A General Mathematical Model Based on Laplace and Modified Z-Transform for Time and Frequency Domain Investigation of Three-Phase Switched Circuits

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Abstract : - A general mathematical method for both the time-domain and frequency domain analysis of power converters with periodic switching cuircuits is proposed .The method is based on mixed using of the Laúlace and modified Z-Transform in linear periodically time-varying system. The model was used for the analysis of three-phase voltage source inverters with Space Vector PWM feeding a three-phase static load, or an induction motor drive but it is applicable for all types of converters with an explicitly determined output voltage (converters with forced commutation) and periodical modulation..From the modulated waveforms we can easily obtained equations for the six step waveforms.The derived equations are validated using a 3 kW three-phase inverter**.**

Key-Words:- Mathematical model,Laplace transform,Modified Z-Transform

1 Introduction

Several methods have been presented for the time analysis of linear circuits containing periodically operated switches in electronic opened-loop systems [1],[2][3].However,the approach used in these methods depends heavily on matrix manipulations as they require matrix inversion as well as exponentiation. Besides,it requires solution of many algebraic equations.
Many electronic systems such as the inverters with Pulse Width Modulation (PWM) can be modeled with periodically varying parameters.In these inverters Space Vector PWM (SVPWM) has attracted great interest in recent years [4] since the harmonic characteristics are better than those of the other methods. At present, most of AC drives use some type of SVPWM.
Recent developments in high switching frequency power devices, such as IGBT, offer the possibility of developing high frequency PWM control techniques. Voltage waveforms of such modulated inverters contain many pulses and gaps. It is important to known current response for such complicated voltage waveforms in a drive design.In order to satisfy the required conditions for differential state equations describing the circuit behavior the continuity conditions due to the steady-state current at the transitions of states are used ,i.e.:

$$i(t_s^+)=i(t_s^-) \tag{1a}$$

also, condition of periodicity must be used:

$$i(0^+)=i(T^-) \quad (1b$$

where t_s are switching instants is a period.

Using (1a) and (1b) for the whole period of PWM we get algebraic equations that must be solved to obtain steady-state solution In case of transient solution when current is not periodic during T we must use (1a) and (1b) for the whole transient duration.The solution of the current response is intricate, as the number of the pulses is increasing.
This paper brings a new mathematical model that uses the Laplace and modified Z transform (mixed p-z approach).The model enables one to determine both steady state and transient state in a relatively simple and lucid formula. Method for finding the Laplace transform of the voltage vector is also presented.The solution is not dependent on the number of the pulses of the PWM pattern.The change of the switching instants is reflected in the solution by a change in only two values

2 Mathematical Model

We are going to investigate the three-phase half-bridge voltage source inverter fed from a DC voltage source and feeding a balanced three-phase Y-connected load. Generally, voltages and currents of three-phase circuits are explained by three variables, respectively. In case of three-phase load fed from a voltage source inverter shown in Fig.1,the phase voltages with respect to neutral point are:

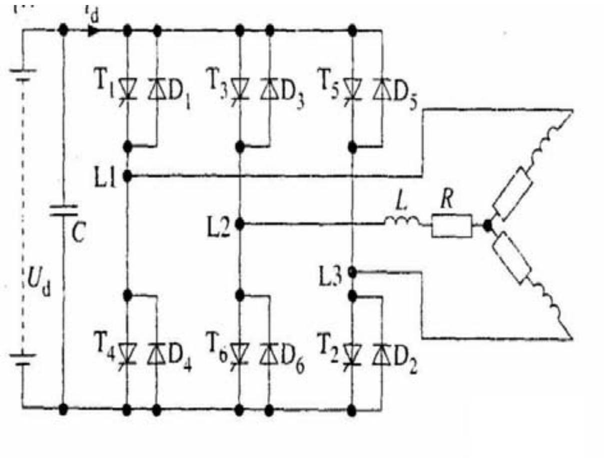

Fig.1 *Three-phase VSI feeding a static load*

$v_{in}(t)$, $i=a,b,c$ (2)
and the phase currents are: $i_i(t)$

Since the neutral point of the load is floating, the sum of three line currents is zero:

$$i_a(t)+i_b(t)+i_c(t)=0 \tag{3}$$

From the phase voltages(currents) we can determine voltage(current) space vector in $\alpha\beta$ coordinate system as follows:

$$\mathbf{V}(t)=\frac{2}{3}[\ v_{an}(t)+a.v_{bn}(t)+a^2.v_{cn}(t)\ ]= \tag{4}$$
$$=V_\alpha(t)+jV_\beta(t)$$
$$a=e^{j2\pi/3}=-\tfrac{1}{2}+j\frac{\sqrt{3}}{2}$$

The same equations are valid for the currents and magnetic fluxes.

2.1 Space Vector Modulation

For next calculations we express time with number of period n and variable within the period ε as follows:

$$t=(n+\varepsilon)T\ ,\quad n=0,1,2,...\quad ,\ 0<\varepsilon\le 1 \tag{5}$$

To obtain the required voltage vector $\mathbf{V}_{AV}$, the conduction times of the selected vectors are modulated according to the amplitude and angle of $\mathbf{V}_{AV}$, as shown in Fig.2

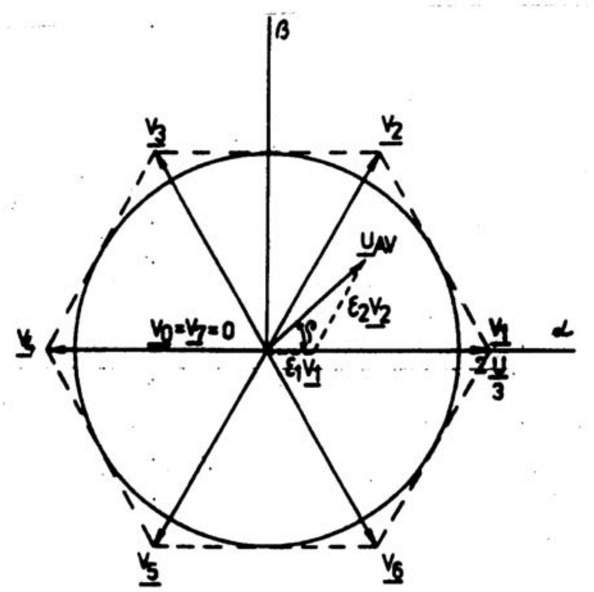

Fig.2 *Voltage Space Vectors in complex plane*

The required voltage vector $\mathbf{V}_{AV}$ is within the sampling period ΔT modulated as follows:

$$e^{j\rho}\mathbf{V}_{AV}\frac{\Delta T}{T}=\mathbf{V}_1\frac{\Delta T_1}{T}+\mathbf{V}_2\frac{\Delta T_2}{T}\ ,$$
$$\Delta T=\Delta T_1+\Delta T_2+\Delta T_0 \tag{6}$$

This type of modulation is called the Space Vector Modulation (SVM).
In (6) ΔT_1 is dwell time of vector $\mathbf{V}_1$, ΔT_2 is dwell time of vector $\mathbf{V}_2$,and ΔT_0 is dwell time of zero vector $\mathbf{V}_0$,or $\mathbf{V}_7$.
ΔT is sampling interval.

$$\Delta T=T/N_1 \tag{7}$$

ρ is an angle that defines position of the reference vector $\mathbf{V}_{AV}$ with respect to real axis in complex $\alpha\beta$ plane .$\mathbf{V}_1$ and $\mathbf{V}_2$ are adjacent to the voltage vector $\mathbf{V}_{AV}$ in a given sector n ,and the conduction per unit times are given from (6) by:

$$\Delta\varepsilon_1=\Delta T_1/T=\varepsilon_{1B}-\varepsilon_{1A}=g\sin(60^0-\rho)/N_1$$

$$\Delta\varepsilon_2=\Delta T_2/T=\varepsilon_{2B}-\varepsilon_{2A}=g\sin\rho/N_1 \tag{8}$$

$$\Delta\varepsilon_0=\Delta T_0/T=1/N_1-g\sin(60^0+\rho)/N_1$$

ε_{1A} and ε_{1B} are respectively, the beginning and end of duration of vector $\mathbf{V}_1$, ε_{2A} and ε_{2B} are respectively, the beginning and end of duration of vector $\mathbf{V}_2$. $\Delta\varepsilon_1$,$\Delta\varepsilon_2$ and $\Delta\varepsilon_0$ are respectively, per unit dwell times(duty ratios) of the applied vectors .

$$g=\frac{V_{AV}}{2\sqrt{3}V_{dc}} \tag{9}$$

g is the transformation (modulation) factor,
V_{dc} is the voltage of DC bus.

By substituting phase voltages for each switching state into (4), the following discrete space vectors are obtained:

$$\mathbf{V}(n) = \frac{2V_{dc}}{3} e^{jn\pi/3}, \qquad n=0,1,2\ldots\ldots \qquad (10)$$

These vectors thus form vertices of hexagon as shown in Fig.2.

As was mentioned, more vectors within sampling period are used.As the SVM is a periodical with T,the voltage vector can be expressed ,in n-th sector ,as

$$\mathbf{V}(n,\varepsilon)=\sum_{k=1}^{M}\frac{2V_{dc}}{3}e^{j\pi n/3}f(\varepsilon,k)e^{j\pi\alpha(k)/3} \qquad (11)$$

M is number of the vectors , which are used within a sector T

From (11) it can be seen, that all vectors are rotated in the next sector through $\pi/3$,and in each sector are vectors modulated with time dependency given by $f(\varepsilon,k)$, and also with the angle dependency given by $e^{j\pi\alpha(k)/3}$.

$f(\varepsilon,k)$ is a switching function which takes values 1 inside of $\Delta\varepsilon_k$, or 0 outside of $\Delta\varepsilon_k$, $\alpha(k)$ defines the sequence of the phase shift of the used vectors , and for SVM with two adjacent vectors has value 1 or 0

3 Laplace Transform of Voltage Vectors

To find the Laplace transform of (11) we can use relation between the Laplace and modified Z transform [8]. Using (5) , and its derivation

$dt=Td\varepsilon$

we can write for the Laplace transform of the periodic voltage vector:

$$\mathbf{V}(p) = \sum_{n=0}^{\infty}\left(\int_0^1 \mathbf{V}(n,\varepsilon)e^{-p(n+\varepsilon)T}Td\varepsilon\right) = T\int_0^1 \mathbf{V}(z,\varepsilon)e^{-pT.\varepsilon}d\varepsilon \qquad (12)$$

Where is noted : $z=e^{pT}$, z is operator of Z-transform.

$\mathbf{V}(z,\varepsilon)$ is the modified Z transform of $\mathbf{V}(n,\varepsilon)$ [7],[8] defined by equation:

$$\mathbf{V}(z,\varepsilon) = \sum_{n=0}^{\infty}\mathbf{V}(n,\varepsilon)z^{-n} \qquad (13)$$

With regard to SVM strategy mentioned, we get from (12) and (13)

$$\mathbf{V}(p) = \frac{2V_{dc}}{3}\frac{1}{p}\left(\frac{e^{pT}}{e^{pT}-e^{j\pi/3}}\right)\sum_{k=1}^{M}e^{j\pi\alpha(k)/3}\left(e^{-pT\varepsilon_{kA}}-e^{-pT\varepsilon_{kB}}\right) \qquad (14)$$

where $\varepsilon_{kA}T$ and $\varepsilon_{kB}T$ are respectively, the beginning and the end of application of k-th non-zero vector.

4 Current Response

Now, we suppose that voltage with the Laplace transform $\mathbf{V}(p)$ is feeding load with admittance:

$$Y(p)=\frac{A(p)}{B(p)}=\sum_{s=1}^{L_s}\frac{A(p_s)}{B'(p_s)}\frac{1}{p-p_s} \qquad (15a)$$

where: $B'(p_s)=\left[\frac{dB}{dp}\right]_{p=p_s}$,and (15b)

p_s are roots of the equation:

$B(p)=0$

L_s is a order of the polynomial B(p).

Thus, using (2) and (3) the Laplace transform of the load current can be expressed as:

$$\mathbf{I}(e^{pT},p)=\mathbf{V}(p)Y(p)=R(e^{pT})Q(p) \qquad (16)$$

As can be seen from (16),the Laplace transform of the current vector consists of two multiplicative parts.One ($R(e^{pT})$) is a function of z-operator,the other (Q(p)) is a function of p-operator .

$$R(e^{pT}) = \frac{e^{pT}}{e^{pT}-e^{j\pi/3}} \qquad (17)$$

$$Q(p)=\frac{2V_{dc}}{3p}\frac{A(p)}{B(p)}\sum_{k=1}^{M}e^{j\pi\alpha(k)/3}\left(e^{-pT\varepsilon_{kA}}-e^{-pT\varepsilon_{kB}}\right)$$

By transforming (16) into modified z -space we get:

$$\mathbf{I}(z,\varepsilon)=R(z).Z_m\{Q(p)\} \qquad (18)$$

In order to find Z_m transform of Q(p) we must use the translation theorem in Z-transform which holds

$$Z_m\{e^{-p.a}.F(p)\} = z^{-x}.F(z,\varepsilon-a+x) \qquad (19)$$

with $Z_m\{\ \}$ denoting the modified Z transform operator.

And where parameter x is given by

$$x = \begin{cases} 1 & \text{for } 0\le\varepsilon<a \\ 0 & \text{for } a\le\varepsilon<1 \end{cases} \qquad (20)$$

If we want to express translation for k-th pulse,with the beginning ε_{kA} and the end ε_{kB} ,

(pulse-width $\Delta\varepsilon_k=\varepsilon_{kB}-\varepsilon_{kA}$) we can use two parameters, namely m_k and n_k to determine per unit time for prepulse,inside-pulse and postpulse,respectively.

m_k is a parameter that defines the beginning of k-the pulse $\varepsilon_{kA,}$ n_k is a parameter that defines the end of the k-pulse ε_{kB} .According to (20) we can write:

$$m_k=\begin{cases}1 & \text{for } 0\le\varepsilon<\varepsilon_{kA}\\ 0 & \text{for } \varepsilon_{kA}\le\varepsilon<1\end{cases}\qquad n_k=\begin{cases}1 & \text{for } 0\le\varepsilon<\varepsilon_{kB}\\ 0 & \text{for } \varepsilon_{kB}\le\varepsilon<1\end{cases}\tag{21}$$

Using parameters m_k,n_k , and Heaviside theorem (15a) we can express (18) with help of (21) and (11) in the modified Z-space:

$$I(z,\varepsilon)=\frac{2V_{dc}}{3}\frac{z}{(z-e^{\frac{j\pi}{3}})}\sum_{k=1}^{M}\left\{\begin{array}{l}\frac{A(0)}{B(0)}e^{\frac{j\pi\alpha(k)}{3}}\frac{z}{z-1}(z^{-m_k}\\ -z^{-n_k})+\\ \sum_{s=1}^{L_s}\frac{A(p_s)}{p_sB'(p_s)}e^{\frac{j\pi\alpha(k)}{3}}\\ \frac{ze^{p_sT\varepsilon}}{z-e^{p_sT}}\\ \left[\begin{array}{l}z^{-m_k}e^{p_sT(m_k-\varepsilon_{kA})}\\ -z^{-n_k}e^{p_sT(n_k-\varepsilon_{kB})}\end{array}\right]\end{array}\right\}\tag{22}$$

Equation (22) has simple poles $e^{j\pi/3}$, 1 ,e^{p_sT}.The inverse Z transform of (22) can be found using the residua theorem .

$$I(n,\varepsilon)=\frac{1}{2\pi j}\oint I(z,\varepsilon)z^{n-1}dz\tag{23}$$

If doing so, we can express the time dependency of the load current by the following formula:

$$\mathbf{i}(n,\varepsilon)=\sum_{k=1}^{M}\left\{\begin{array}{l}\frac{2V_{dc}}{3}e^{j\pi\alpha(k)/3}\left[\begin{array}{l}\frac{A(0)(e^{-j\pi m_k/3}-e^{-j\pi n_k/3})}{B(0)(e^{j\pi/3}-1)}+\\ \sum_{s=1}^{L_s}\frac{A(p_s)e^{p_sT\varepsilon}}{p_s.B'(p_s)(e^{j\pi/3}-e^{p_sT})}\\ (e^{-j\pi m_k/3+p_sT(m_k-\varepsilon_{kA})}-\\ -e^{-j\pi n_k/3+p_sT(n_k-\varepsilon_{kB})})\end{array}\right]\\ e^{j\pi/3(n+1)}+\end{array}\right\}$$

$$+\sum_{k=1}^{M}\left\{\frac{2V_{dc}}{3}e^{j\pi\alpha(k)/3}\sum_{s=1}^{L_s}\begin{array}{l}\frac{A(p_s)}{p_sB'(p_s)}\frac{e^{p_sT}}{(e^{j\pi/3}-e^{p_sT})}\\ (e^{-p_sT\varepsilon_{kB}}-\\ -e^{-p_sT\varepsilon_{kA}})e^{p_sT(n+\varepsilon)}\end{array}\right\}\tag{24}$$

$=\mathbf{i}_S(n,\varepsilon)+\mathbf{i}_T(n,\varepsilon)$

The solution contains two parts.

Since p_s includes a negative real part (we consider stable systems), the second portion of (24) consisting $e^{p_sT(n+\varepsilon)}$ attenuates, for $n\rightarrow\infty$, forming the transient component of the current space vector $\mathbf{i}_T(n,\varepsilon)$.The term

$e^{j\pi(n+1)/3}=\cos\pi(n+1)/3+j.\sin\pi(n+1)/3$

therefore, the first part of (24) is the steady-state component of the current space vector $\mathbf{i}_S(n,\varepsilon)$.

As an example ,let us consider three-phase R,L series load. Equation (24) has only one simple root:

$$p_1=\frac{-R}{L}\tag{25}$$

By substituting p_1 into (14) we can write for the load current components:

a)steady-state component:

$$\mathbf{i}_S(n,\varepsilon)=\sum_{k=1}^{M}\frac{2V_{dc}}{3}e^{j\pi\alpha_k/3}\frac{1}{R}\left\{\begin{array}{l}(\frac{e^{-j\pi m_k/3}-e^{-j\pi n_k/3}}{e^{j\pi/3}-1})-\\ (\frac{e^{-RT\varepsilon/L}}{e^{j\pi/3}-e^{-RT/L}})\\ (e^{-j\pi m_k/3-RT(m_k-\varepsilon_{kA})/L}\\ -e^{-j\pi n_k/3-RT(n_k-\varepsilon_{kB})/L})e^{j\pi(n+1)/3}\end{array}\right\}\tag{26}$$

b)transient component:

$$\mathbf{i}_T(n,\varepsilon)=\sum_{k=1}^{M}\left\{\frac{2V_{dc}}{3}e^{j\pi\alpha_k/3}\frac{1}{R}e^{-RT/L}\left(\frac{e^{RT\varepsilon_{kA}/L}-e^{RT\varepsilon_{kB}/L}}{e^{j\pi/3}-e^{-RT/L}}\right)e^{-RT(n+\varepsilon)/L}\right\}\tag{27}$$

Fig.3 shows trajectory of the steady-state current vector in complex $\alpha\beta$ plane. This trajectory is given by (26).The parameters of the modulation are:N_1=7,g=0.8

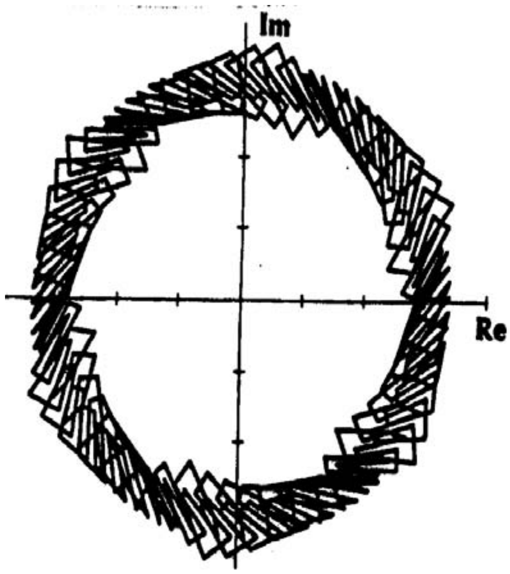

Fig.3 *Trajectory of stator current space vector-steady state*

.Fig.4 shows the phase A steady-state current given by the real part of Fig.3 and phase A voltage given by real part of 11.

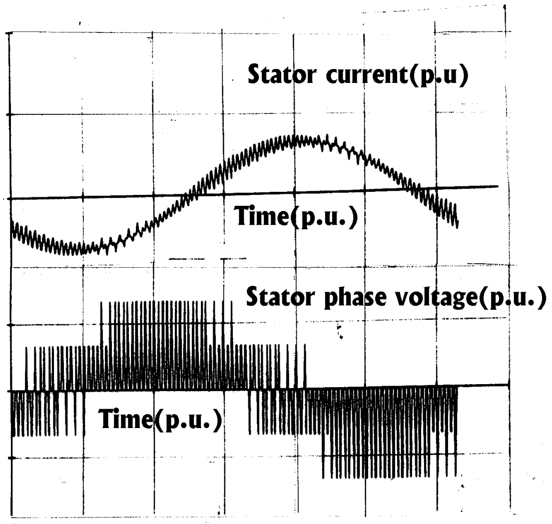

Fig.4 *Phase A current (upper trace) and phase A voltage(bottom trace)-steady state*

From (24) we can derive easily the solution for six-step waveform (without modulation).

Analytical expressions for the steady-state currents of the system with a three-phase VSI with six-step waveform feeding a three-phase static inductive load were presented in [9] (Equations (1)-(8)).These expressions were derived by existing methods using (1a) and (1b), which necessitates solving algebraic equations to express the initial value of the load phase current i_0. From the proposed mathematical model we can determine the solution in a very simple form.

In Eq (26), which is valid for the steady-state, we substitute:

M=1 (one pulse per sector) ,$\varepsilon_{1A}=0$, ,$\varepsilon_{1B}=1$,$m_1=0$,$n_1=1$.

By substituting these values into (26) we obtain for the steady-state vector current of the RL load:

$$\mathbf{i}_S(n,\varepsilon)=\frac{2V_{dc}}{3R}e^{\frac{j\pi n}{3}}\left[1-e^{\frac{-RT}{L}\varepsilon}\left(\frac{e^{\frac{j\pi}{3}}-1}{e^{\frac{j\pi}{3}}-e^{\frac{-RT}{L}}}\right)\right] \quad (28)$$

Putting n=0 and $0<\varepsilon\leq1$,we get solution for the first sixth of the period,for n=1 and $0<\varepsilon\leq1$, we get solution for the second sixth of the period ,etc.

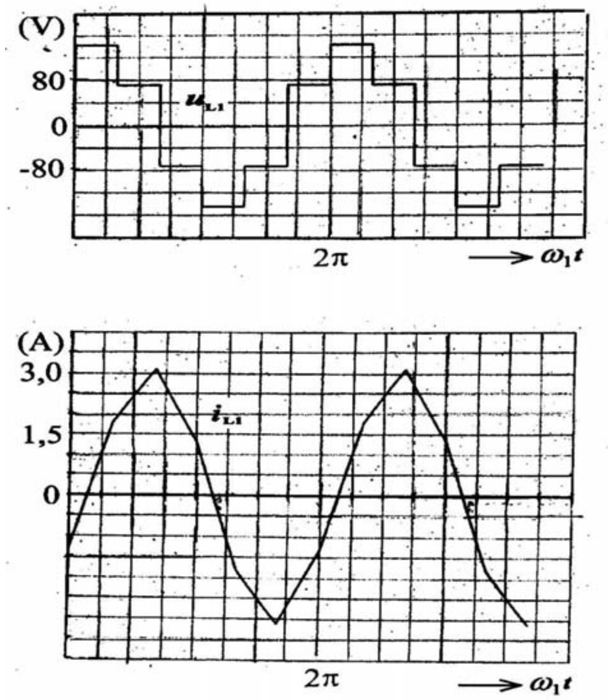

Fig.5 *Phase A voltage and current-steady-state ,six step waveforms*

The A-phase current is given by real part of (28)

$$i_A(n,\varepsilon)=\mathrm{Re}\{\,\mathbf{i}_S(n,\varepsilon)\} \quad (29)$$

For the voltage vector with six-step waveform we can write :

$$\mathbf{V}\,(n,\varepsilon)=2/3(V_{dc}.e^{jn\pi/3})=\mathbf{V}\,(n) \quad (30)$$

And the A phase voltage is given by a real part:

$$v_A\,(n,\varepsilon)=v_A(n)=\mathrm{Re}\{\,\mathbf{V}\,(n,\varepsilon)\}=2/3(V_{dc}.\cos\pi.n/3) \quad (31)$$

Fig.5 shows phase A current given by (29) and phase A voltage),for six-step waveforms (without modulation)

If we compare Fig.5 with the waveforms in [9].we can see that the results are identical. But the presented. mathematical model contains only one equation (28) which is valid for the whole output period (n=0,1,2,3,4,5, , $0<\varepsilon\leq1$) The model in [9] necessitates solution for every sixth of the period, which means six equations per one period . Besides, it requires solving the initial value of the load phase current.

5 Experimental Results

Validation of the derived analytical equations was also carried out using measurements with a 3 kW three-phase inverter supplying RL load. A three-phase static inductive load has the parameters: $R=623\Omega, \omega_1 L=502\Omega$. An IGBT inverter utilized Space Vector PWM with sampling intervals $N_1=7$,modulation factor g=0,8,and with a fundamental frequency of the output voltage of 50 Hz.

Fig.6 shows experimental waveforms of the phase A steady-state load current (upper trace)and the phase A load voltage (lower trace).

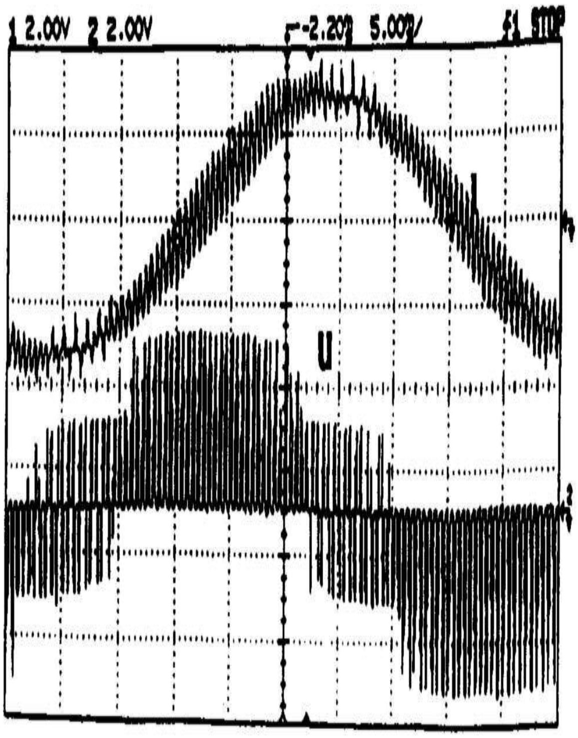

Fig.6 *Experimental results.Phase A current and voltage-steady-state*

The corresponding theoretical phase A steady-state current and phase voltage given are shown in Fig.4.As can be seen, there is very good agreement between measured and theoretical results, with correlation being better than 5% over most of the load range.

Fig.7 shows phase A voltage and current measured in the inverter without modulation-six step waveforms.

If we compare Fig.7 with theoretical waveforms given in Fig.5 we can see very high correlation.

The simple form of equations (28) and (31) can be used directly to assess the system performance.

All the dependencies were graphical visualized by the programme MATHCAD [11]

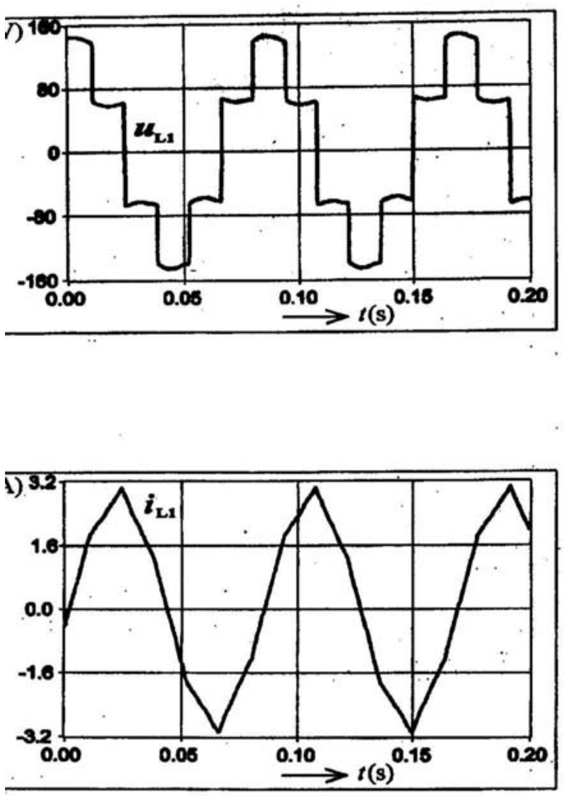

Fig.7 *Experimental results.Phase A voltage and current-six step waveforms*

6 Frequency-domain analyze6*4.a*

6.a) Fourier series for the stator voltage vectors

We shall calculate the Fourier series of the periodic variation of the stator voltage space vector [6]:

$$\mathbf{V}(n,\varepsilon) = \sum_{k=-\infty}^{\infty}\left[\ \mathbf{C}_k e^{(jk\omega_1(n+\varepsilon)T}\ \right] \tag{33}$$

where $\omega_1=2\pi/T_1$ is the angular frequency of the fundamental harmonic. From (33) , the phase voltages can be expressed as:

$$v_{An}(n,\varepsilon) = \mathrm{Re}\left\{\sum_{\nu=-\infty}^{\infty}(\mathbf{C}_\nu\, e^{j\nu\omega_1(n+\varepsilon)T})\right\} \tag{33a}$$

$$v_{Bn}(n,\varepsilon) = \mathrm{Re}\left\{e^{j4\pi/3}\sum_{\nu=-\infty}^{\infty}(\mathbf{C}_\nu\, e^{j\nu\omega_1(n+\varepsilon)T})\right\} \tag{33b}$$

$$v_{Cn}(n,\varepsilon) = \mathrm{Re}\left\{e^{j2\pi/3}\sum_{\nu=-\infty}^{\infty}(\mathbf{C}_\nu\, e^{j\nu\omega_1(n+\varepsilon)T})\right\} \tag{33c}$$

To derive the coefficients of the Fourier series, we can use the relationship between the Laplace transform of the periodic waveform and Fourier coefficients:

$$\mathbf{C}_k = \left\langle \frac{1}{T_1}\left[\, 1-(e^{-pT_1})\,\mathbf{V}(p)\right] \right\rangle_{p=jk\omega_1} \quad (34)$$

$\mathbf{V}(p)$ is given by (28).
By substituting (28) into (44) we obtain the Fourier coefficients as follows:

$$\mathbf{C}_\nu = \mathbf{C}_{(1+6\nu)} = \frac{2V_{dc}}{3\pi j(1+6\nu)}\sum_{k=1}^{M} e^{(\mathbf{j}\pi\alpha(k)/3)}\left[\, e^{(-\mathbf{j}(1+6\nu)\frac{\pi}{3}\varepsilon_{k\mathbf{A}})} - e^{(-\mathbf{j}(1+6\nu)\frac{\pi}{3}\varepsilon_{k\mathbf{B}})}\right] \quad (35)$$

where

$$\nu = 0, \pm 1, \pm 2, \ldots \quad (36)$$

6. b Fourier series for the phase voltages

From voltage-space expression (35) we obtain the phase voltages as a real part of the complex equation (35) as:

$$v_{An}(n,\varepsilon) = \sum_{\nu=-\infty}^{\infty}\left[\frac{2V_{dc}}{\pi 3(1+6\nu)}\sum_{k=1}^{M}\left[\sin\frac{\pi}{6}(1+6\upsilon)\Delta\varepsilon_k\right] \sin\left[\,(1+6\nu)\left[\omega_1(n+\varepsilon)T-\frac{\pi}{6}(\varepsilon_{kA}+\varepsilon_{kB})\,\right] + \frac{\pi\alpha(k)}{3} - 2\pi/3 \right]\right] \quad (37)$$

As an example we can see from Fig.8 the Fourier approximation of the voltage space-vector with space-vector modulation. We take into account first 10 harmonics

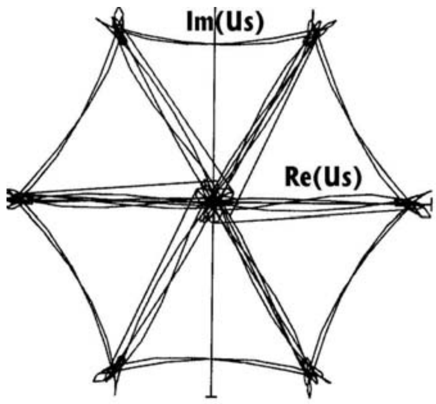

Fig.8 *Fourier series approximation of the voltage space vector*

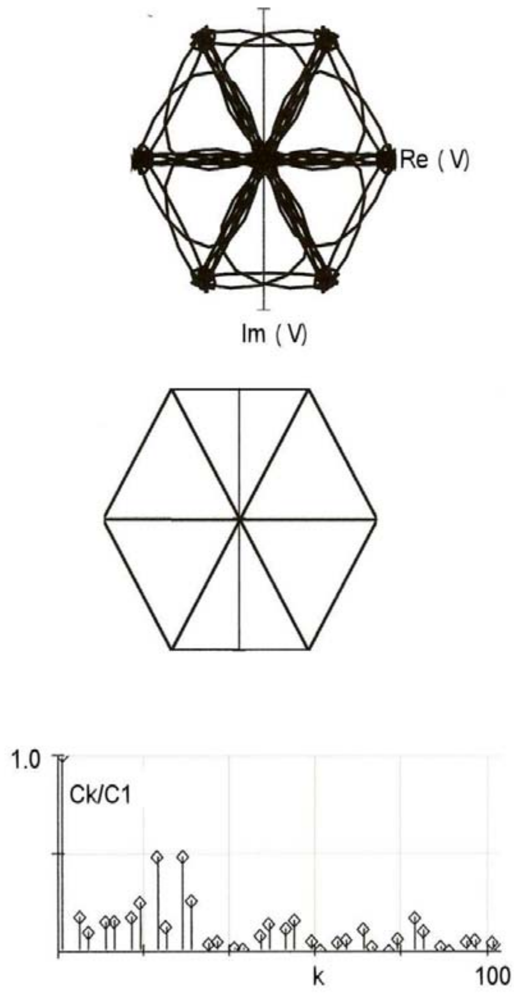

Fig.9 *Voltage space vectors and harmonic spectrum. g=0.3,N_1=4 (f_{SW}=1200 Hz) ,f_1=50 Hz. Top: Fourier series approximation of voltage space vectors for ν=20.Middle: Ideal voltage space vectors. Bottom: Harmonic voltage spectrum.*

From Fig.9 we can see the Fourier series approximation of the voltage space-vector (upper trace); ideal trajectory (middle trace) and Fourier spectrum (bottom trace) again for the space-vector PWM modulated voltage.

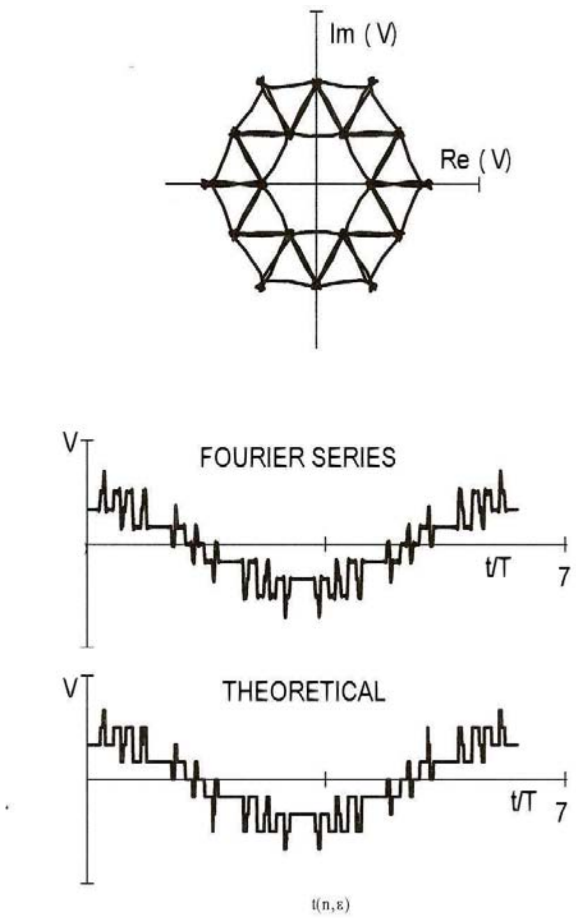

Fig.10 *Voltage space vector and phase voltages*

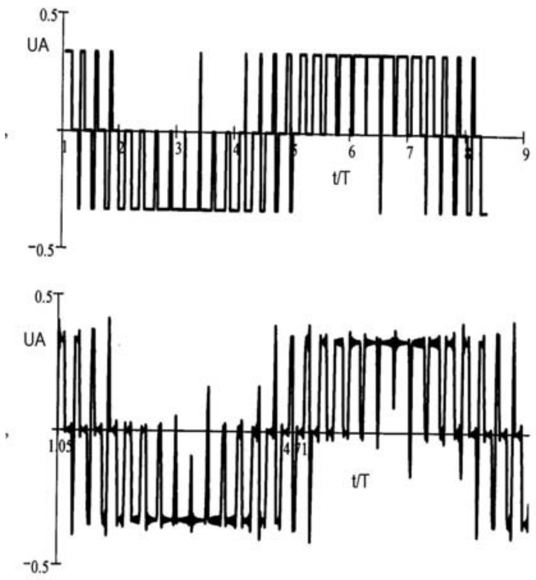

Fig.*11 Phase voltage and its approximation*

Fig.10 shows the voltage-space vector given by the proposed analytical model and its phase-voltage approximation.

Again,phase voltage and its analytical approximation are shown in Fig.11.As we can see,there is good correlation between analytical and theoretical results.

6 Conclusion

An approach for the analysis of linear system containing periodically operated switches is described. The approach was demonstrated for the inverter with Space Vector PWM ,but it is applicable for all types of converters with explicitly determined output voltage. The mathematical model uses the Laplace and modified Z transforms.The steady -state and transient components of the load current are determined in a simple and lucid form that it avoids involved matrix inversion as well as exponentiation. Experimental results prove the feasibility of the proposed mathematical model as compared with the conventional methods. The theory is based on a relatively simple model,but correlation between measurements and calculations is very good.

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