Systems for Move Stabilisation and for Aircrafts and Rockets Flight Optimization

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Abstract: - This paper deals with models of the rockets' motion around mass centre, command laws of vertical and horizontal planes trajectory, stabilisation laws of the roll motion of the rockets and target control laws [1], [2]. One also studies the way to choose of the parameters of general command laws which assures state variables simultaneous cancellation. Some automatic control systems, using the previous laws, are also presented. Also, one presents some extremal systems for optimization of aircrafts and rockets' flight using test signals for extreme's search [3], [4].

Key-Words: - rocket, command law, target, control, extremal system, testing signal.

1 Models of the rocket's motion around mass centre in vertical and horizontal planes

In fig.1 one presents the rocket's motion (A) around mass centre in vertical plane block diagram, with transfer operators, where: $\theta -$ the trajectory slope in vertical plane, $\alpha -$ attack angle, $V -$ flight velocity, $\delta -$ the ribbed deflection in horizontal plane (vertical plane command), $T_v -$ the time constant of A [5],

$$k_\delta = \frac{m_\delta}{J_z},$$
$$2\xi\omega_0 = \frac{m_{\dot{\vartheta}}}{J_z} + \frac{V}{T_v}, \qquad (1)$$
$$\omega_0^2 = \frac{m_\alpha}{J_z} + \frac{m_{\dot{\vartheta}}}{J_z T_v} + \frac{g\sin\theta}{VT_v} + \frac{\dot{T}_v}{T_v^2},$$

where $m_\delta -$ command coefficient, $J_z -$ inertial moment with respect to lateral axis, $m_{\dot{\vartheta}} -$ dynamic damp couple coefficient, $m_\alpha -$ static stabilisation coefficient, $\xi -$ damp coefficient, $\omega_0 -$ proper pulsation.

Similar to the block diagram in fig.1, one can obtain the block diagram, with transfer operators, in fig.2.

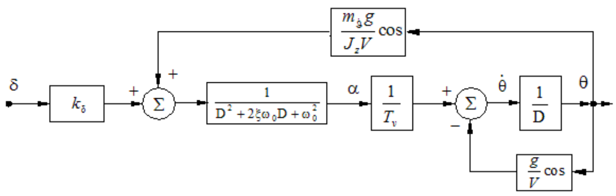

Fig.1 Block diagram of the system

This block diagram corresponds to the plane's horizontal motion; $\delta \to \delta', \alpha \to \alpha', \theta \to \theta'$; the two feedback links are missing here, because of the missing of the proportional terms with respect to gravitational acceleration (g) miss. Thus, transfer operator for the horizontal plane motion is

$$H_\theta(\mathrm{D}) = \left(\frac{\theta'}{\delta'}\right)(\mathrm{D}) = \frac{k_\delta}{T_v \mathrm{D}\left(\mathrm{D}^2 + 2\xi\omega_0 \mathrm{D} + \omega_0^2\right)}. \qquad (2)$$

2 Automatic command of the flight trajectory

In fig.2 one presents the block diagram, with transfer operators, of the automatic control system of the trajectory slope (θ and θ'), where $\overline{\theta}$ and $\overline{\theta}'$ expresses the desired slope angles for the two planes (vertical and horizontal); E.E.1 and E.E.2 are actuators (execution elements) for the commands δ and δ', U_1 and U_2 – command variables.

$$U_1 = \ddot{\theta} - k\frac{\dot{\theta}^2}{\Delta\theta}, \tag{3}$$

$$U_2 = \dot{\theta}' - ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}; \tag{4}$$

$\Delta\theta = \overline{\theta} - \theta,\ \Delta\theta' = \overline{\theta}' - \theta'$ and a, k – coefficients whose variation domains are presented below. For a steady state regime

$$\ddot{\theta} = k\frac{\dot{\theta}^2}{\Delta\theta}, \tag{5}$$

$$\dot{\theta}' = ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}, \tag{6}$$

which express command laws of the trajectory for two planes (vertical and horizontal). For an appropriate choosing of the parameter k, the variables $\Delta\theta, \dot{\theta}$ and $\ddot{\theta}\left(\theta \to \overline{\theta}, \dot{\theta} \to 0, \ddot{\theta} \to 0\right)$ becomes simultaneously null. Simultaneously with $\Delta\theta$'s cancellation, $\Delta\theta'$ and $\ddot{\theta}\left(\theta' \to \overline{\theta}', \dot{\theta}' \to 0\right)$ becomes null. All these statements will be demonstrated below for a general case.

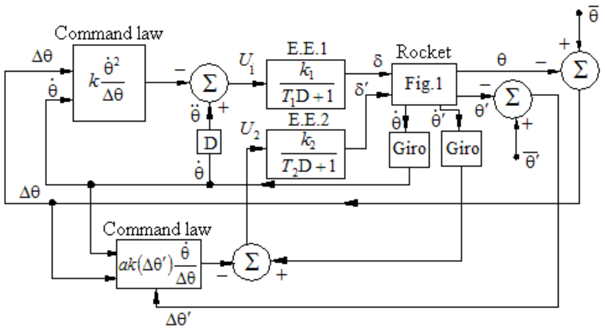

Fig.2 Block diagram of the automatic control system of the trajectory slope

3 Rocket's roll motion stabilisation

For an appropriate precision of the automatic control process, rockets with symmetrical distribution of the empennage and wings have stabilisation and maintaining φ to the zero value systems [6],[7]. Under disturbances' action, the rocket may rotate around its longitudinal axis. Thus, the vertical and horizontal channel may interfere, the control errors increases [5], [8]. The systems in fig.3 and in fig.4 assures simultaneous cancellation of the variables $\varphi, \dot{\varphi}$ and $\ddot{\varphi}$. The command law for the system in fig.3 is the following one

$$\ddot{\varphi} = k\frac{\dot{\varphi}^2}{\varphi}. \tag{7}$$

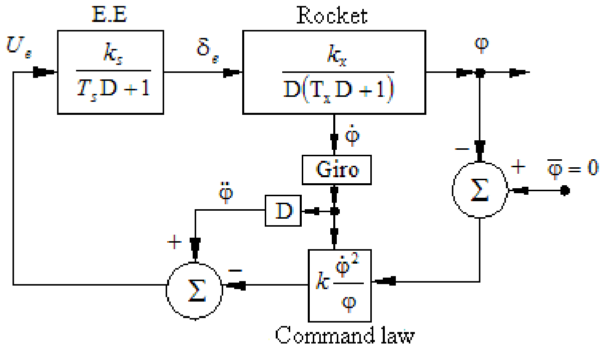

Fig.3 Stabilisation system of the rocket (variant 1)

Command law for the system in fig.4 is equivalent to the one given by Eq. (7); a supplementary phase coordinate z_0 is introduced [5].

Thus, if $T_x \to T_M$ (T_x – time constant of the rocket's motion around longitudinal axis, while T_M – model time constant), then the variable becomes

$$z_1 = \frac{1}{T_M D + 1}U_e = \frac{1}{T_M D + 1}\frac{T_x D + 1}{k_x k_s}\dot{\varphi} \cong \frac{1}{k_c}\dot{\varphi}, \tag{8}$$

where $k_c = k_x k_s$ and

$$z_0 = \frac{1}{D}(U_e - z_1) = \frac{1}{D}U_e - \frac{1}{k_c}\varphi \tag{9}$$

or

$$\dot{z}_0 = U_e - \frac{1}{k_c}\dot{\varphi}. \tag{10}$$

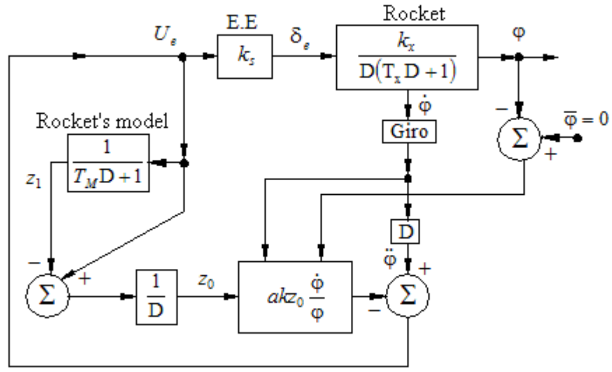

Fig.4 Stabilisation system of the rocket (variant 2)

From the equation that expresses the rocket's motion around its longitudinal axis

$$T_x\ddot{\varphi} + \dot{\varphi} = k_c U_e \tag{11}$$

it results

$$\frac{T_x}{k_c}\ddot{\varphi} = U_e - \frac{1}{k_c}\dot{\varphi}. \qquad (12)$$

Comparing equations (10) and (12), it results

$$\dot{\varphi} = \frac{k_c}{T_x} z_0 = a z_0, \quad a = \frac{k_c}{T_x}. \qquad (13)$$

Thus, command law (7) becomes

$$\ddot{\varphi} = a k z_0 \frac{\dot{\varphi}}{\varphi}. \qquad (14)$$

Variables $\varphi, \dot{\varphi}, \ddot{\varphi}, z_0$ simultaneously tend to zero.

4 Rocket's flight trajectory command and roll motion stabilisation

Such a control system is the one in fig.5. Command laws are

$$\ddot{\theta} = k\frac{\dot{\theta}^2}{\Delta\theta}, \qquad (15)$$

$$\dot{\theta}' = ak(\Delta\theta')\frac{\dot{\theta}}{\Delta\theta}, \qquad (16)$$

$$\dot{\varphi} = bk\varphi\frac{\dot{\theta}}{\Delta\theta}. \qquad (17)$$

The variables $\varphi, \dot{\varphi}, \dot{\theta}, \dot{\theta}'$ simultaneously tend to zero together with $\Delta\theta$ and $\Delta\theta'\left(\theta \to \overline{\theta}, \theta' \to \overline{\theta'}\right)$.

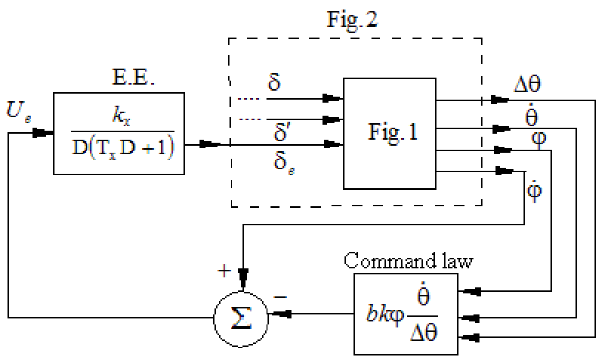

Fig.5 System for rocket's flight trajectory command and roll motion stabilisation

5 Rocket's flight trajectory command and target control

The structure of such a system is presented in fig.6. δ_m is the engine input command signal, r – the distance from A to T (target).

Command laws are (5), (6) and

$$\dot{r} = ckr\frac{\dot{\theta}}{\Delta\theta}. \qquad (18)$$

For φ and $\dot{\varphi}$ cancellation, a supplementary law (17) is needed.

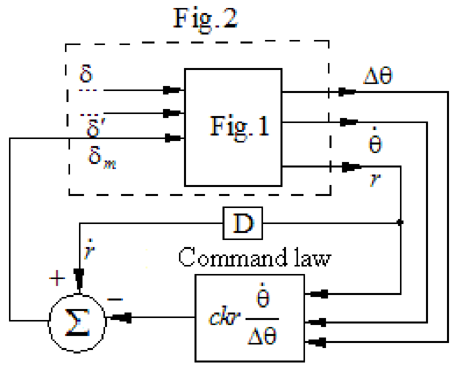

Fig.6 System for rocket's flight trajectory command and target control

6 Command laws analysis

One considers the variable z, which describes the equation

$$\ddot{z} = f(z, \dot{z}), \qquad (19)$$

where f has such a form that $\ddot{z}$ tends to zero when $z = \dot{z} = 0$; f may have the form $f = k\dot{z}^2 / z$ and, consequently, equation (19) becomes [9]

$$\ddot{z} = k\frac{\dot{z}^2}{z}, \qquad (20)$$

where k is a non-dimensional proportionality coefficient .

Integrating equation (20), one obtains

$$\frac{\dot{z}}{\dot{z}_0} = \left(\frac{z}{z_0}\right)^k, \quad \frac{z}{z_0} = \left[(1-k)\frac{\dot{z}_0}{z_0}(t_f - t)\right]^{\frac{1}{1-k}}, \qquad (21)$$

where $z_0 = z(0), \dot{z}_0 = \dot{z}(0)$ and t_f – time moment when the three variables become null. If one chooses $k > 1$, then the time interval till the three variables become null is very big and if one chooses $k \le 1/2$ then, if $t \to t_f, z^2$ tends to zero, while $\dot{z}$ (linear) tends to zero with a big slope $\frac{1}{2}\frac{\dot{z}_0}{z_0}$ [5]. If one chooses $k \in \left(\frac{1}{2}, 1\right)$ one obtains some changes. Thus, for $k = 0{,}7 \cong 2/3$, one observes that z^3 tends to zero, $\dot{z}^2$ tends to zero, while $\ddot{z}$ tends to zero (with a slope equal to 2/3) [9].

The laws (5), (7) have the form (20), where z is $\Delta\theta$, for equation (5), respectively φ for Eq. (7).

Another command law, correlated with (19), has the form

$$\dot{q} = g(q, z, \dot{z}), \tag{22}$$

which may be chosen [9]

$$\dot{q} = akq\frac{\dot{z}}{z}, \tag{23}$$

with $a = \text{const.}$ Integrating (for $z(0) = z_0$ and $q(0) = q_0$) it results the equation

$$\frac{q}{q_0} = \left(\frac{z}{z_0}\right)^{ak}, \tag{24}$$

which expresses that q and z are simultaneously null, together with $\dot{q}$ and $\dot{z}$.

The laws (6) and (16), (17), (18) have the form (23) where z is $\Delta\theta, \varphi, r$.

Other command law's forms are presented in [10] and [11].

7 Extremal systems for flight's optimization

Some equipments, installations and systems for which the dependence input – output is described in stationary regime by nonlinear functions with extreme points (maximum or minimum) may be met on aircrafts; perturbations may modify these extreme points. In order to maintain the working point around the extreme point one uses regulation contours (command laws); these are named extremal systems.

Some optimal criteria are presented bellow:

1) minimum displacement time of A (flying object) from one point to another with minimum fuel consume [12];
2) maximum flight distance with a set fuel consume;
3) minimum deviation of A from the set trajectory;
4) arrive in a point with a minimum error.

Structural diagram of such an extremal system (automat optimization) is presented in fig.7.

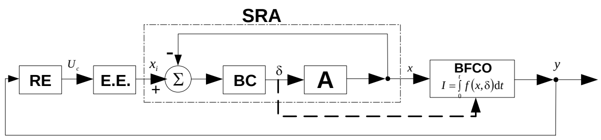

Fig.7 Structural diagram of an extremal system

The system contains an automat regulation system (SRA) in rapport with variable x (state variable or state vector); it consists of: control object (for example a flying object A), command block BC with regulation block and performing element (E.E.) and negative feedback in rapport with measured variable x. Reference variable x_i is obtained by the mean of extremal system (optimization system) which consists of the following blocks: BFCO – optimization criteria obtaining block, RE – extremal regulator (consisting of BME – block for extreme index measurement and BFC – block for command signal forming) and performing element (E.E.)

Generally the optimization criteria has a quadratic form [13]

$$I = \int_0^t f(x, \delta)\mathrm{d}t, \tag{25}$$

where δ is the command variable applied to A or BFCO.

If, for example, $x = V -$ flying velocity of A, then optimization criterion is the fuel consume

$$I = C_c = \frac{g}{a_x}\frac{G_T}{V} = \frac{g}{Va_x}\frac{\mathrm{d}G}{\mathrm{d}t}, \tag{26}$$

where: $G_T -$ fuel consume in time unity, $m -$ mass and $G = mg -$ the weight force at moment t and $a_x -$ longitudinal acceleration of A. Extremal characteristics family $C_c = C_c(V)$ is represented in fig.8, for $G_1 < G_2 < G_3$; along with fuel quantity decrease, extreme point $(V_{\min}, C_{c\,\min})$ moves to origin of the coordinates axis system.

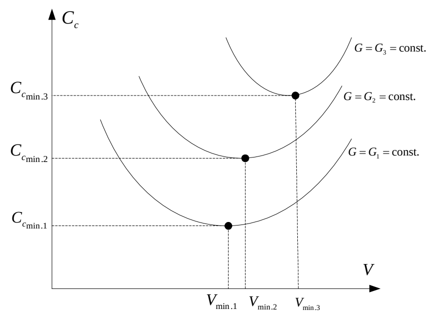

Fig.8 Extremal characteristics family $C_c = C_c(V)$

In many cases, for the mono variable extremal systems, optimization criterion may approximated by a parabola [14]

$$y = a(x - x_m)^2 + y_m, \tag{27}$$

where x_m and y_m are the coordinates of the extreme point (minimum or maximum). If $a < 0$, the parabola has a maximum point and, if $a > 0$, then this has a minimum point. Extreme point is a fix one if a, x_m, y_m don't modify in time; otherwise the disturbances lead to its movement.

At the beginning one knows that exist an extreme for characteristics $y = I(x)$ and its aspect (minimum or maximum). The main action of the extremal regulator RE is to find the extreme. This is obtained by the mean of BME (marking block of the extreme index).

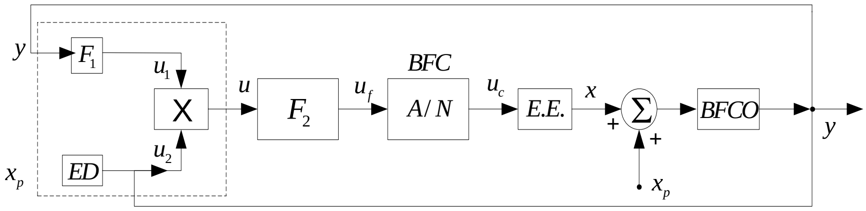

Fig.9 Application of the testing signal

Depending on this index's sign and value, BFC (block for command signal forming) forms signal u_c, which is applied to the performing element (E.E.), so that current operational point moves towards extreme. The most important searching methods are:

1) determination of output variable (Δy)'s increase;
2) determination of output variable's derivative $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$;
3) use of testing (searching) signals.

Testing signal is a deterministic one, being given by a sinusoidal generator, which is placed before BFCO and before synchrony detector DS (fig.9) [15];

$$x_p = A\sin\omega t. \quad (28)$$

Replacing in equation (27) x with $(x + x_p)$, one obtains

$$y = y(x + x_p) = y(x) + y_1(x_p) + y_2(x_p), \quad (29)$$

where

$$y_1(x_p) = \frac{\mathrm{d}y}{\mathrm{d}x} x_p = \frac{\mathrm{d}y}{\mathrm{d}x} A\sin\omega t, \quad (30)$$

$$y_2(x_p) = \frac{1}{2} a x_p^2 = \frac{1}{2} a A^2 (1 - \cos 2\omega t). \quad (31)$$

Filter F_1 is accorded to pulsation ω, so that these filters' components $y(x)$ and $y_2(x_p)$, let pass only component $y_1(x_p)$. Taking into account the decrement and phase difference φ introduced by filter F_1, at its output one obtains the signal

$$u_1 = \frac{\mathrm{d}y}{\mathrm{d}x} A_1 \sin(\omega t + \varphi). \quad (32)$$

Phase difference element ED introduces a phase difference φ and a decrement. Thus, one obtains the signal

$$u_2 = A_2 \sin(\omega t + \varphi). \quad (33)$$

Filter F_2 filters the alternative component of u and its output is

$$u_f = \frac{A_1 A_2}{2} \left|\frac{\mathrm{d}y}{\mathrm{d}x}\right| \quad (34)$$

for the ascendant branch of the extremal characteristic $y(x_p)$, respective

$$u_f = -\frac{A_1 A_2}{2} \left|\frac{\mathrm{d}y}{\mathrm{d}x}\right| \quad (35)$$

for the descendent branch. The signal u_f is applied to the block for command signal forming (BFC); this block has an amplifier or non linear form.

In order to optimize different flight regimes one uses move equations of flying object's mass centre [16]

$$\begin{aligned}
\dot{V} &= \frac{T\cos\alpha - X}{m} - g_0\left(\frac{R}{R+h}\right)^2 \sin\theta, \\
V\dot{\theta} &= \frac{T\sin\alpha + Y}{m} + \frac{V^2\cos\theta}{R+h} - g_0\left(\frac{R}{R+h}\right)^2 \cos\theta, \quad (36) \\
\dot{h} &= V\sin\theta, \\
\dot{L} &= \frac{R}{R+h} V\cos\theta,
\end{aligned}$$

where: V is the flight speed, $T-$ the thrust force, $X-$ the drag force, $Y-$ lift force, $m-$ the mass of the flying object, $R-$ the Earth radius, $h-$ flight altitude, $L-$ flight distance, g_0- the gravitational acceleration somewhere on Earth, $\theta-$ trajectory slope (measured in rapport with the horizontal line), $\alpha-$ the attack angle.

Forces X and Y have the forms

$$\begin{aligned} X &= X(V, h, Y), \\ Y &= Y(V, h), \end{aligned} \quad (37)$$

with $c_x = c_{x0} + k_y c_y^2, c_{x0} = c_{x0}(M), M -$ Mach number; the thrust force has the expression

$$T = T(V, h, \delta_m); \quad (38)$$

δ_m expresses the place of the engine's regulator element.

For $\alpha = 0, R \cong R + h(g_0 \cong g)$, by elimination of $V\sin\theta$ between first and the forth equation (36), one obtains

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{T-X}{m} - \frac{g}{V}\frac{\mathrm{d}h}{\mathrm{d}t}. \quad (39)$$

Taking into account equation (39)

$$\dot{V} = a_x - \frac{g}{V}\dot{h}, \quad (40)$$

the relation for the fuel consume is

$$C_c = \frac{g}{a_x}\frac{G_T}{V}. \quad (41)$$

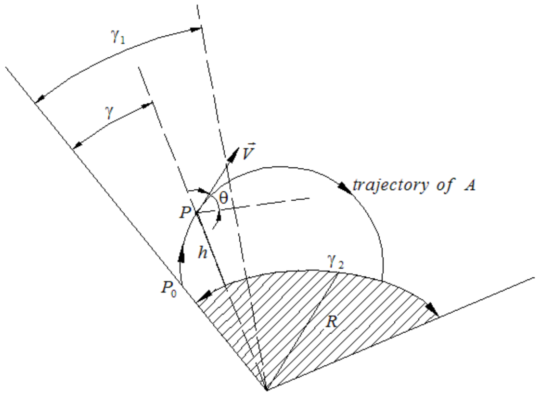

Fig.10 Elements for problem's formulation

Now one presents some flight parameters' optimization. Considering the flight of a ballistic rocket, launched in point P_0, the flight distance is

$$\gamma_t = \gamma_1 + \gamma_2, \quad (42)$$

where γ_1 is the launch angular distance and γ_2 – angular distance of flight without thrust force. The drag force at re-entrance in atmosphere is neglected and gravitational acceleration is considered to be constant. For the problem's formulation one uses the elements from fig.10.

Horizontal component of the flight velocity (in the period of launching) in a point P is $V\cos\theta$. Angular displacement is

$$d\gamma = \frac{V\cos\theta \cdot dt}{R+h} \quad (43)$$

and the angular velocity is

$$\dot{\gamma} = \frac{V\cos\theta}{R+h}. \quad (44)$$

Vertical velocity in the current point P is described by the third equation (36). Thus, first two equations (36) become

$$\dot{V} = \frac{T\cos\alpha - X}{m} - g\left(\frac{R}{R+h}\right)^2 \sin\theta,$$
$$V(\dot{\theta} - \dot{\gamma}) = \frac{T\sin\alpha + Y}{m} - g\left(\frac{R}{R+h}\right)^2 \cos\theta. \quad (45)$$

Maximum flight distance is obtained when function

$$\gamma_t = \int_0^{t_1} \frac{V\cos\theta}{R+h} dt + \gamma_2 \quad (46)$$

has the maximum value. In order to obtain the maximum flight distance, one modifies the command variable (the attack angle) during the active flight. This function may be calculated only after the flight finish while the command must be modified every moment. That's why, instead of criterion (46) one uses criterion

$$I = \gamma = \int_0^{t} \frac{V\cos\theta}{R+h} dt. \quad (47)$$

Optimization system has the form presented in fig.11 and it is based on block diagrams from fig.7 and fig.9; the testing signal is $x_p = A\sin\omega t$.

Variables V, H, θ and α are measured or are calculated using an adequate navigation system. The system has two control loops (for flight velocity and for attack angle) and an optimization loop. Variables V_0 and ϑ_0 are the calculated values for V and ϑ.

The optimization loop works as follows: if $\gamma \neq \gamma_m$ (maximum value of γ) or $\frac{\partial\gamma}{\partial\alpha} \neq 0$, then command u_f has the form (34) or (35); u_f is a signal proportional with gradient $\frac{\partial\gamma}{\partial\alpha}$, which is processed by BFC and, by the mean of E.E., one operates in the commands link of the α's control loop so that α tends to value α_m (value of α for which $\gamma = \gamma_m$).

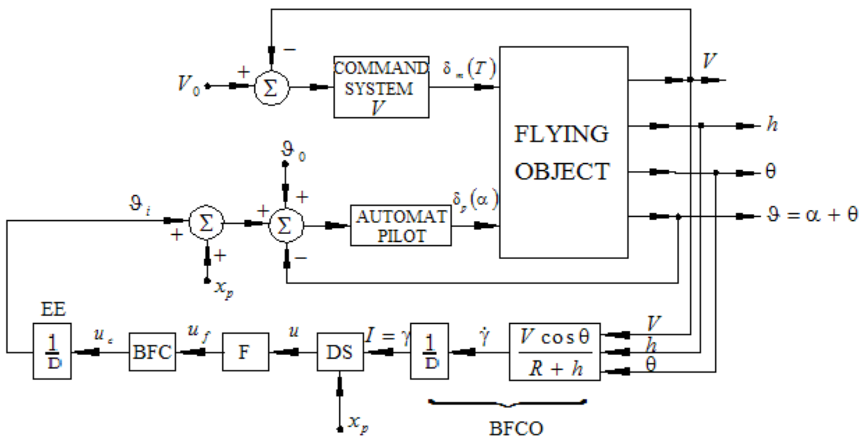

Fig.11 Block diagram of an automat optimization system with one optimization loop

In fig.12 one presents block diagram of an automat optimization system with 2 optimization loops; the difference between this block diagram and the one from fig.11 is that the same signal γ processes even in the control loop of the flight velocity. Testing signals have different frequencies; $x_{p_1} = A_1 \sin \omega_1 t, x_2 = A_2 \sin \omega_2 t$.

It results that $u_{f_1} \cong \frac{\partial \gamma}{\partial \alpha}$ and $u_{f_2} \cong \frac{\partial \gamma}{\partial V}$.

Now one presents a **flight automat optimization system for a short period of time**. Optimization criterion is described by equation

$$I = t = \int_{V_0}^{V} \frac{dV}{a_x} + g \int_{h_0}^{h} \frac{dh}{Va_x}. \qquad (48)$$

Acceleration a_x is obtained by the mean of thrust force T and drag force X.

For obtaining optimal operating regimes of the engine, one must modify the inlet position in rapport with the flight regime. As optimization criterion may be chosen: minimum flight time, maximum longitudinal acceleration and so on. Here, criterion (48) will be used. Because integration variables are V and h, for integration, mechanical equipments will be used; variables V and h are converted in rotation angular velocities which are multiplied with values $\frac{1}{a_x}$ and $\frac{g}{Va_x}$. The obtained output signals are proportional with integrals from equation (48). The structure of the optimization system is presented in fig.13.

By the mean of E.E., the command of extremal testing regulator is applied to inlet which is brought to the position that assures engine and flight optimal regimes.

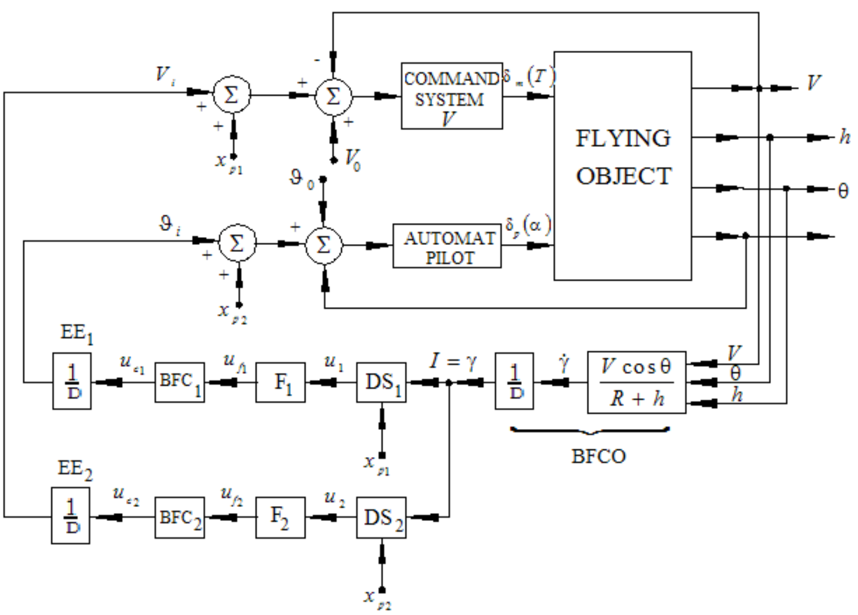

Fig.12 Block diagram of an automat optimization system with 2 optimization loops

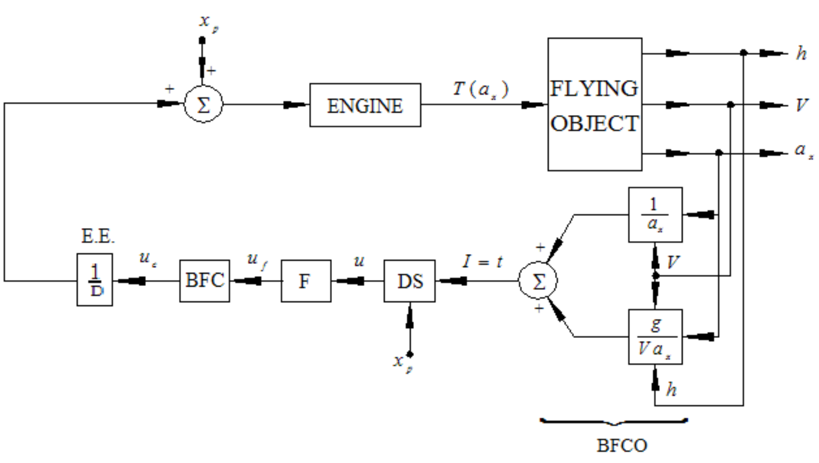

Fig.13 Flight automat optimization system for a short period of time

Another flight optimization system is based on **criterion of minimum fuel consume** expressed by equation

$$I = C_c = \frac{g}{a_x}\frac{G_T'}{V} = \frac{mg}{T - X}\frac{G_T'}{V}, G_T' = G_T + G_F, \quad (49)$$

where G_T is the primary fuel consume in time (flight without post combustion) and G_F – supplementary consume with post combustion. Thus, this is the case of a turbo-reactor with firing chamber.

In the block diagram of the system (presented in fig.14) the engine and the flying object are seen as two interconnected dynamic systems. Input variables of the dynamic model of the aircraft are thrust force T and rudder deflection δ_p; output variables are: pitch angle ϑ, attack angle α, flight velocity V, longitudinal acceleration a_x and flight altitude h. For the engine dynamic model, the input variables are: flight altitude h, flight velocity V, attack angle α, supplementary consume with post combustion G_F, primary fuel consume in time (flight without post combustion) G_T and nozzle's section S_n.

The output variables are: the rev of the low pressure rotor n, the temperature of the engine's working fluid at the entrance of the turbine T_3^*, the tempera ture of the fluid in the post combustion chamber T_F and the thrust force T.

The control of the engine's working regime is made by control of rev and control of the temperature T_3^*, while flight regime's control is made using the automat pilot.

The optimization loops affect post combustion fuel's consume and pitch angle. The frequencies and the amplitude of the testing signals are different.

Modification of the velocity V may be made by modification of thrust force T. Mathematical model of A is described by differential equation (with non dimensional variables)

$$\dot{V} = a_{11}V + b_m H_m(D)\delta_m + \dot{\nu}_x, \quad (50)$$

where $H_m(D)$ is the transfer operator of the engine, δ_m – command law and ν_x – disturbance that appears due to longitudinal wind blast. Command law may have the following form

$$\delta_m = k_m^V(\overline{V} - V) - k_m^{\dot{V}}\dot{V}, \quad (51)$$

where k_m^V and $k_m^{\dot{V}}$ are transmission ratios.

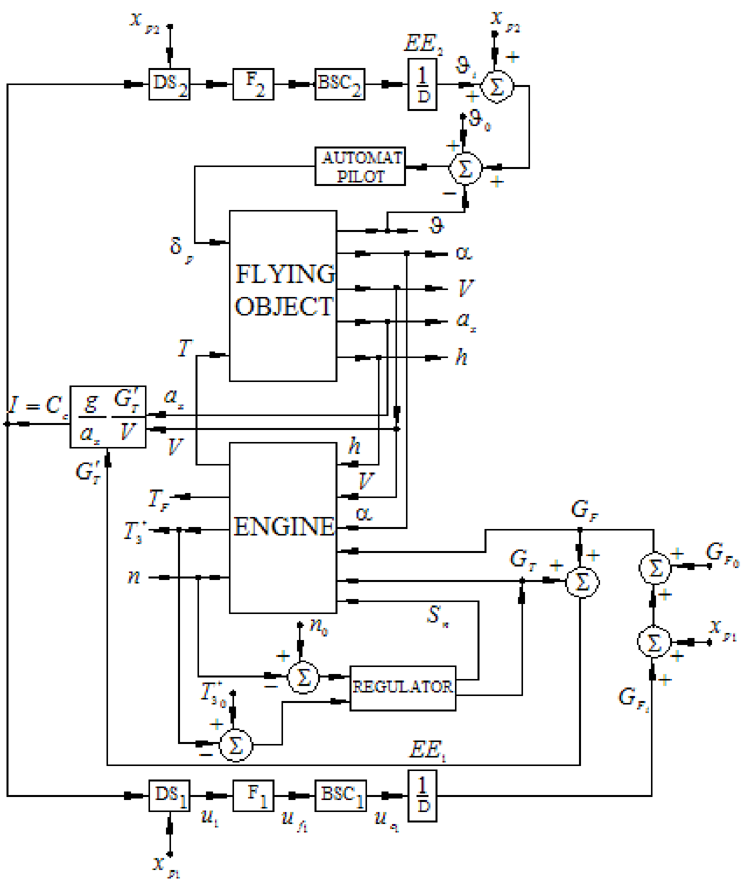

Fig.14 Optimization system based on criterion of minimum fuel consume

By elimination of δ_m from equations (50) and (51) one obtains the equation of the system

$$\dot{V} = [a_{11} - b_m H_m(\mathrm{D}) k_m(\mathrm{D})] V + b_m H_m(\mathrm{D}) k_m^V \overline{V} + \mathrm{D}\nu_x, \quad (52)$$

where

$$k_m(\mathrm{D}) = k_m^V + k_m^{\dot{V}} \mathrm{D} \quad (53)$$

represents transfer operator of the signal from command law.

Transfer operator $H_m(\mathrm{D})$ has one of the following forms (the first corresponds to the static system and the second one corresponds to the a static system)

$$H_m(\mathrm{D}) = \frac{1}{\tau \mathrm{D} + 1}, \quad H_m(\mathrm{D}) = \frac{1}{\mathrm{D}(\tau \mathrm{D} + 1)}, \quad (54)$$

with $\tau = \tau_m / \tau_a, \tau_m -$ engine time constant in rapport with thrust force and $\tau_a -$ aerodynamic time constant of the aircraft.

By introducing in equation (53) $H_m(\mathrm{D})$ for the static command system of the flight velocity, one obtains

$$(a_2 \mathrm{D}^2 + a_1 \mathrm{D} + a_0) V(\mathrm{D}) = b_V \overline{V}(\mathrm{D}) + \mathrm{D}(\tau \mathrm{D} + 1)\nu_x, \quad (55)$$

where

$$a_2 = \tau, a_1 = 1 - a_{11}\tau + b_m k_m^{\dot{V}}, \quad a_0 = -a_{11} + b_V, b_V = b_m k_m^V. \quad (56)$$

Stability conditions are $a_1 > 0, a_0 > 0.$ Generally, $a_{11} < 0$, but, in case of supersonic aircrafts, $a_{11} > 0$ and, anyway, $a_1 > 0$ because $a_{11}\tau << 1$. From the second condition $(a_0 > 0)$, one results $k_m^V > a_{11} / b_m$.

Optimal values of the transmission ratios may be obtained by approximation of the closed loop system's transfer operator in rapport with $\overline{V}$, with an imposed transfer operator

$$H_0(\mathrm{D}) = \frac{V(\mathrm{D})}{\overline{V}(\mathrm{D})} = \frac{\frac{b_V}{a_0}\frac{a_0}{a_2}}{\mathrm{D}^2 + \frac{a_1}{a_2}\mathrm{D} + \frac{a_0}{a_2}} = \frac{\frac{b_V}{a_0}\omega_0^2}{\mathrm{D}^2 + 2\xi\omega_0 \mathrm{D} + \omega_0^2}. \quad (57)$$

Thus,

$$\frac{a_0}{a_2} = \omega_0^2, \frac{a_1}{a_2} = 2\xi\omega_0. \quad (58)$$

8 Conclusion

One uses the models of rocket's motion around mass centre in vertical and horizontal plane and around longitudinal axis. Different structures of automatic command and stabilisation of the rocket's motion, using control laws that assure simultaneous cancellation of some state variables, are presented in this paper. Also, the author presents some optimization extremal systems for the aircrafts and rockets' flight using testing signals for the search of extreme point.

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