Bifurcation and Chaotic Aspects in Peak Current Controlled Buck-Boost Converters

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Abstract: - The paper investigates the bifurcation and chaotic behavior of a current-mode controlled Buck-Boost converter operating in continuous conduction mode (CCM). The analysis comprehends both open loop and proportional closed loop operation cases. Classical averaged dynamic models cannot predict the chaotic converters behavior which leads to typical phenomenology like period-doubling, quasiperiodic and chaotic operation. The most important dynamic behavior aspects are revealed by simulation, together with a rigorous mathematical description that exactly predicts the converters operation modes.

Key-Words: - Non-Linear dynamics, bifurcation, chaos, subharmonic oscillation, dc-dc converters, buck-boost converter, current-mode control, power electronics

1 Introduction
As it is known, current-mode control is one of the most popular control methods used for achieving fast output regulation in switching converters [1]. The goal of this control is to force the inductor current to follow a reference value which is provided by an output feedback circuit. The inner current loop can become unstable under certain conditions. The basic phenomenology associated with this inner loop is the period-doubling which is not detectable by the averaged dynamical model.

The block diagram of a current-mode controlled buck-boost converter operated in CCM is represented in Fig.1. The operation of the inner current loop can be briefly described as follows. Transistor current, \( i_n \), is sensed and compared to a reference current \( I_{ref} \) generating the on-off driving signal for the switch \( S \). The switch \( S \) is turned on by a clock at the beginning of each switching cycle and the inductor current increases until it reaches the value of \( I_{ref} \). At this point the switch \( S \) is turned off and remains in this state until the next period begins. The control equation for this operation mode can be derived from the first topological state when \( S \) is on, as follows:

\[
L \frac{di_n}{dt} = \frac{I_{ref} - i_n}{d_n T_s} = v_{L,on}
\]  

where \( v_{L,on} \) is the inductor voltage in the first topological state. From (1) the duty cycle results as:

\[
d_n = \frac{I_{ref} - i_n}{v_{L,on} / L T_s}
\]  

The converter can be operated in an open loop mode when the output feedback loop is not present or in a closed loop mode, when the output feedback loop is added.
2 Open Loop Operation Mode

As far as dynamics of the inner loop is concerned, the converter can be operated open loop. Because the output feedback loop is usually much slower and its purpose is to adjust the reference value $I_{ref}$ in the event of input voltage and load variations, the omission of the output voltage loop should not alter the high frequency dynamics of the inner current loop.

For investigating the chaotic behavior for a current-mode controlled buck-boost converter in open loop operation mode, the following circuit parameters were considered: $V_g = 12V$, $L = 1.1mH$, $R_L = 0.47Ω$, $f_s = 20kHz$, $R = 50Ω$, $C = 4.4μF$. The circuit operation was simulated using the CASPOC Simulation Research Software [2].

In Fig.2 are presented the inductive current waveform, the phase portrait and the inductive current harmonic spectrum in the case of period 1 operation. It can be observed that the converter operates in a stable and periodical mode due to the small value of the bifurcation parameter $I_{ref}$.

By increasing the value of $I_{ref}$, the converters operation mode can change. In Fig.3 are presented the same waveforms but in the case of period 2 operation. It can be observed that the amplitude of the current harmonic at half of the switching frequency is higher than the harmonic corresponding to the switching frequency. Although subharmonic, this operation mode is still stable and periodical.

The case of quasiperiodic operation is presented in Fig.4. Due to its nonperiodic nature, this operation mode leads to presence of noises, especially at low frequencies as it can be observed from the inductors current spectrum in Fig.3(c).

![Inductive current waveform](image1)

![Phase portrait](image2)

![Inductive current harmonic spectrum](image3)

Fig.2 Open loop operation with period 1 ($I_{ref} = 0.4A$). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum
Fig. 3 Open loop subharmonic operation with period 2 ($I_{ref} = 0.6A$). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum.

Fig. 4 Open loop quasiperiodic operation ($I_{ref} = 0.95A$). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum.
In Fig.5 the chaotic operation mode waveforms are presented. This operation mode can be also observed from the irregular form of the Poincaré section in Fig.5(d) and it must be avoided in the all applications involving switching power supplies.

By solving the state equations that describes the dynamics of a second order dc-dc converter with CCM operation, the iterative maps can be obtained in the following form [3]:

\[ x(t_{n+1}) = f(x(t_n), d) \]  

where:

\[ f(x,d) = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} x + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} v_s \]  

In the case of the investigated buck-boost converter, the analytical approximate expressions for the \( f \) and \( g \) functions involved in the iterative maps, were determined by the help of MATHEMATICA program. Inductor resistance, \( R_L \), was taken into account, resulting in [4]:

Fig.5 Open loop chaotic operation (\( I_{ref} = 1.3A \)). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum, (d) Poincaré section
The first bifurcation can be precisely determined using the characteristic multipliers. These can be obtained as solutions of the characteristic equation, the results being presented in Table 1.

\[
\begin{align*}
    f_{s1} &= \left\{1 - \frac{dT}{L}R_t + \frac{1}{2}\left(\frac{dT}{L}R_t\right)^2\right\} \left\{1 - \frac{(1-d)T_s R_t}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \left(\frac{R_t}{L}\right)^2\right)\right\} \\
    f_{s2} &= \left\{1 - \frac{dT}{CR} + \frac{1}{2}\left(\frac{dT}{CR}\right)^2\right\} \left\{1 - \frac{(1-d)T_s R_t}{C} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \frac{1}{C^2 R}\right)\right\} \\
    f_{s1} &= \left\{1 - \frac{dT}{L}R_t + \frac{1}{2}\left(\frac{dT}{L}R_t\right)^2\right\} \left\{1 - \frac{(1-d)T_s R_t}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \left(\frac{R_t}{L}\right)^2\right)\right\} \\
    f_{s2} &= \left\{1 - \frac{dT}{CR} + \frac{1}{2}\left(\frac{dT}{CR}\right)^2\right\} \left\{1 - \frac{(1-d)T_s R_t}{C} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \frac{1}{C^2 R}\right)\right\} \\
    g_1 &= \frac{dT}{6L^2} (6L^2 - 3dT_s LR_t + (dT_s R_t)^2) \left\{1 - \frac{(1-d)T_s R_t}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \left(\frac{R_t}{L}\right)^2\right)\right\} \\
    g_2 &= \frac{dT}{6L^2} (6L^2 - 3dT_s LR_t + (dT_s R_t)^2) \left\{1 - \frac{(1-d)T_s R_t}{C} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \frac{1}{C^2 R}\right)\right\}
\end{align*}
\]

From the buck-boost circuit topology and from the inductors current waveform the duty-cycle value results:

\[
d_n = \frac{i_{ref} - i_{Ls}}{\frac{L}{T_s}}
\]  

(6)

The above relation was obtained taking in consideration a linear shape of current waveform during the first topological state. This approximation leads only to an error of 0.02% in the duty-cycle value.

Based on the iterative map the theoretical bifurcation diagram presented in Fig. 6 was obtained in MATLAB. As the current reference value increases it can be observed the occurrence of bifurcations with period-doubling, quasiperiodic operation and chaotic operation.

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<table>
<thead>
<tr>
<th>(I_{ref})</th>
<th>Characteristic Multipliers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>-0.8235; 0.6819</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.9792; 0.6838</td>
</tr>
<tr>
<td>0.51</td>
<td>-0.9942; 0.6839</td>
</tr>
<tr>
<td>0.511</td>
<td>-1.0034; 0.6513</td>
</tr>
<tr>
<td>0.515</td>
<td>-0.9781; 0.6908</td>
</tr>
<tr>
<td>0.52</td>
<td>-0.9364; 0.7036</td>
</tr>
<tr>
<td>0.54</td>
<td>-0.8762; 0.7246</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.7985; 0.7638</td>
</tr>
</tbody>
</table>

It can be observed that for small \(I_{ref}\) values the characteristic multipliers absolute values are less than -1, denoting a stable period 1 operation. As \(I_{ref}\) value increases, one of the characteristic multipliers is moving towards -1. At an approximate value of 0.511A for \(I_{ref}\) one of the characteristic multipliers equals -1, indicating a bifurcation with period-doubling. From this point forward the converter still operates in a stable mode but with period 2 operation.

The converters behavior can be also observed from the graphical representation of the highest Lyapunov exponent versus the current reference \(I_{ref}\) performed in MATLAB and presented in Fig. 7.

The first bifurcation can be precisely determined using the characteristic multipliers. These can be obtained as solutions of the characteristic equation, the results being presented in Table 1.
The same behavior type (cascades of period-doubling until a chaotic operation) can be obtained for other values of the $\gamma = T/RC$ parameter. In Fig.9 are presented the bifurcation diagrams obtained by CASPOC simulation in the cases of $\gamma = 0.454$ and respectively $\gamma = 0.162$.

3 Closed Loop Operation Mode

In the presence of the output voltage feedback loop, with $K$ being the feedback gain, the converter operates in a closed loop mode. The converter model requires an additional equation in order to describe the relationship between the output voltage and the reference current. The bifurcation parameter is chosen to be the voltage feedback loop gain $K$. In these conditions the current reference value is no longer a fixed one.

Because of the linear proportional feedback, the equation for the current reference is:

$$i_{ref} = I_{ref} - K \left[ \frac{d_i}{CR} - V_{ref} \right]$$

In equation (7), $V_{ref}$ is the steady-state reference output voltage and $I_{ref}$ is the steady-state reference current which can be chosen as the secondary bifurcation parameter.

The inductors current waveform, for closed loop operation of the buck-boost converter under current-mode control, is represented in Fig.10.

In a similar way to the open loop case, the converters behavior results from the bifurcation diagram, obtained from CASPOC simulation and presented in Fig.11.
It can be observed that until the feedback gain value $K$ of 0.03, the converter operates with period 1. As around this value the first bifurcation occurs, this leads to the period-doubling phenomenon. Furthermore by increasing the value of the voltage loop gain, the period-doubling processes are repeating until the converter operates in a chaotic way. By increasing the value of the bifurcation parameter over the value of 0.11, on the chaotic behavior will be over-imposed the on-limit collision because of the discontinuous operation mode on certain time intervals.

The waveforms for the inductor current and phase portrait from CASPOC simulation, corresponding to the period 1, period 2, period 4 and chaotic operations are presented in Fig.12.
Fig.12 Current-mode controlled buck-boost converter, with closed loop operation, CASPOC simulation waveforms (inductor current and phase portrait).
(a) Period 1 operation, $K=0.01$, (b) Period 2 operation, $K=0.05$,
(c) Period 4 operation, $K=0.092$, (d) Chaotic operation, $K=0.106$, 
4  Simulation results
As it was mentioned in the above sections, the buck-boost converter behavior was simulated with the CASPOC Simulation Research software.

Next, in Fig.15 is presented the closed loop operation simulation circuit for the buck-boost converter.

For the open loop operation the simulation circuit from Fig.13 was used. In Fig.14 is presented the open loop operation simulation circuit used for obtaining the bifurcation diagram.

The inductor current harmonic spectrum was represented in MATLAB using the simulation results of the circuit from Fig.13.
The bifurcation diagram for open loop operation was obtained in MATLAB using the simulation results of the circuit from Fig.14. The iterative map was determined with a MATHEMATICA program and the characteristic multipliers were determined in MATLAB. In the closed loop operation case, the bifurcation diagram was determined in MATLAB using the simulation results of the circuit from Fig.15.

5 Conclusion

In closed loop operation the bifurcation phenomenon is basically the same as in the open loop case with period-doubling and border collision interplaying to organize the bifurcation patterns. Similar transitions in the appearance of the bifurcation diagram can be observed in both cases. As load time constant increases the distance to chaos along the \( I_{ref} \) axis is progressively shortened, with a reduced number of bifurcations after the border collision. The iterative maps developed accurately predict the converter behaviour. From the presented case of the buck-boost converter and from other examples contained in the literature, [9], [10], [11], it can be revealed that bifurcations with period-doubling and chaotic behavior are characteristic to all current-mode control operated converters, regardless of the presence of the output voltage feedback loop.

References: