ by replacing these relations in (6), after computations, it is obtained:

$$\Delta i_{dr}^{/*} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k(\Delta\omega^* - \Delta\omega_s^*) \tag{21}$$

where the following notation has been used:

$$\varepsilon = (1 - k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{/*}}\,. \tag{22}$$

When $\omega_s^* \geq 0{,}1$ it results that it can be considered (with approximation):

$$\left(\underline{\Psi}_s^*\right)^* = 1 \quad \text{and} \quad \underline{\Psi}_r^{/*} = -jk \tag{23}$$

In these conditions, by applying Laplace transformation to the relation (9), it is obtained:

$$hs \cdot \Delta\omega^* = -\frac{k}{x_{st}^*}\operatorname{Re}(\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*)\,, \tag{24}$$

or, equivalently

$$hs \cdot \Delta\omega^* = -\frac{k}{x_{st}^*}\operatorname{Re}(\Delta\underline{\Psi}_{dr}^{/*} - k\Delta\underline{\Psi}_{ds}^*)\,, \tag{25}$$

respectively

$$hs \cdot \Delta\omega^* = -k\Delta i_{dr}^{/*}\,. \tag{26}$$

5 Quantitative Results

Further on, for the study of the induction motor stability, the relations (7) and (12) established before are used. The first relation can be written in the form:

$$\Delta\omega^* = -\frac{k}{hs}\cdot \Delta i_{dr}^{/*} \Leftrightarrow \Delta\omega^* = G_1(s)\cdot \Delta i_{dr}^{/*}, \qquad (27)$$

with

$$G_1(s) = -\frac{k}{hs} \qquad (28)$$

The second relation is processed analogously:

$$\Delta i_{dr}^{/*} = G_2(s)\cdot(\Delta\omega_s^* - \Delta\omega^*), \qquad (29)$$

where

$$G_2(s) = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot k \qquad (30)$$

By using (13) and (15) the following configuration can be drawn.

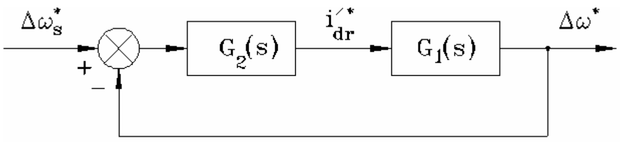
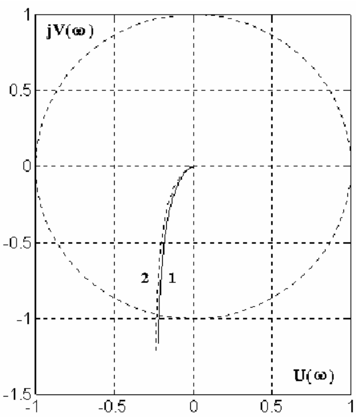


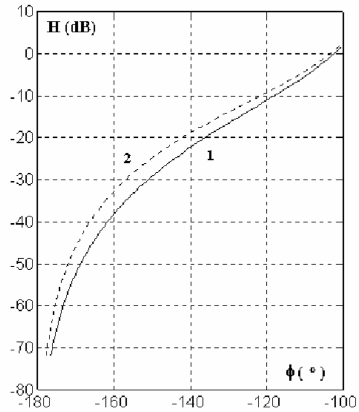
Fig. 6. Block scheme of the machine in the mentioned situation.

Further on it is possible to pass to the stability study in our concrete case by using all these introductive notions. This analysis will be made with the help of a Matlab program achieved on the basis of the scheme depicted in the figure 1 and of the relations (13), (14), (15) and (16).

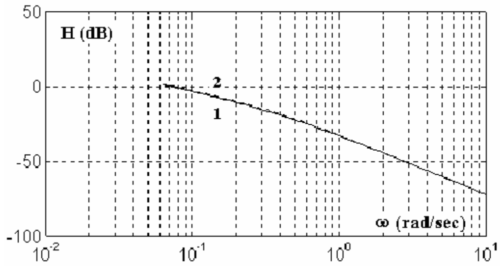
By running this program the following graphics have been obtained.



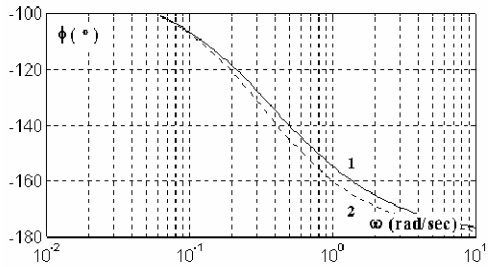
a) Transfer locus.



b) Amplitude – phase characteristics.

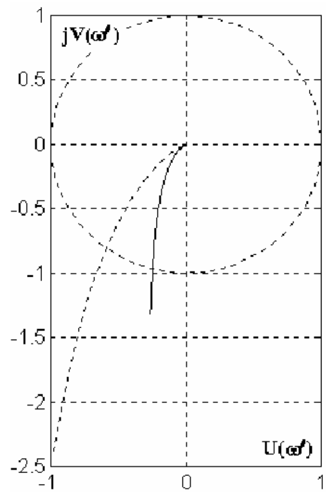


c) Pulsation-amplitude characteristics.

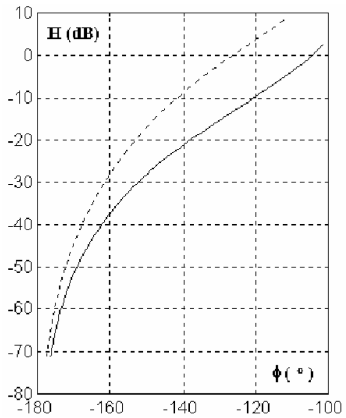


d) Phase-pulsation characteristics.

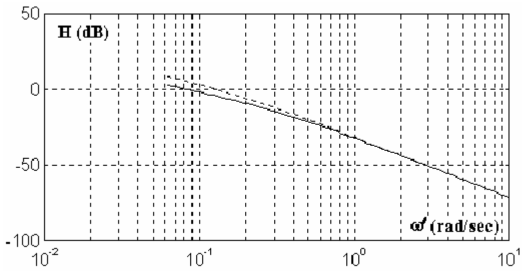
Fig. 7. Graphic dependences corresponding to the cases R_S=7,5 Ω (continuous line) and R_S=2,5 Ω (dot line).



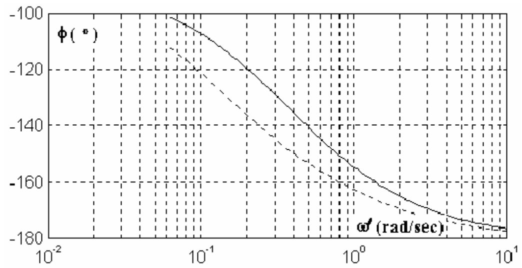
a) Transfer locus.



b) Amplitude – phase characteristics.



c) Pulsation-amplitude characteristics.



d) Phase-pulsation characteristics.

Fig. 8. Graphics dependences corresponding to the cases R'_r=5,5 Ω (continuous line) and R'_r=4,5 Ω (dot line).

Observation

In order to determine the characteristics depicted in the previous figures it has been considered that the induction motor has the following parameters:

R_s=7,5 Ω; $R_r^{/}$ =5,5 Ω; Ls=0,529 H;

$L_r^{/}$ =0,528 H; L_{sh}=0,498 H; J=0,004 kgm^2,

that leads to the following per unit values:

r_s^* =0,0989; $r_r^{/*}$ =0,0725; x_s^* =2,1907; $x_r^{/*}$ =2,1865;

x_{1m}^* =2,0623; x_{st}^* =0,2456; x_{rt}^* =0,2451; s_{ks}=0,4026;

s_{kr}=0,2958; k=0,9414; h=32,4; ε =0,0458.

Observation

With the help of a special achieved Matlab program and of the characteristics corresponding to the cases when a parameter from the ones depicted in the second column of the Table 5 is successively modified (over the initial case), the margins of phase depicted in the third column of the same table are obtained.

Table 5

Par.	Abs. value [Ω], [H], [kgm^2]	Per unit par.	Per unit value	Phase margin [degree]
R_s	7,5	r_s^*	0,0988	75,54
	2,5		0,0330	74,20
$R_r^{/}$	5,5	$r_r^{/*}$	0,0725	75,54
	4,5		0,0593	53,71
L_s	0,529	x_s^*	2,1907	75,54
	0,549		2,2735	69,13
$L_r^{/}$	0,528	$x_r^{/*}$	2,1865	75,54
	0,548		2,2694	67,31
L_{sh}	0,498	x_{1m}^*	2,0623	75,54
	0,438		1,8138	75,76
J	0,004	h	32,4	75,54
	0,003		24,3	47,65

In the Table 6 is presented the per cent variation of the parameters and the per cent variation of the phase margins.

Table 6

Parameter	Per cent variation of the parameter	Per cent variation of the phase margin
R_s	66,6	2,04
$R_r^{/}$	18,2	28,89
L_s	3,64	8,48
$L_r^{/}$	3,93	10,89
L_{sh}	12,04	0,29
J	25	36,92

6 Conclusion

The following conclusions can be emphasized, by analyzing the previous results:

- the decrease of the stator winding resistance leads to the stability decrease;
- the rotor resistance decrease has also as an effect, the decrease of the machine stability and conversely;
- the increase of the stator winding inductivity leads to the stability decrease;
- at the same time with the rotor inductivity increase the system stability decreases;
- the main inductivity increase has a non-stabilizing effect;
- the inertia moment increase contributes to the stability increase.

In order to catch quantitatively these interdependences, the following table can be filled.

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