Bifurcation and Chaotic Aspects in Peak Current Controlled Buck-Boost Converters

DAN NEGOITESCU, DAN LASCU, VIOREL POPESCU, CORINA IVAN
Department of Applied Electronics
Politehnica University of Timisoara, Faculty of Electronics and Telecommunications
Bd. Vasile Parvan 2, 300223 Timisoara
ROMANIA
dan.negoitescu@etc.upt.ro, dan.lascu@etc.upt.ro,
viorel.popescu@etc.upt.ro, corina_m_i@yahoo.com

Abstract: - The paper investigates the bifurcation and chaotic behavior of a current-mode controlled Buck-Boost converter operating in continuous conduction mode (CCM). The analysis comprehends both open loop and proportional closed loop operation cases. Classical averaged dynamic models cannot predict the chaotic converters behavior which leads to typical phenomenology like period-doubling, quasiperiodic and chaotic operation. The most important dynamic behavior aspects are revealed by simulation, together with a rigorous mathematical description that exactly predicts the converters operation modes.

Key-Words: - Non-Linear dynamics, bifurcation, chaos, subharmonic oscillation, dc-dc converters, buck-boost converter, current-mode control, power electronics

1 Introduction

As it is known, current-mode control is one of the most popular control methods used for achieving fast output regulation in switching converters [1]. The goal of this control is to force the inductor current to follow a reference value which is provided by an output feedback circuit. The inner current loop can become unstable under certain conditions. The basic phenomenology associated with this inner loop is the period-doubling which is not detectable by the averaged dynamical model.
The block diagram of a current-mode controlled buck-boost converter operated in CCM is represented in Fig.1. The operation of the inner current loop can be briefly described as follows. Transistor current, i_s, is sensed and compared to a reference current I_{ref}, generating the on-off driving signal for the switch *S*. The switch *S* is turned on by a clock at the beginning of each switching cycle and the inductor current increases until it reaches the value of I_{ref}. At this point the switch *S* is turned off and remains in this state until the next period begins.
The control equation for this operation mode can be derived from the first topological state when *S* is on, as follows:

$$L\frac{di_L}{dt}=\frac{I_{ref}-i_{Ln}}{d_nT_s}=v_{L,on} \qquad (1)$$

where $v_{L,on}$ is the inductor voltage in the first topological state. From (1) the duty cycle results as:

$$d_n=\frac{I_{ref}-i_{Ln}}{\frac{v_{L,on}}{L}T_s} \qquad (2)$$

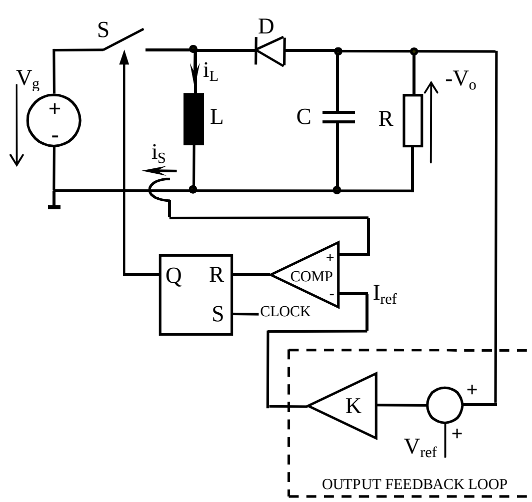

Fig.1 Schematic diagram of a current-mode controlled buck-boost converter

The converter can be operated in an open loop mode when the output feedback loop is not present or in a closed loop mode, when the output feedback loop is added.

2 Open Loop Operation Mode

As far as dynamics of the inner loop is concerned the converter can be operated open loop. Because the output feedback loop is usually much slower and its purpose is to adjust the reference value I_{ref} in the event of input voltage and load variations, the omission of the output voltage loop should not alter the high frequency dynamics of the inner current loop.

For investigating the chaotic behavior for a current-mode controlled buck-boost converter in open loop operation mode, the following circuit parameters were considered: $V_g = 12V$, $L = 1{,}1mH$, $R_L = 0{,}47\Omega$, $fs = 20kHz$, $R = 50\Omega$, $C = 4{,}4\mu F$. The circuit operation was simulated using the CASPOC Simulation Research Software [2].

In Fig.2 are presented the inductive current waveform, the phase portrait and the inductive current harmonic spectrum in the case of period 1 operation. It can be observed that the converter operates in a stable and periodical mode due to the small value of the bifurcation parameter I_{ref}.

By increasing the value of I_{ref}, the converters operation mode can change. In Fig.3 are presented the same waveforms but in the case of period 2 operation. It can be observed that the amplitude of the current harmonic at half of the switching frequency is higher than the harmonic corresponding to the switching frequency. Although subharmonic, this operation mode is still stable and periodical.

The case of quasiperiodic operation is presented in Fig.4. Due to its nonperiodic nature, this operation mode leads to presence of noises, especially at low frequencies as it can be observed from the inductors current spectrum in Fig.3(c).

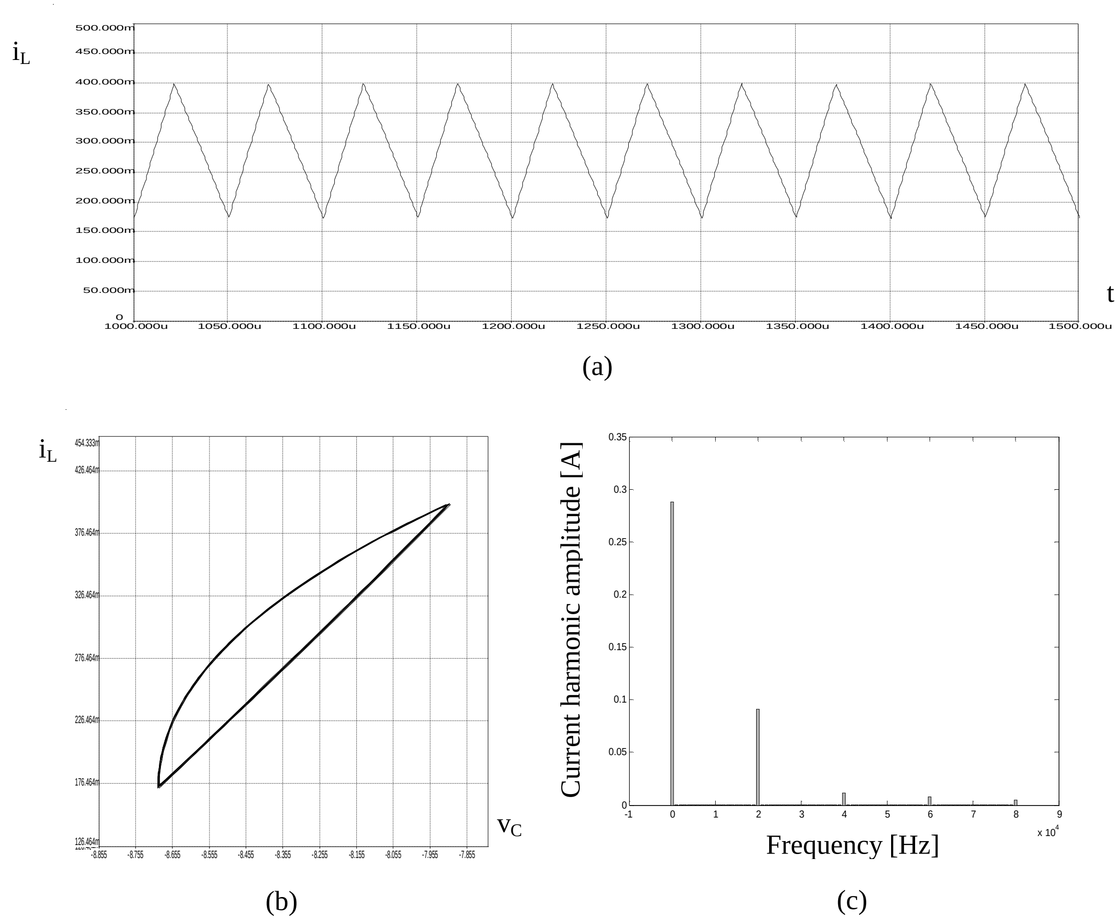

Fig.2 Open loop operation with period 1 ($I_{ref} = 0.4A$). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum

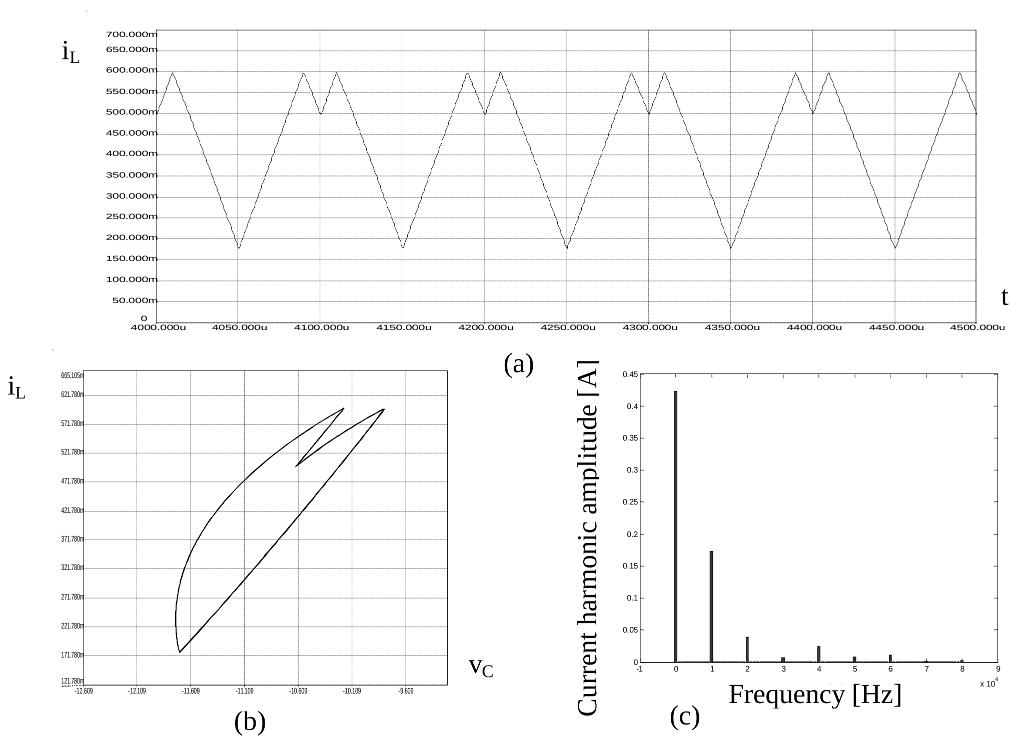

Fig.3 Open loop subharmonic operation with period 2 (I_{ref} = 0.6A). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum

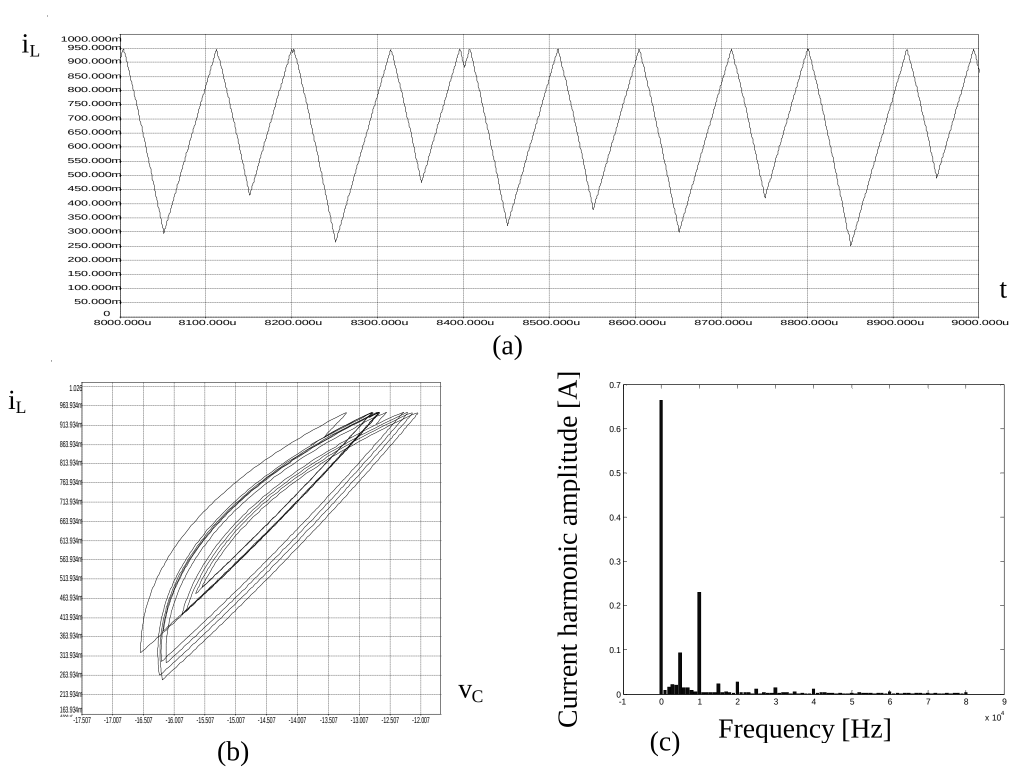

Fig.4 Open loop quasiperiodic operation (I_{ref} = 0.95A). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum

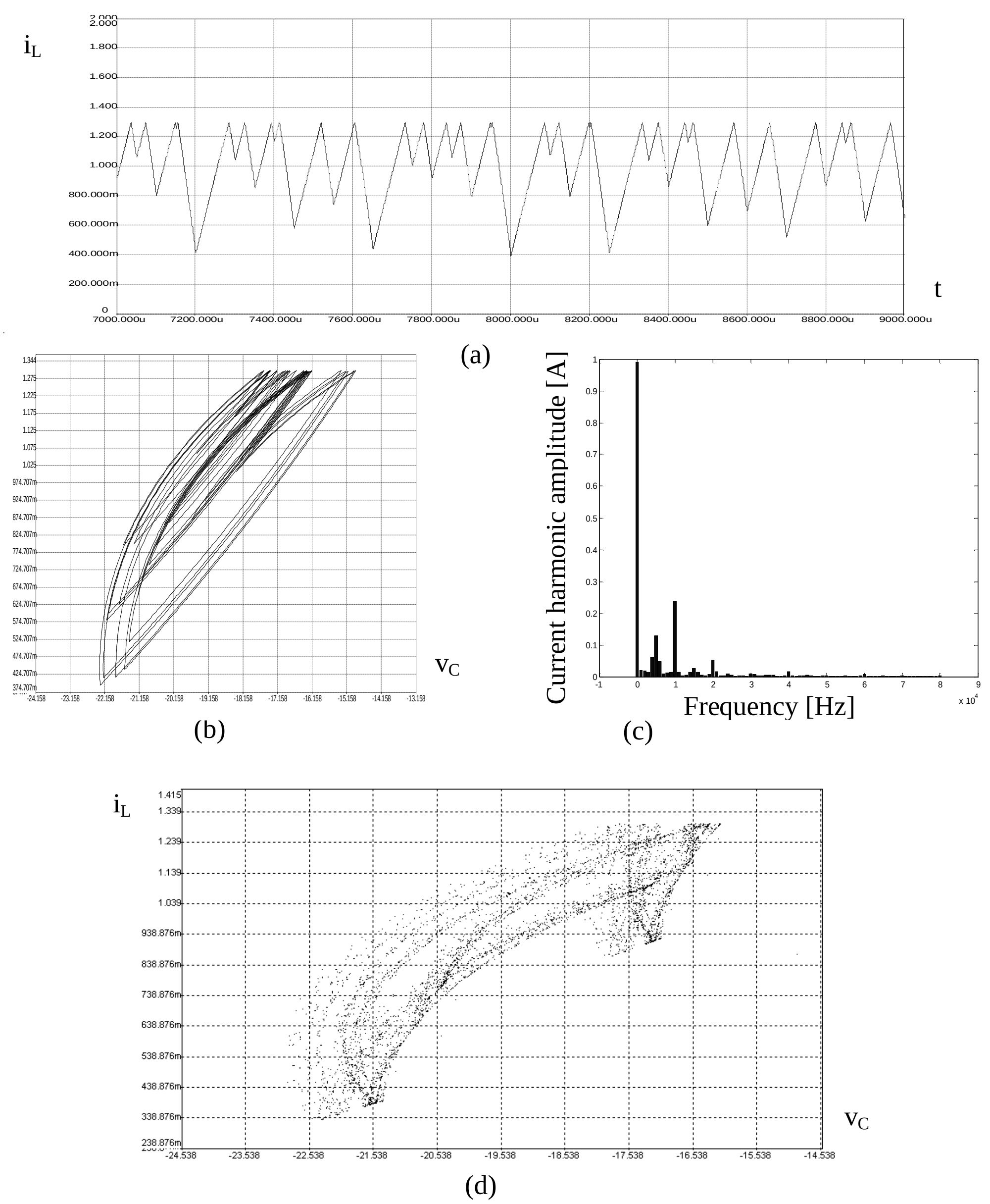

Fig.5 Open loop chaotic operation (I_{ref} = 1.3A). (a) Inductor current waveform, (b) Phase portrait, (c) Inductor current harmonic spectrum, (d) Poincaré section

In Fig.5 the chaotic operation mode waveforms are presented. This operation mode can be also observed from the irregular form of the Poincaré section in Fig.5(d) and it must be avoided in the all applications involving switching power supplies.

By solving the state equations that describes the dynamics of a second order dc-dc converter with CCM operation, the iterative maps can be obtained in the following form [3]:

$$x(t_{n+1}) = f(x(t_n), d) \qquad (3)$$

where:

$$f(x,d) = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} x + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} v_g \qquad (4)$$

In the case of the investigated buck-boost converter, the analytical approximate expressions for the *f* and *g* functions involved in the iterative maps, were determined by the help of MATHEMATICA program. Inductor resistance, R_L, was taken into account, resulting in [4]:

$$\begin{cases} f_{11} = \left(1 - \frac{dT_sR_L}{L} + \frac{1}{2}\left(\frac{dT_sR_L}{L}\right)^2\right)\left(1 - \frac{(1-d)T_sR_L}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \left(\frac{R_L}{L}\right)^2\right)\right) \\ f_{12} = \left(1 - \frac{dT_s}{CR} + \frac{1}{2}\left(\frac{dT_s}{CR}\right)^2\right)\left(-\frac{(1-d)T_s}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(\frac{1}{CLR} + \frac{R_L}{L^2}\right)\right) \\ f_{21} = \left(1 - \frac{dT_sR_L}{L} + \frac{1}{2}\left(\frac{dT_sR_L}{L}\right)^2\right)\left(\frac{(1-d)T_s}{C} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{C^2R} - \frac{R_L}{CL}\right)\right) \\ f_{22} = \left(1 - \frac{dT_s}{CR} + \frac{1}{2}\left(\frac{dT_s}{CR}\right)^2\right)\left(1 - \frac{(1-d)T_s}{CR} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \frac{1}{C^2R^2}\right)\right) \\ g_1 = \frac{dT_s}{6L^3}\left(6L^2 - 3dT_sLR_L + \left(dT_sR_L\right)^2\right)\left(1 - \frac{(1-d)T_sR_L}{L} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{CL} + \left(\frac{R_L}{L}\right)^2\right)\right) \\ g_2 = \frac{dT_s}{6L^3}\left(6L^2 - 3dT_sLR_L + \left(dT_sR_L\right)^2\right)\left(\frac{(1-d)T_s}{C} + \frac{1}{2}\left((1-d)T_s\right)^2\left(-\frac{1}{C^2R} - \frac{R_L}{CL}\right)\right) \end{cases} \tag{5}$$

From the buck-boost circuit topology and from the inductors current waveform the duty-cycle value results:

$$d_n = \frac{i_{ref} - i_{Ln}}{\frac{V_g}{L}T_s} \tag{6}$$

The above relation was obtained taking in consideration a linear shape of current waveform during the first topological state. This approximation leads only to an error of 0.02% in the duty-cycle value.

Based on the iterative map the theoretical bifurcation diagram presented in Fig.6 was obtained in MATLAB. As the current reference value increases it can be observed the occurrence of bifurcations with period-doubling, quasiperiodic operation and chaotic operation.

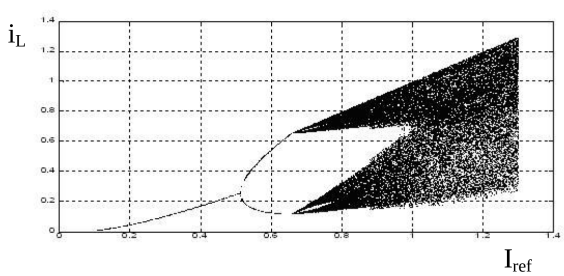

Fig.6 Bifurcation diagram of a current-mode controlled buck-boost converter with open loop operation

The first bifurcation can be precisely determined using the characteristic multipliers. These can be obtained as solutions of the characteristic equation, the results being presented in Table 1.

Table 1

I_{ref}	Characteristic Multipliers	
0,4	-0,8235;	0,6819
0,5	-0,9792;	0,6838
0,51	-0,9942;	0,6839
0,511	-1,0034;	0,6513
0,515	-0,9781;	0,6908
0,52	-0,9364;	0,7036
0,54	-0,8762;	0,7246
0,6	-0,7985;	0,7638

It can be observed that for small I_{ref} values the characteristic multipliers absolute values are less than -1, denoting a stable period 1operation. As I_{ref} value increases, one of the characteristic multipliers is moving towards -1. At an approximate value of *0.511A* for I_{ref} one of the characteristic multipliers equals -1, indicating a bifurcation with period-doubling. From this point forward the converter still operates in a stable mode but with period 2 operation.

The converters behavior can be also observed from the graphical representation of the highest Lyapunov exponent versus the current reference I_{ref}, performed in MATLAB and presented in Fig.7.

Next, the bifurcation diagram for the investigated buck-boost converter was obtained through simulation in CASPOC. Except for some small discrepancies caused by the approximations used in the iterative maps it can be observed that the simulated results, presented in Fig.8, are in accordance with the theoretical expectations.

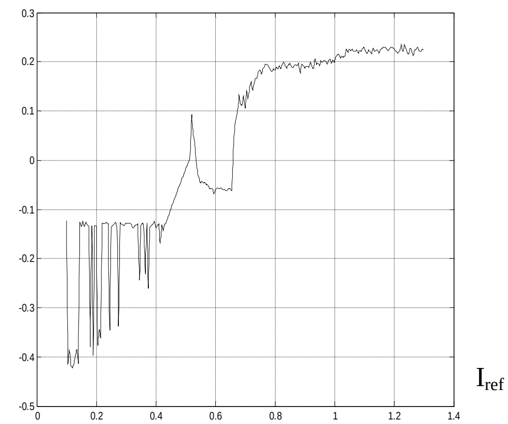

Fig.7 The highest Lyapunov exponent versus I_{ref}

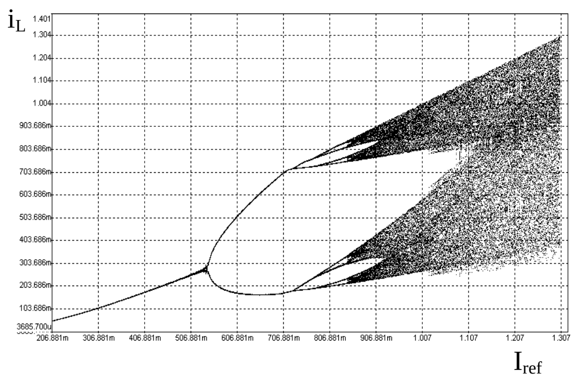

Fig.8 Bifurcation diagram obtained by CASPOC simulation of a current-mode controlled buck-boost converter with open loop operation, for γ=0,227.

The same behavior type (cascades of period-doubling until a chaotic operation) can be obtained for other values of the $\gamma=T_s/RC$ parameter. In Fig.9 are presented the bifurcation diagrams obtained by CASPOC simulation in the cases of $\gamma=0.454$ and respectively $\gamma=0.162$.

3 Closed Loop Operation Mode

In the presence of the output voltage feedback loop, with K being the feedback gain, the converter operates in a closed loop mode. The converter model requires an additional equation in order to describe the relationship between the output voltage and the reference current. The bifurcation parameter is chosen to be the voltage feedback loop gain K. In these conditions the current reference value is no longer a fixed one.

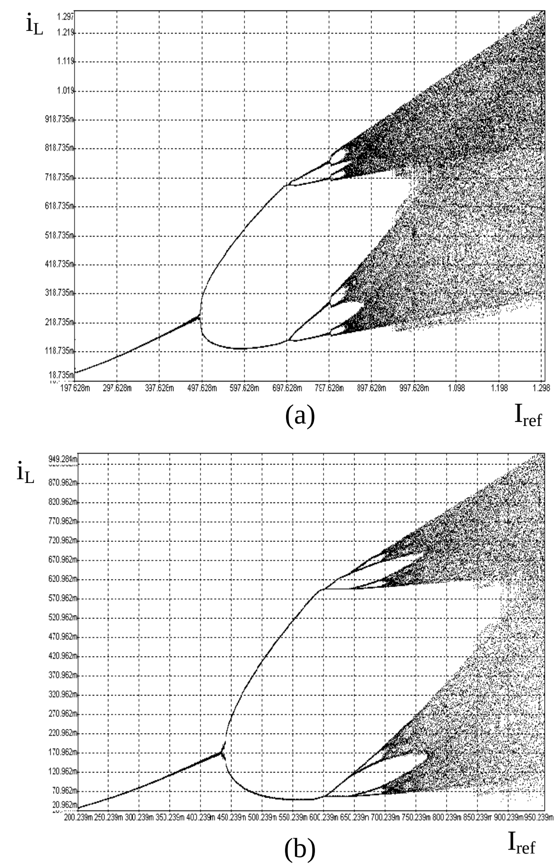

Fig.9 Bifurcation diagram obtained by CASPOC simulation of a current-mode controlled buck-boost converter with open loop operation, (a) γ=0,454, (b) γ=0,162.

Because of the linear proportional feedback, the equation for the current reference is:

$$i_{ref} = I_{ref} - \kappa\left[v_{Cn} e^{-\frac{d_n T_s}{CR}} - V_{ref} \right] \qquad (7)$$

In equation (7), V_{ref} is the steady-state reference output voltage and I_{ref} is the steady-state reference current which can be chosen as the secondary bifurcation parameter.

The inductors current waveform, for closed loop operation of the buck-boost converter under current-mode control, is represented in Fig.10.

In a similar way to the open loop case, the converters behavior results from the bifurcation diagram, obtained from CASPOC simulation and presented in Fig.11.

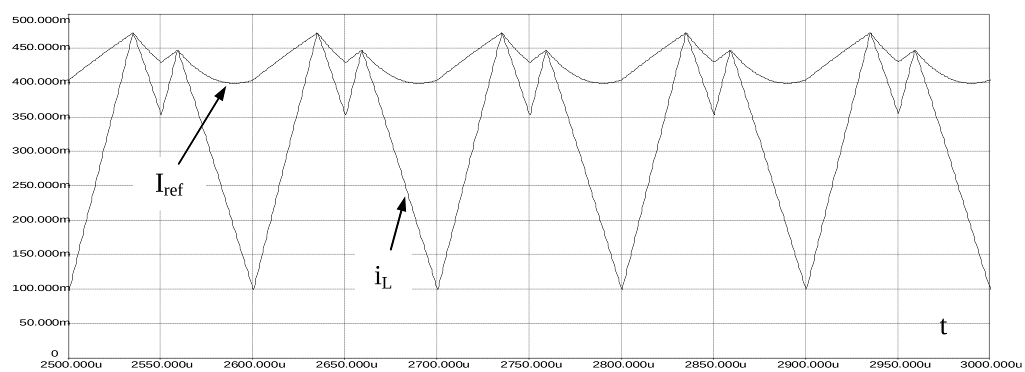

Fig.10 Inductor current waveform in closed loop operation mode

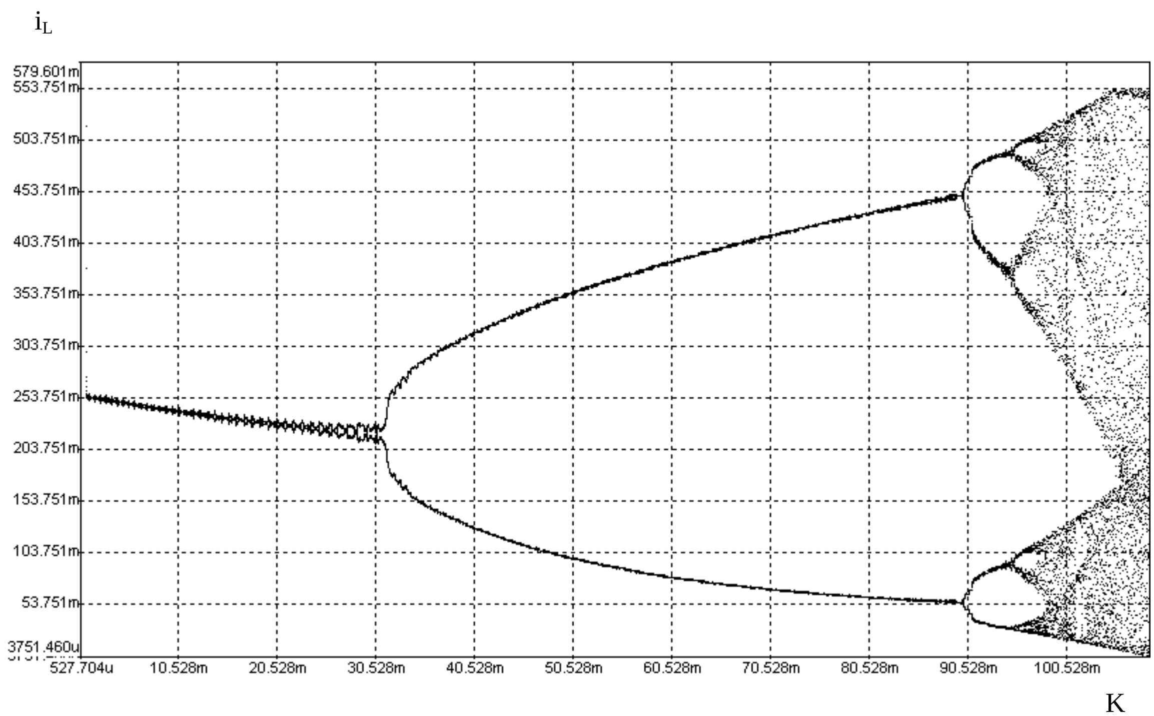

Fig.11 Bifurcation diagram obtained by CASPOC simulation of a current-mode controlled buck-boost converter with closed loop operation.

It can be observed that until the feedback gain value K of *0.03*, the converter operates with period 1. As around this value the first bifurcation occurs, this leads to the period-doubling phenomenon. Furthermore by increasing the value of the voltage loop gain, the period-doubling processes are repeating until the converter operates in a chaotic way. By increasing the value of the bifurcation parameter over the value of *0.11*, on the chaotic behavior will be over-imposed the on-limit collision because of the discontinuous operation mode on certain time intervals.

The waveforms for the inductor current and phase portrait from CASPOC simulation, corresponding to the period 1, period 2, period 4 and chaotic operations are presented in Fig.12.

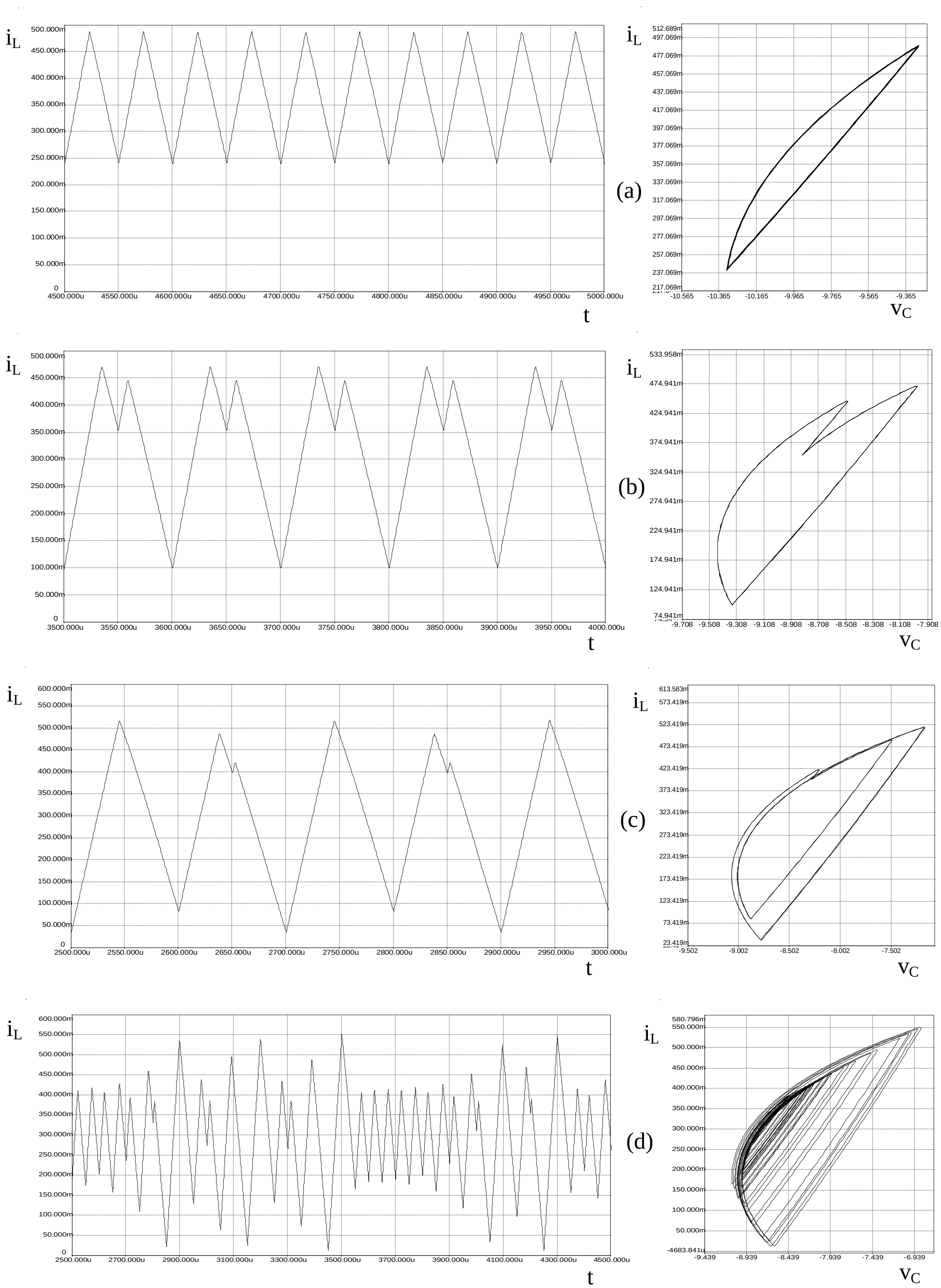

Fig.12 Current-mode controlled buck-boost converter, with closed loop operation, CASPOC simulation waveforms (inductor current and phase portrait).
(a) Period 1 operation, K=0,01, (b) Period 2 operation, K=0,05,
(c) Period 4 operation, K=0,092, (d) Chaotic operation, K=0,106,

4 Simulation results

As it was mentioned in the above sections, the buck-boost converter behavior was simulated with the CASPOC Simulation Research software.

Next, in Fig.15 is presented the closed loop operation simulation circuit for the buck-boost converter.

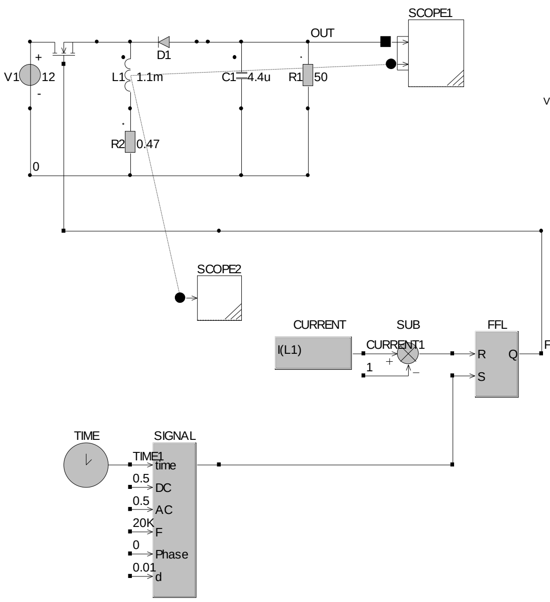

Fig.13 CASPOC simulation circuit of a current-mode controlled buck-boost converter open loop operated.

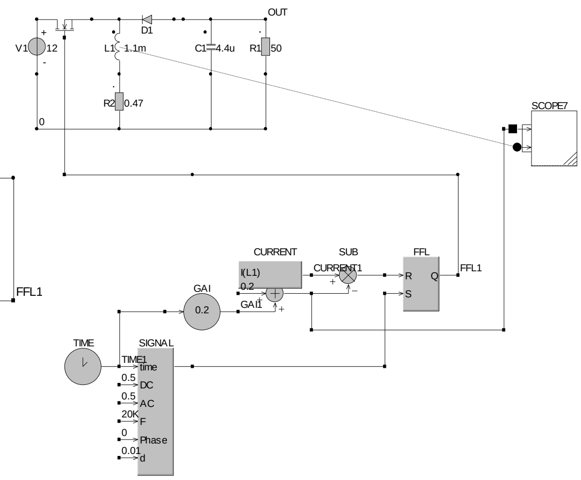

Fig.14 CASPOC simulation circuit of a current-mode controlled buck-boost converter open loop operated used for obtaining the bifurcation diagram.

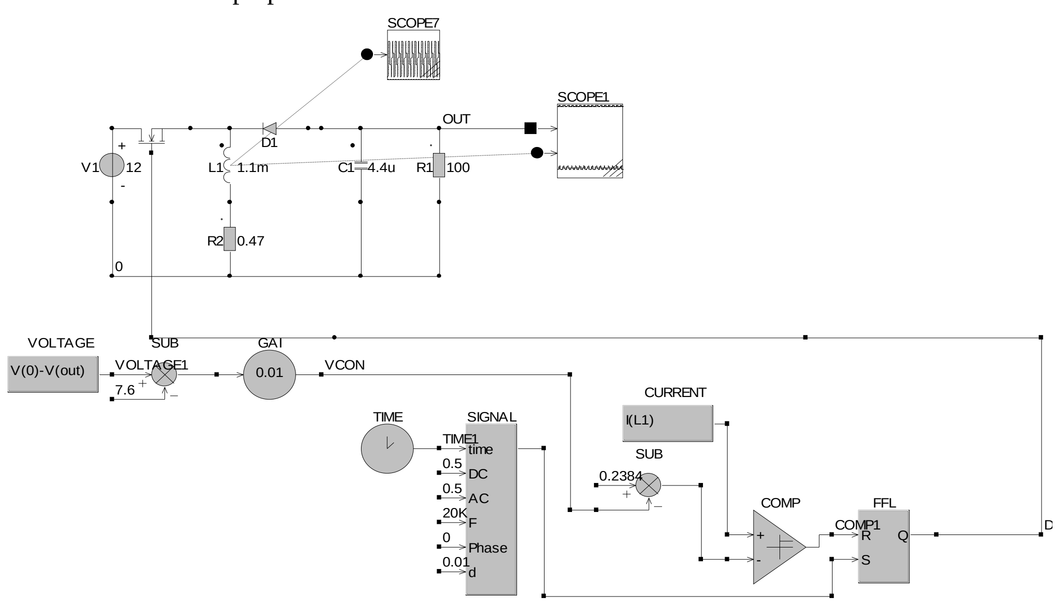

Fig.15 CASPOC simulation circuit of a current-mode controlled buck-boost converter in closed loop operation used for obtaining the bifurcation diagram.

For the open loop operation the simulation circuit from Fig.13 was used. In Fig.14 is presented the open loop operation simulation circuit used for obtaining the bifurcation diagram.

The inductor current harmonic spectrum was represented in MATLAB using the simulation results of the circuit from Fig.13.

The bifurcation diagram for open loop operation was obtained in MATLAB using the simulation results of the circuit from Fig.14.
The iterative map was determined with a MATHEMATICA program and the characteristic multipliers were determined in MATLAB.
In the closed loop operation case, the bifurcation diagram was determined in MATLAB using the simulation results of the circuit from Fig.15.

5 Conclusion

In closed loop operation the bifurcation phenomenon is basically the same as in the open loop case with period-doubling and border collision interplaying to organize the bifurcation patterns.
Similar transitions in the appearance of the bifurcation diagram can be observed in both cases.
As load time constant increases the distance to chaos along the I_{ref} axis is progressively shortened, with a reduced number of bifurcations after the border collision. The iterative maps developed accurately predict the converter behaviour.
From the presented case of the buck-boost converter and from other examples contained in the literature, [9], [10], [11], it can be revealed that bifurcations with period-doubling and chaotic behavior are characteristic to all current-mode control operated converters, regardless of the presence of the output voltage feedback loop.

References:

[1] Erickson, R. W., Maksimović, D., *Fundamentals of Power Electronics*, Second Edition, Kluwer Academic Publishers, 2001.

[2] CASPOC - *Reference Manual*, Simulation Research, 1999.

[3] Tse, C. K., *Complex Behavior of Switching Power Converters*, CRC Press LLC, 2004.

[4] Ivan, C.M., *Contributions regarding the analysis and modelling of power switching converters*, Ph.D. Thesis, “Politehnica” University of Timisoara, 2008.

[5] Hamill, D. C., Deane, J. H. B., Aston, P. J., *Some applications of chaos in power converters*, IEE Colloquium: Update on new power electronic techniques, ref. no. 1997/091, London, May 1997, pp. 5/1-5/5.

[6] Tse, C. K., Lai, Y. M., *Control of Bifurcation in Current-programmed DC/DC Converters: A Reexamination of Slope Compensation*, IEEE International Symposium on Circuits and Systems, (ISCAS'00), Geneva Switzerland, June 2000, pp. I-671-674.

[7] Hamill, D. C., Jefferies, D. J., *Subharmonics and chaos in a controlled switched-mode power converter*, IEEE Trans. on Circuits and Systems, vol. 35, no. 8, Aug. 1988, pp. 1059-1061.

[8] Tse, C. K., *Recent Development in the Study of Nonlinear Phenomena in Power Electronics Circuits*, IEEE Circuits and Systems Society Newsletter, March Issue, 2000, pp. 14-48.

[9] Cafagna, D., Grassi, G., *Complex Dynamic Phenomena in Power Converters: Bifurcation Analysis and Chaotic Behavior*, WSEAS TRANSACTIONS on CIRCUITS AND SYSTEMS, Issue 4, Volume 3, June 2004, ISSN1109-2734, pp. 735-740.

[10] Hong-Zhong, L., Hao-Zhong, C., Cheng-Min, W., *A novel approach for determining the hopf bifurcation point of dynamic voltage stability in power system*, WSEAS TRANSACTIONS on CIRCUITS AND SYSTEMS, Issue 7, Volume 4, July 2005, ISSN1109-2734, pp. 691-698.

[11] Dragan, D., *Controlling Chaos in DC/DC Converters using Ott-Grebogi-Yorke and Pyragas Methods*, WSEAS TRANSACTIONS on CIRCUITS AND SYSTEMS, Issue 6, Volume 5, June 2006, ISSN1109-2734, pp. 849-854.