Mathematical Models of High-Speed Trains Movement

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Abstract: - In the paper it is presented the mathematical model of the useful movement of the high-speed trains. On the mathematical model basis it is built the structural diagram associated of the useful movement. The obtained structural scheme can be easily implemented in SIMULINK. An immediately example of the utilization of the useful movement model it constitutes the drawing of the movement diagrams, which it illustrates the dynamics aspect of the useful movement. Like application, by means of the SIMULINK model they are traced, on the basis of the traction and braking characteristics and of the imposed conditions of the route, the movement diagram of the high-speed train.

Key-Words: - modeling, simulation, electric vehicle, high-speed train, useful movement

1 Introduction

In any electric vehicle, the developed electromagnetic torque of the traction electric motors has been transmitted of the motor wheels. These, through turning, establish the translation movement of the vehicle on the rail.

Because the rotation movement of the motor wheels it is "attached", through transmission, of the traction motor rotor movement, it results that the motor wheels movement equation can be deduced from the motor equation with which these are coupled. Thus, the motor torque transmitted of the motor wheels it is $M_R=i\cdot\eta_t\cdot M_2$, where M_2 is the developed useful torque of the traction motor. At the running radius $r=D_r/2$, the motor torque M_R it corresponds it the motor force F_o[N] at wheels [1], [2], [3], [4]

$$Fo = \frac{M_R}{D_r/2} = \frac{2}{D_r}\cdot i\cdot\eta_t\cdot M_2 \quad (1)$$

In the slip absence, the peripheral speed v of the motor wheels (which they are turned with the angular speed Ω_0)

$$v = \Omega_0\cdot\frac{D_r}{2} \quad (2)$$

it is the same with the translation movement speed (on the rail) of the vehicle. As $\Omega_0=\Omega_m/i$, where Ω_m it is the angular speed of the traction motor rotor, it results that

$$v = \frac{D_r\cdot\Omega_m}{2\cdot i} \quad (3)$$

The relations (1) and (3) are fundamentals in the traction calculations. They permit the establishment of the vehicle characteristics depending on the useful torques quantity M_2 and on the angular speed of the shafts of the all its traction motor (in the equality case of the diameters of the all motor wheels).

In the running, both under its traction motors action and under the rail resistance influence, it achieves the useful translation movement of the all vehicle in the long rail.

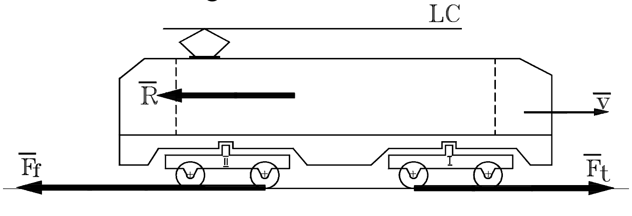


Fig.1 External forces which they establish the movement

Moreover, the useful movement of the train it is established only of the external forces action. These can be (fig.1):

- motor active forces (of traction), with the resultant $\overline{F_t}$ (of controllable magnitude, which they operate on the sense and direction of the useful movement);

- braking active forces, with the resultant $\overline{F_f}$ (of controllable magnitude, which they operate on the useful movement direction, but on the contrary sense of the speed vector $\bar{v}$) and

- train resistance forces with resultant $\bar{R}$.

The active forces $\overline{F_t}$ (of traction) and $\overline{F_f}$ (of braking) they operate not simultaneously in none running regime (presence of one it is equivalent to the exclusion of other), while the train resistance $\bar{R}$ it is presented all the time, even in the active forces absence (in the coasting regime without current).

2 Modelling of traction motor

In the power schemes of motor electric vehicles, the traction induction motor it is the final element from the equipments chain by conversion of energy. After all, it achieves the electromechanical conversion of energy making thus possibly the movement. Like complex electromechanical system, the induction motor will have decomposed functionally between an electromagnetic part and a mechanical part (fig.2).

Both the *M* electromagnetic torque and the Ω_m mechanical speed of rotor, they intervene like internal variables, by interaction, between those two functional

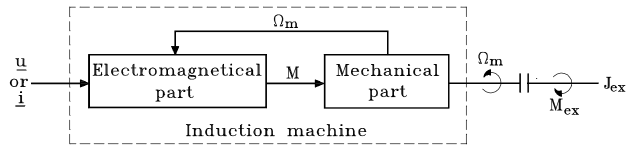


Fig.2 Functional parts of traction induction motors

parts. In the motor vehicle case, the mechanical part of traction induction motor it is coupled (through the transmission intermediation) with the motive axle and can be modelled in the shape of the useful movement or/and the elastic mechanical transmissions [1]. For can be connected, the models must be achieved in accordance to the same principles, indifferently that they describe the phenomenon by electric nature or mechanical nature. In the choice case of fixed reference system, connected at stator, the electromagnetic part of induction motor it will have described by the equations [5], [6], [7], [8], [9]:

$$\frac{d\underline{\psi}_s}{dt} = \underline{u}_s - R_s \cdot \underline{i}_s$$

$$\frac{d\underline{\psi}_{r'}}{dt} = j \cdot p \cdot \Omega_m \cdot \underline{\psi}_{r'} - R_{r'} \cdot i_{r'} \tag{4}$$

$$\underline{i}_s = \frac{\underline{\psi}_s - \frac{L_u}{L_{r'}} \cdot \underline{\psi}_{r'}}{\sigma L_s}; \quad \underline{i}_{r'} = \frac{\underline{\psi}_{r'} - \frac{L_u}{L_s} \cdot \underline{\psi}_s}{\sigma L_{r'}}$$

$$M = \frac{3}{2} \cdot p \cdot Im\{\underline{i}_s \cdot \underline{\psi}_s^*\}$$

where

$\underline{u}_s$= stator voltage vector
$\underline{i}_s$= stator current vector
$\underline{i}_r'$= rotor current vector
$\underline{\Psi}_s$= stator flux vector
$\underline{\Psi}_r'$= rotor flux vector
L_u= magnetizing inductance
L_s= stator inductance
L_r'= rotor inductance
p= number of pole pairs
R_s= stator resistance
R_r'= rotor resistance and

$\sigma = 1 - \frac{Lu^2}{Ls \cdot Lr'}$ = leakage total coefficient of motor.

On the equations basis (4) they are represented the structural diagram and mask block of the electromagnetic part of induction motor (fig.3).

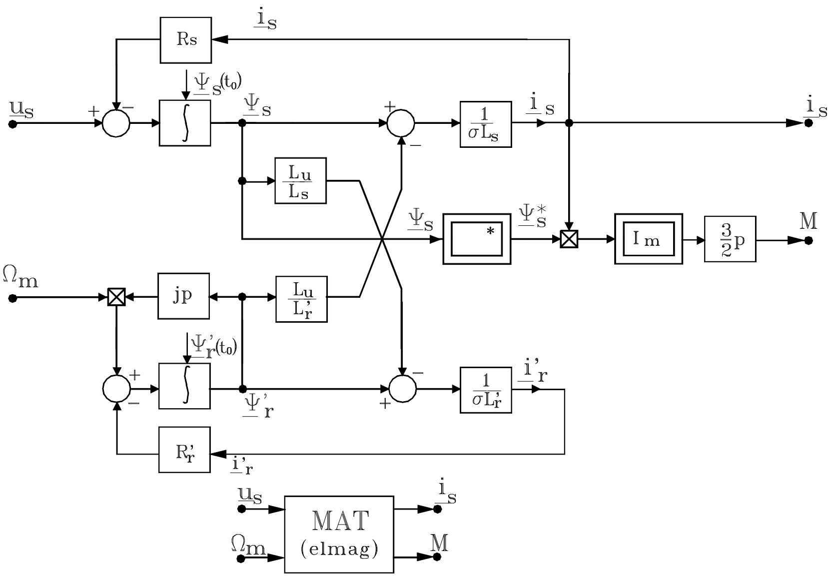


Fig.3 Structural diagram and mask block for electromagnetic part of induction motor

The structural diagram of electromagnetic part can be coupled both with the structural diagram of machine converter through the intermediation of the input variables $\underline{u}_s$ and output variables $\underline{i}_s$ and with the structural diagram of mechanical part through the intermediation of input quantities Ω_m and output quantities M.

3 Modelling of Useful Movement

The train resistance forces, between with the traction characteristics of the high-speed train, they permit the useful movement study of the vehicle. In these conditions the useful movement equations is [1], [2], [3], [9]

$$m^* \cdot \frac{dv}{dt} = F - R; \quad m^* = m \cdot \xi \tag{5}$$

where F it is F_t in traction regime or - F_f in braking regime, and ξ is the coefficient of increase the mass of the train that take account to the presence and weight of the turning parts from the train structure (ξ=1.06…1.2).

Thus, through the coefficient of increase the mass agency it can make abstraction of the turning parts presence, replacing the real mass "m" of the train with a "fictitious mass" $m^*=m\cdot\xi$ found in translation movement with the speed "v", the same of the considered train. From the physical viewpoint, this is equivalently with fictitious replace of the mechanical system of rigid solid parts through a material point with inertial mass $m^*=m\cdot\xi$.

For the dynamic aspect approach of the useful movement it is necessary a mathematical model. In this purpose it is considered a electric vehicle of mass $m[t]$ and coefficient of increase the mass ξ having the train resistance r[daN/t]. On the useful movement duration, the speed v(t) and the distance x(t) they are ruled at the equations

$$m \cdot \xi \cdot \frac{dv}{dt} = F - R\,; \quad \frac{dx}{dt} = v \tag{6}$$

If the movement has been made under the useful torques action M_2 (identical), developed of those "z" traction motors of the electric vehicle, then in accordance with the relations (1) and (3)

$$\Omega_m = \frac{2 \cdot i}{D_r} \cdot v\,; \quad F = z \cdot \frac{2}{D_r} \cdot i \cdot \eta_t \cdot M_2 \tag{7}$$

Moreover, if the mass m of the train it is expressed in [t], the total train resistance R[N] it is established with

$$R[\mathrm{N}] = r[\mathrm{daN/t}] \cdot m[\mathrm{t}] \cdot 10 \tag{8}$$

The equations ensemble (6), (7), (8) they form the mathematical model of the useful movement. Written together, in the shape of

$$v = \frac{1}{m \cdot \xi} \int (F-R)dt\,; \quad \Omega_m = \frac{2 \cdot i}{D_r} \cdot v; \quad x = \int v \cdot dt\,;$$
$$F = z \cdot \frac{2}{D_r} \cdot i \cdot \eta_t \cdot M_2\,; \quad R = (r_{ps}(v) \pm i_{de}(x) + r_c(x)) \cdot m \cdot 10 \tag{9}$$

they permit the structural diagram construction of useful movement (fig.2).

For the mask block they have been considered like input quantity the M torque and like output quantity the Ω_m speed, time variable quantities on the useful movement duration.

By means of this scheme ("coupled" at the structural diagram of electromagnetic part of traction motor fig.3) can be simulated the useful movement of any electric vehicle as compared with the concrete modality by leadership (or by control) of this. Accordingly they are obtained the running diagrams v(t) and x(t), too. The modification of vehicle mass, of dependences i_{de}(x) or r_c(x), specific to certain vehicle or route, can be easily operated, obtaining an exact mathematical model, which it respects all the running conditions.

In the motor wheels diameters inequalities case, the scheme suffers a minor change, the total force F resulting like sum of partial forces developed by each motor partly.

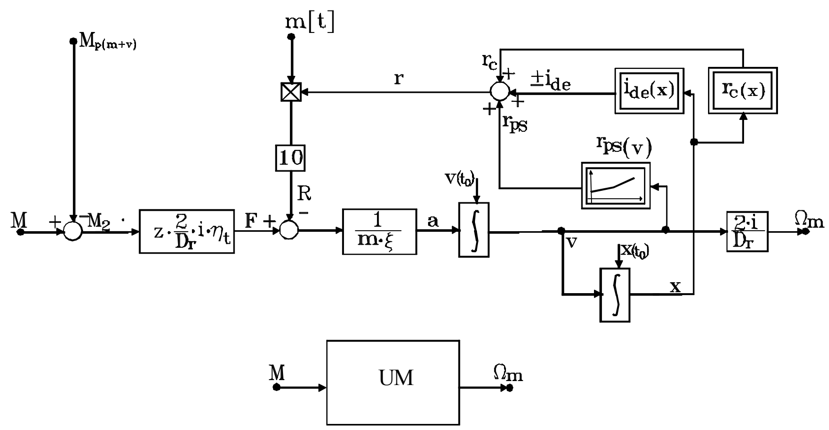


Fig.4 Structural diagram and mask block of useful movement

4 Simulink Model of Useful Movement

The SIMULINK model corresponding to the useful movement of the electric vehicle can be easily implemented, having with a view the topological comparison with the associate structural diagram. In the traction electric motor consideration case, too, at the electromagnetic part model *"MAT"* it is coupled the block *„UM"* (fig.5). The two blocks, for the electromagnetical part and for the useful movement part, has been achieved on the corresponding structural diagrams basis (fig. 6).

Within the framework of the useful movement (fig.4), the transmission has been supposed ideal. In simulations in which they have been desired to be taken account the other mechanical phenomenous (like example the stick-slip [10]), too, can be interspersed the afferent elastic mechanical transmission model [1], [2], [3].

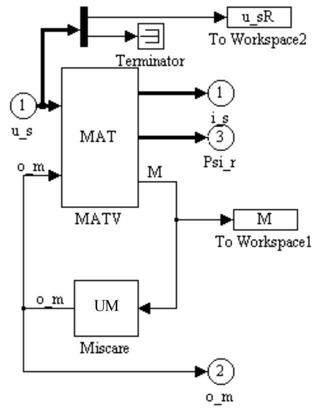
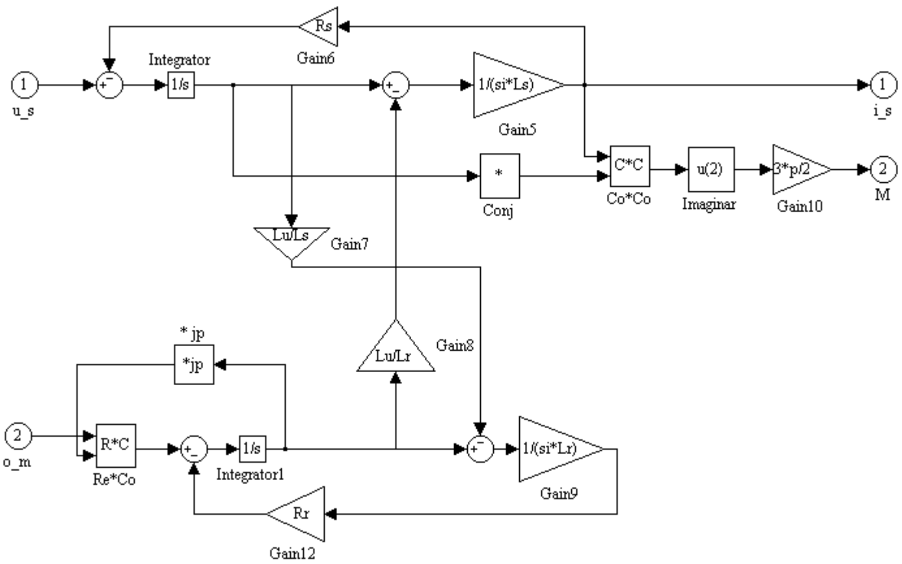
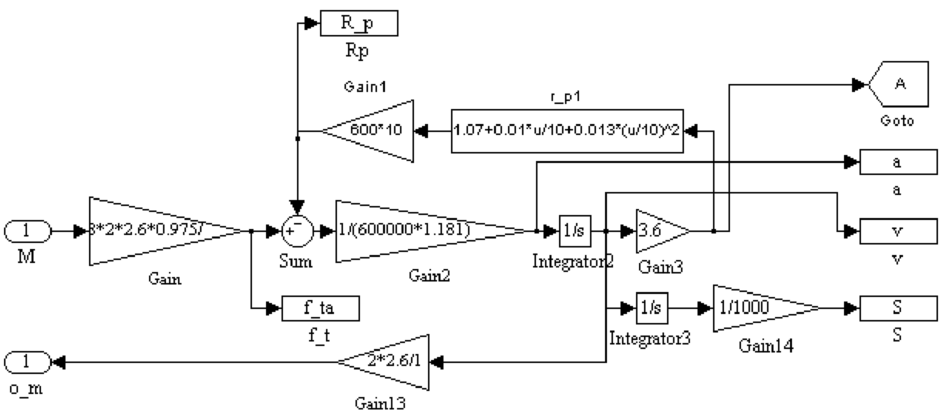


Fig.5 Model of traction induction motor at consideration useful movement of train



a) mask block „MAT"



b) mask block „UM"

Fig.6 SIMULINK models
a) for electromagnetic part of traction induction motor; b) for useful movement

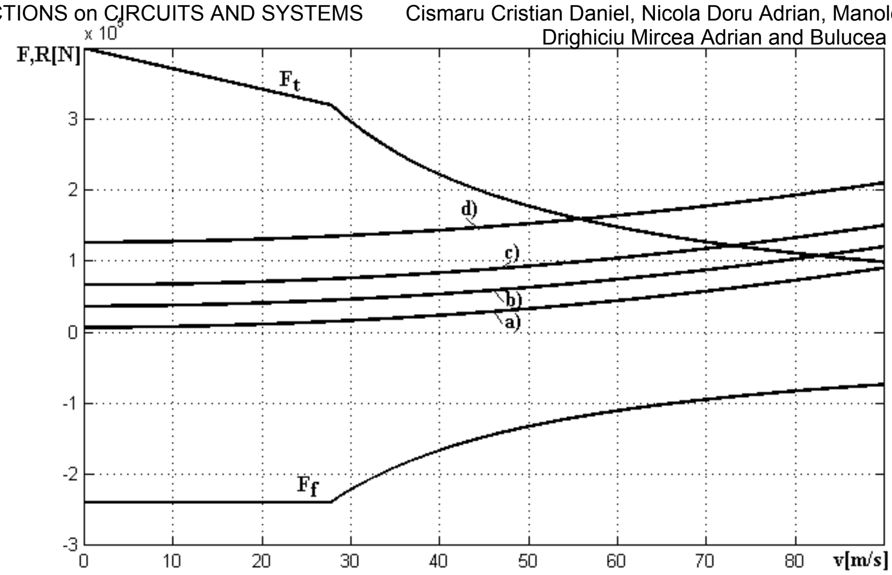


Fig.7 Traction and braking characteristics and 4 values of train resistance of ETR 500 italian high speed train resistance, m=600 t i_{de}=0; b) i_{de}=5; c) i_{de}=10; d) i_{de}=20

5 Simulations

An immediately example of the useful movement model it constitutes it the running diagrams drawing, which they illustrate the dynamic aspect of the dynamic aspect of the electric vehicle.

The running diagrams of the electric train they are drawn on the traction and braking characteristics basis and of the conditions imposed of route.

The traction and braking characteristics consort any presentation, however summarily would be, of a high-speed train. Like example, it has been considered the ETR 500 italian high speed train case, for which, except the traction and braking characteristics, they have more represented the train resistance, too, corresponding to the different specific features of the route (through the declivities consideration) (fig.7).

For the running diagrams drawing corresponding to a route of the ETR500 it is used a SIMULINK model based on the useful movement model (fig.8). In the blocks *„$F_t(v)$"* and *"$F_f(v)$"* they are implemented the traction and braking characteristics (fig.7). The model is based on the useful movement model, at which the main input variable it is supplied of the block *„F"*, which it models the useful movement phases:

- starting phase,
- running at constant speed phase,
- coasting phase and
- braking phase.

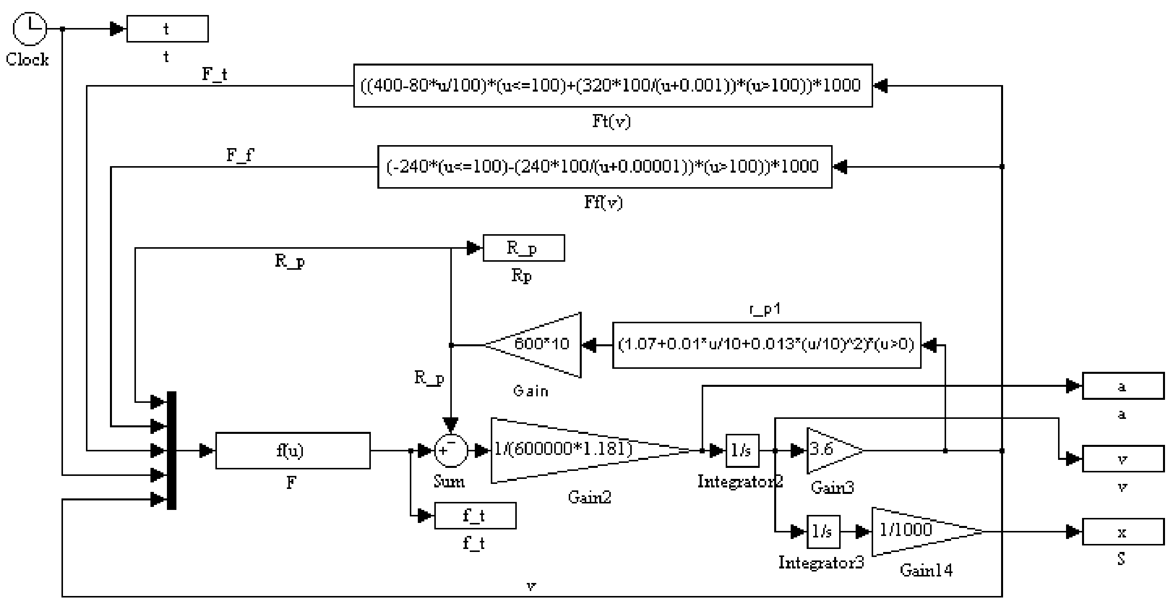


Fig.8 SIMULINK model used for running diagrams drawing

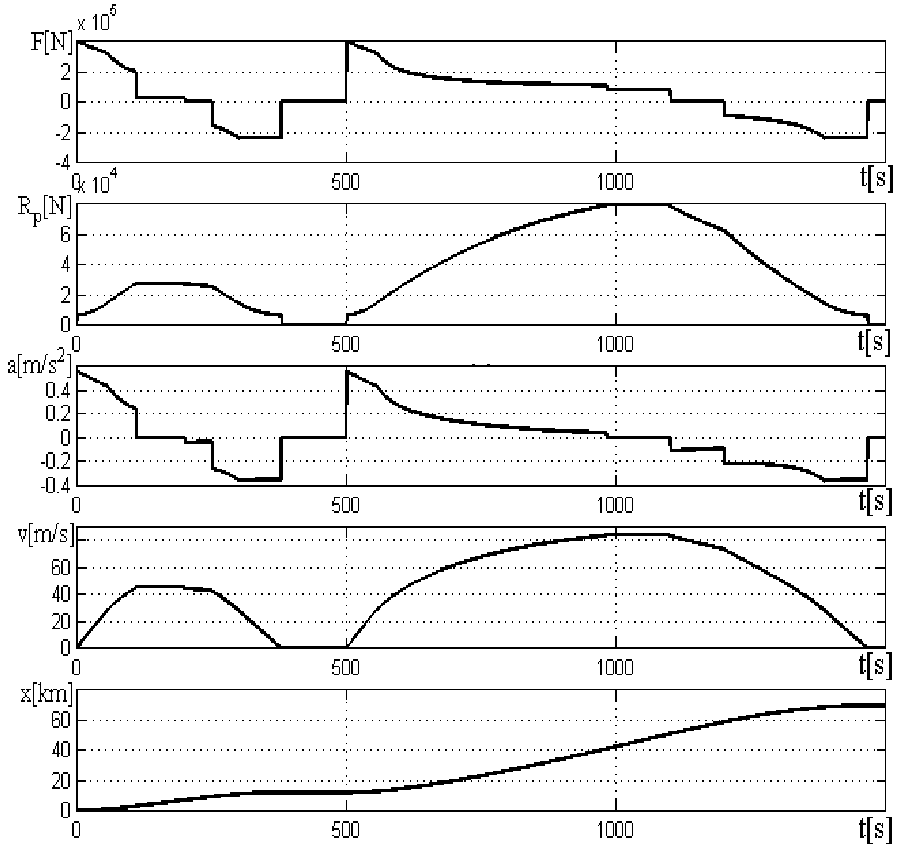


Fig.9 Running diagrams of high speed train

By means of this SIMULINK model (fig.8) they have been drawn the running diagrams corresponding to a test route of the high-speed train, on its route has been reached the speed of 44,44 m/s (160 km/h) and respectively of 83,33 m/s (300 km/h). They have been drawn the variations of the force, of the train resistance, of the acceleration, of the speed and of the distance (fig.9).

In the previous case has been taken account only the main train resistance corresponding to a route with conditions from case a, fig.7.

The next simulations referred to the same high-speed train, with the same performances (traction and braking characteristics), for which it is desired the speed variation obtainment on a distance of 100 km. They are considered another values of the train resistance established of the different declivities values existence, too (fig.7). The maximum reached speeds they correspond the conjunctions between the traction force and adequate train resistances, excepting the case a) (fig.7) where the maximum speed has been limited at 83,33 m/s (300 km/h).

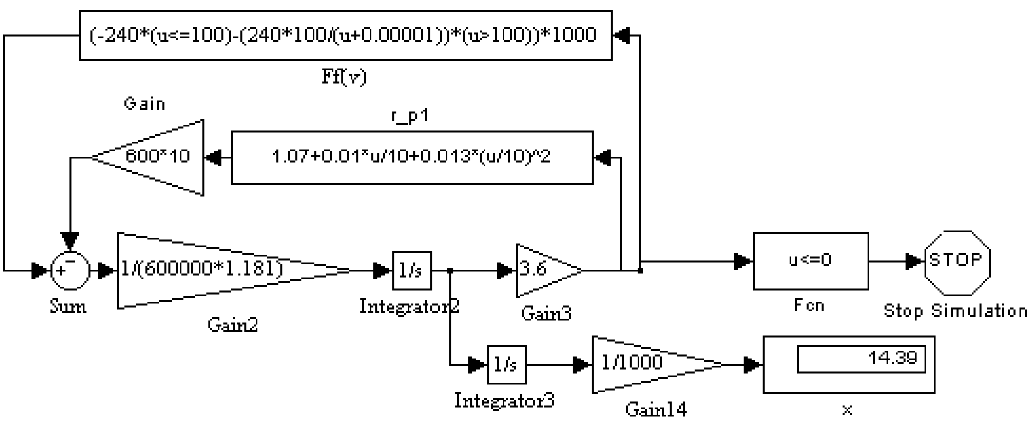


Fig.10 SIMULINK model used establishment of braking distance

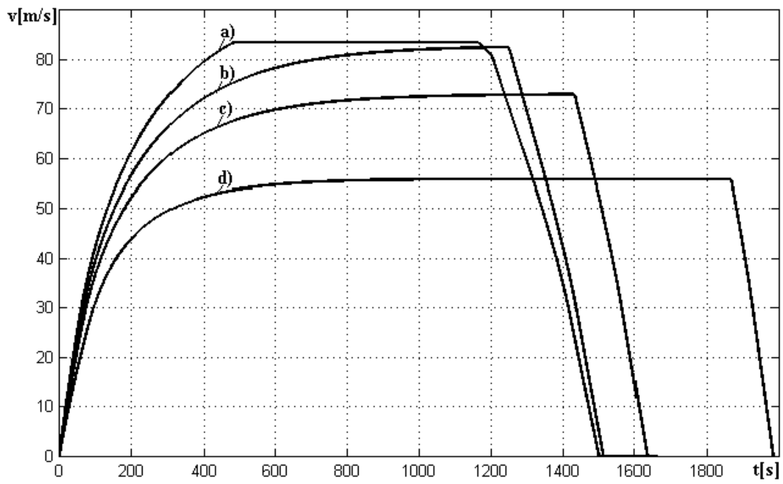


Fig.11 Speed diagram for different values of train resistance (m=600 t)
a) i_{de}=0; b) i_{de}=5; c) i_{de}=10; d) i_{de}=20

For the respecting of 100 km distance must achieved an obtainment algorithm of the point from which the train start to brake. For this it is used a SIMULINK model (fig.8), what it permits the determination of the braking distance when it is known: the braking beginning speed, the braking force (variable depending on speed), the mass m of the train, the train resistance (variable depending on speed and distance). It is thus possibly the knowledge of the point from which it begins the braking and the stopping at fixed point.

For those, for cases (fig.7), by means those two SIMULINK models (fig.9 and fig.10), they have been simulated the speed diagrams (fig.11), all corresponding to a distance of 100km. They have been obtained the time of 1500 s (case a), of 1512 s (case b), of 1632 s (case c) and of 1982 (case d).

5 Conclusions

As compared to other models of the useful movement of the electric vehicles, which they are especially for the traffic control, [13], [14], [15], [16], [17] the mathematical model presented in this paper has a high generality degree.

The structural diagram of useful movement (fig.4) can be easy coupled at the structural diagram of the electromagnetic part of the traction motor, being thus possibly a formed construction associated of railway vehicle. Here it is illustrated the traction induction motor case but, only through the modification of the electromagnetic part, can be studied the movement of the high speed trains with D.C. traction motors and even synchronous traction motors. Also, the model can be easily adapted of the concrete conditions of the electric vehicle or the route.

The structural diagram (fig.4), relatively easy of implemented and of used, it has a great importance in traction, can be useful in the establishment of a control methods, on a special route, based on the best utilization of the installed load. The respective methods are implemented then in the computer control system on the electric vehicles, contributing at the circulation safety increase, at the consumptions decrease and permitting even a possibly ATC (Automatic Train Control).

The model permits, through the specific features introduction of the routes from any country within the framework of the useful movement model, the testing and the comparison of the performances of different possible variants of the electric vehicles.

The topological similitude between the structural diagram and the SIMULINK model it permits an easy implementation in SIMULINK, both for the presented structural diagram and for the simulations of a concrete situations.

In conclusion, the presented model is useful in:
- construction of the running diagrams;
- the traffic control;
- the implementation of a new control methods of the train moving;
- the implementation of a diagnostic systems;
- construction of a main electric circuits models of the vehicles [18];
- the study of a anti-slip control methods;
- the route influence about the electric components working of the train.
- the achievement of the calculus models of the consumed energy of train etc.;
- optimization of the train movement.

These elements show the advantages of the proposed method and open a new direction for future research.

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