About Stability of Mechatronic Systems Driven by Asynchronous Motors

MONICA ADELA ENACHE, SORIN ENACHE, MIRCEA DOBRICEANU
Electromechanical Faculty
University of Craiova
Decebal Street, Craiova, 200440
ROMANIA
menache@em.ucv.ro http://www.em.ucv.ro

Abstract: - This paper presents two methods for stability analysis of mechatronic systems driven by asynchronous motors: an analytical method and a numerical one, original, conceived by authors. The presentation is accompanied by simulations and experimental results leading to credible conclusions.

Key-Words: - stability, mechatronic system, asynchronous motor, parameters, simulation, Matlab, experiment.

1 Introduction

The stability problem for mechatronic systems driven by asynchronous motors is widely approached in literature [1], [2], [3], [6], [14], [15], [16] etc. This paper develops only two methods of analysis. The first one, an analytical method, analyzes the machine inductivities influences on stability. The second one, an original numerical method, conceived by authors [9], analyzes the effects of the resistance and inertia moment modifications.

2 Analytical method

Further on we aim that, by using known representations of the transfer locus and of the amplitude-phase characteristics, to analyze the influences of the machine windings resistances on stability. In this purpose, it is imposed, at the beginning, to establish an adequate mathematical model of the system which is referred to.

Further on the induction motor is supposed to be supplied by means of a sinusoidal voltage source having variable frequency. In order to obtain the relations necessary to carry out the proposed study the following matrix equation, written in accordance with [12], will be used:

$$s\begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} = \begin{vmatrix} -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & \omega_r & \frac{1}{T_r\sigma} & \frac{\omega}{\sigma} \\ -\omega_r & -\left(\frac{1}{T_s}-\frac{1}{T_r}\right)\cdot\frac{1}{\sigma} & -\frac{\omega}{\sigma} & \frac{1}{T_r\sigma} \\ -\frac{1}{T_s} & 0 & 0 & \omega_s \\ 0 & -\frac{1}{T_s} & -\omega_s & 0 \end{vmatrix} \cdot \begin{vmatrix} I_{ds}(s) \\ I_{qs}(s) \\ I_{df}(s) \\ I_{qf}(s) \end{vmatrix} + \begin{vmatrix} \frac{1}{L_s\sigma} \\ 0 \\ \frac{1}{L_s} \\ 0 \end{vmatrix} \cdot U_{ds}(s), \qquad (1)$$

where the following notations have been used:

$T_s = \frac{L_s}{R_s}$ - time constant of the stator winding;

$T_r' = \frac{L_r'}{R_r'}$ - time constant of the rotor winding;

$\sigma = 1 - \frac{L_{sh}^2}{L_s L_r'}$ - leakage coefficient of the machine windings;

$$I_{df}(s) = I_{ds}(s) + \frac{L_{sh}}{L_s} I_{dr}'(s),$$
$$I_{qf}(s) = I_{qs}(s) + \frac{L_{sh}}{L_s} I_{qr}'(s). \qquad (2)$$

The motion equation is added to the relation (1):

$$\frac{J}{p}\frac{d\omega}{dt}=\frac{3}{2}pL_{sh}(i_{qs}i^{/}_{dr}-i_{ds}i^{/}_{qr})-m_r\,. \qquad (3)$$

If the electrical transient process is considered to be much faster than the mechanical one (J is very great), it results that the mathematical model which will be used further on, is limited to the matrix equation.

This one may be also written as:

$$s[Y(s)]=[A]\cdot[Y(s)]+[B]\cdot[U(s)], \qquad (4)$$

or, equivalently:

$$[Y(s)]=(s[I]-[A])^{-1}\cdot[B]\cdot[U(s)], \qquad (5)$$

which is the input-output operational equation for the analyzed multi-variable system.

The transfer matrix is obtained from this equation:

$$[H(s)]=(s[I]-[A])^{-1}\cdot[B]=\frac{adj(s[I]-[A])}{\det(s[I]-[A])}\cdot[B], \qquad (6)$$

where I is the unit matrix.

The transfer function denominator will have the form:

$$n(s)=a_0+a_1s+a_2s^2+a_3s^3+a_4s^4, \qquad (7)$$

with

$$a_0=\frac{1}{T_s^2T_r^2\sigma^2}+\frac{\omega_s^2}{T_r^2\sigma^2}+\frac{\omega_r^2}{T_s^2\sigma^2}+2\frac{\omega_s\omega_r}{T_sT_r\sigma^2}(1-\sigma)$$

$$a_1=\frac{2}{T_sT_r\sigma^2}\cdot\left(\frac{1}{T_s}+\frac{1}{T_r}\right)+\frac{2\omega_r}{T_s\sigma}+\frac{2\omega_s}{T_r\sigma}, \qquad (8)$$

$$a_2=\omega_s^2+\omega_r^2+2\frac{\sigma+1}{T_sT_r\sigma^2}+\frac{1}{T_s^2\sigma^2}+\frac{1}{T_r^2\sigma^2}$$

$$a_3=\frac{2}{\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right),$$

$$a_4=1.$$

Further on the following approximation will be used:

$$\mu=\frac{1}{2\sigma}\left(\frac{1}{T_s}+\frac{1}{T_r}\right)\cong\frac{1}{T_s\sigma}\cong\frac{1}{T_r\sigma}\,. \qquad (9)$$

For a concrete case of an induction motor (R_s =7,5 Ω, $R_r^{/}$=5,5 Ω, $L_s=0{,}529$ H, $L_r^{/}=0{,}528$ H, $L_{sh}=0{,}498$ H) the following data are obtained:

$$\sigma=0{,}112, \qquad \mu=110. \qquad (10)$$

In order to determine the transfer function poles for the studied system, its denominator will be made equal to zero:

$$n(s)=0, \qquad (11)$$

with n(s) given by the relations (7) and (8).

The following poles have been obtained by solving this equation:

a) If $\mu\geq\frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4}=-\mu\pm\sqrt{\mu^2(1-\sigma)-\frac{\omega^2}{4}}\pm j(\omega+2\omega_r)/2\,; \qquad (12)$$

b) If $\mu<\frac{\omega}{2\sqrt{1-\sigma}}$,

$$s_{1,2,3,4}=-\mu\pm j\left[\sqrt{\frac{\omega^2}{4}-\mu^2(1-\sigma)}\pm(\omega+2\omega_r)/2\right]. \qquad (13)$$

As one can observe, in the relation (12), $\mu>\sqrt{\mu^2(1-\sigma)-\frac{\omega^2}{4}}$, irrespective of the machine parameters or the operation point. So, the real part of the transfer function poles is always negative. In conclusion, the studied system is always stable.

Since the inductances L_s and $L_r^{/}$ are inverse proportional with μ, these ones have a non-stabilizing effect on the induction motor operating at variable frequency.

3 Numerical method

This method, detailed in [10], has as starting point the following relations, written in per unit values:

$$hs\cdot\Delta\omega^{*}=-k\Delta i^{/*}_{dr}, \qquad (14)$$

$$\Delta i'_{dr} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot$$
$$\cdot k(\Delta\omega^* - \Delta\omega_s^*), \qquad (15)$$

where

$$\varepsilon = (1 - k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{*\prime}}. \qquad (16)$$

At the same time it must be mentioned that the per unit quantities used in the previous equations have been noted with "*", the quantities depicted here having the significance from [9].

The equation (15) may also be written in the following form:

$$\Delta\omega^* = -\frac{k}{hs} \cdot \Delta i_{dr}^{\prime *}, \qquad (17)$$

or, equivalently:

$$\Delta\omega^* = G_1(s) \cdot \Delta i_{dr}^{\prime *}, \text{ with } G_1(s) = -\frac{k}{hs}. \qquad (18)$$

Similarly, the equation (16) becomes:

$$\Delta i_{dr}^{\prime *} = G_2(s) \cdot (\Delta\omega_s^* - \Delta\omega^*), \qquad (19)$$

where

$$G_2(s) = -\frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \cdot k \qquad (20)$$

The previous relations lead to the equivalent scheme of the induction machine operating at variable frequency.

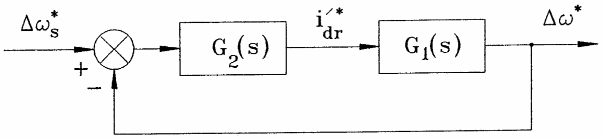
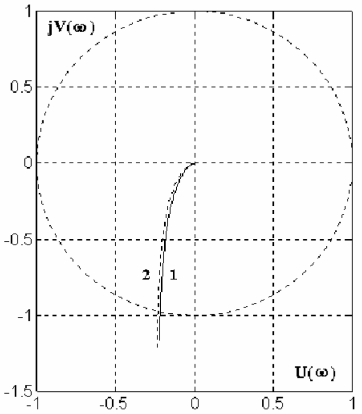


Fig.1. Machine block scheme in the mentioned situation.

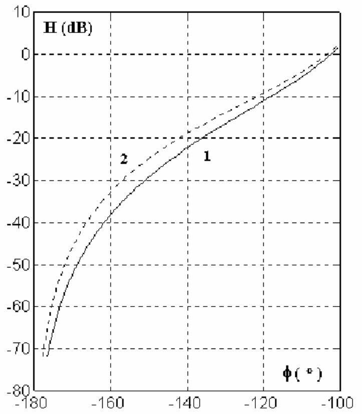
4 Simulations

A MATLAB program for the stability analysis has been conceived, by using the previous scheme. The representations from figures 2, 3 and 4 have been obtained by running this program, for the concrete case of a motor rated at 1,1 kW.

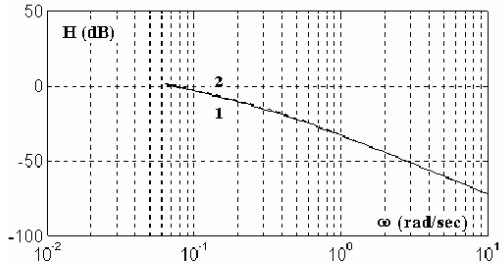
There must be also mentioned the importance of the introduced method resulting from the possibility to emphasize the machine parameters influence and especially the inertia moment influence, on stability when operating at variable frequency, fact that provides originality to this method.



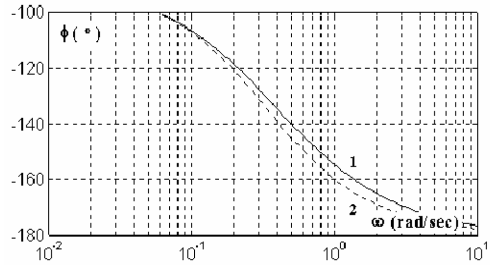
a) Transfer locus.



b) Amplitude – phase characteristics.

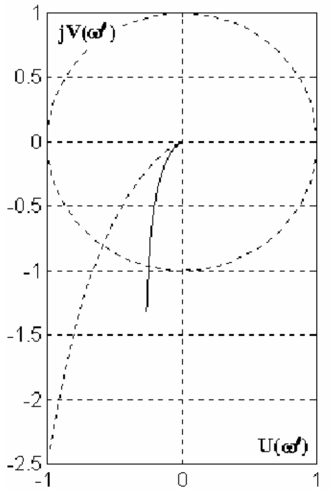


c) Pulsation - amplitude characteristics.

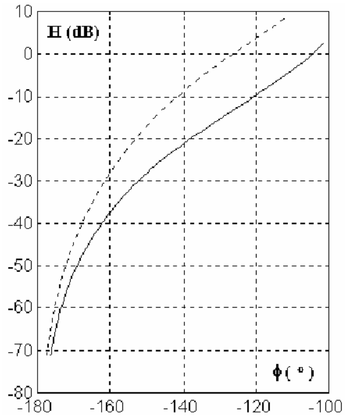


d) Phase - pulsation characteristics.

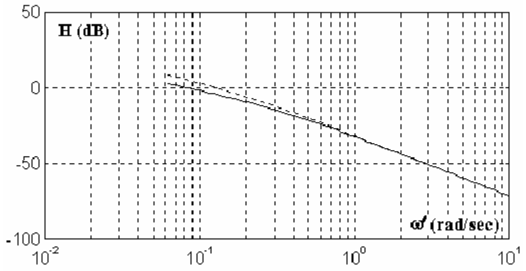
Fig.2. Graphic dependences corresponding to the cases R_S=7,5 Ω (continuous line) and R_S=2,5 Ω (dotted line).



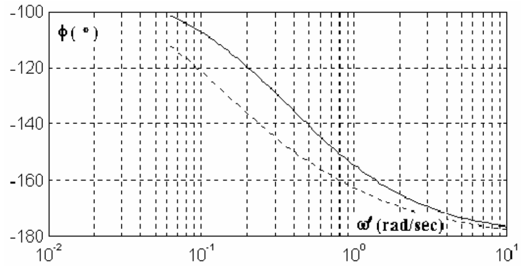
a) Transfer locus.



b) Amplitude – phase characteristics.



c) Pulsation - amplitude characteristics.



d) Phase - pulsation characteristics.

Fig.3. Graphic dependences corresponding to the cases R'_r=5,5 Ω (continuous line) and R'_r=4,5 Ω (dotted line).

In order to catch quantitatively these interdependences the following table has been filled.

Table 1

Par.	Abs. value [Ω], [H], [kgm^2]	Per unit par.	Per unit value	Phase margin [degree]
R_s	7,5	r_s^*	0,0988	75,54
	2,5		0,0330	74,20
$R_r^/$	5,5	$r_r^{/*}$	0,0725	75,54
	4,5		0,0593	53,71
J	0,004	h	32,4	75,54
	0,003		24,3	47,65

These results help us to emphasize a few important **conclusions** regarding the resistances influence on the studied system stability:

- the decrease of the stator winding resistance leads to the stability decrease;
- the rotor resistance decrease has also as an effect, the decrease of the machine stability and conversely;
- the inertia moment increase contributes to the stability increase.

5 Experimental circuit

In order to confirm the previous conclusions, a series of experimental tests have been performed; a few of them are detailed further on.

Thus, the experimental circuit has the structure depicted in the following figure [11].

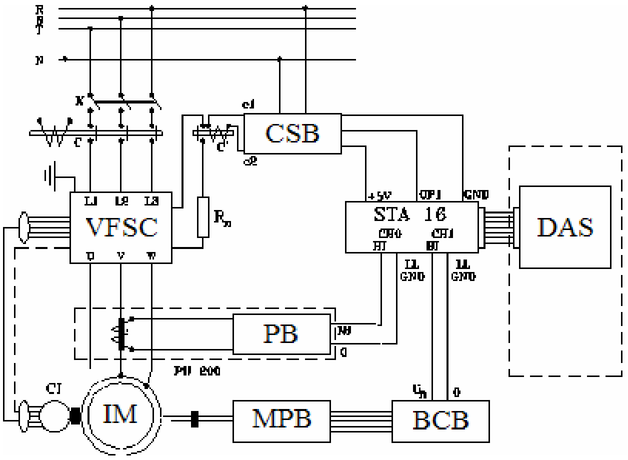


Fig.4. Scheme of the experimental circuit.

The notations have the following meaning:

IM – induction motor;
VFSC – voltage and frequency static converter;
DAS – data acquisition board;

CSB – command and synchronization block;
PB – protection block;
MPB – magnetic powder break;
BCB – brake command block;
STA 16 – connection block.

A picture of this circuit is depicted for conformity.

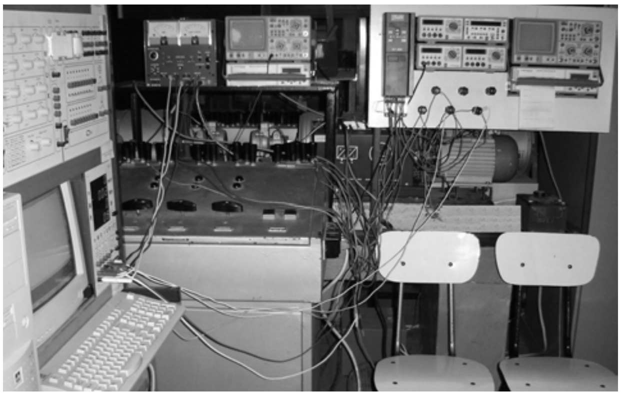


Fig.5. Picture of experimental circuit.

In order to obtain the determinations in dynamic regime the experimental circuit depicted before has been carried out, having an acquisition board DAS as a central element. This high speed analogical and digital interface has been assembled inside a computer. The acquisition and the adequate data processing are ensured with the help of a program conceived in Matlab.

The main characteristics of the board are presented in the following table.

Table 2

	Characteristic	Value
1	Number of analogical inputs	16 unipolar inputs or 8 differential
2	Resolution of the analogical-numerical converter	12 bit
3	Inputs: unipolar bipolar	0 ÷ +10 V ± 10 V
4	Domains selection	By the program
5	Amplifications of the input domains	1, 10, 100, 500
6	Channels D/A (12 bit)	2
7	Digital lines I/O	32 bit
8	Maximum sampling frequency	100 kHz
9	Acquisition time	1,4 ms

The correspondence between the amplification of the input domains, the input type and the maximum rate for scanning several channels so that to obtain the same results as in the case when a single channel is scanned, is emphasized in the following table.

Table 3

Amp	Unipolar	Bipolar	Frequency
1	0 ÷ +10 V	± 10 V	100 kHz
10	0 ÷ +1 V	± 1 V	100 kHz
100	0 ÷ 100 mV	± 100 mV	70 kHz
500	0 ÷ +20 mV	± 20 mV	30 kHz

Data transfer can be made in three ways:
- by direct transfer into the memory without the intermediary micro-processor DMA (Direct Memory Access);
- by subroutine of interruptions;
- by program.

The command and synchronization block CSB ensures the data acquisition start before the motor starting. The delay occurring between the two moments is then corrected by software.

The module PI 200 has been used for adapting the measured currents to the values required by the board. It contains a current transformer in whose secondary there is connected a calibrated resistance; the voltage drop occurring on this resistance is of maximum ±10 V.

6 Experimental Results

The graphics depicted in the following figures have been obtained with the help of the previous circuit, for the case of an indirect voltage and frequency static converter.

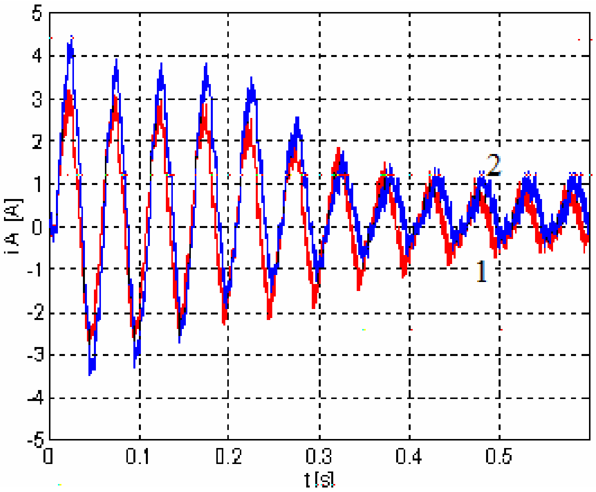


Fig.6. Graphic dependences corresponding to the cases R_S=7,5 Ω (1) and R_S=2,5 Ω (2).

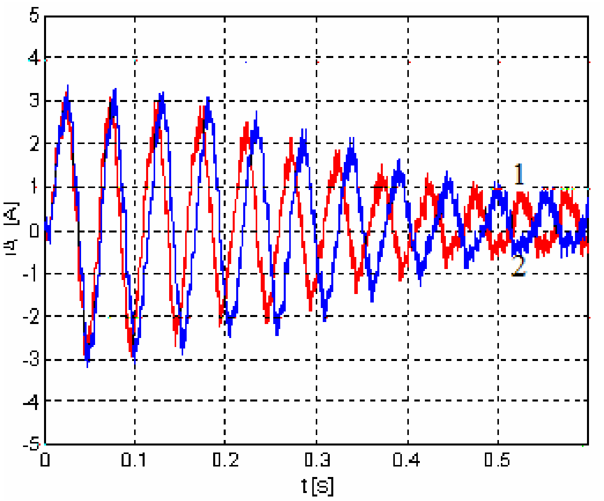


Fig.7. Graphics dependences corresponding to the cases R'_r=5,5 Ω (1) and R'_r=4,5 Ω (2).

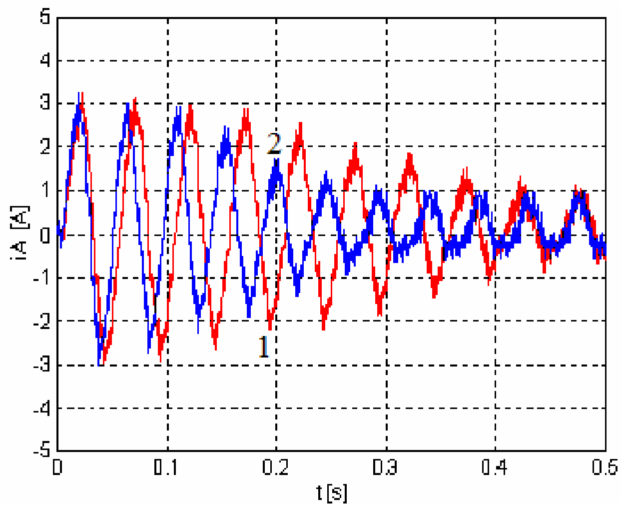


Fig.8. Graphics dependences corresponding to the cases J=0,006 kg.m^2 (1) and J=0,004 kg.m^2 (2).

A few interesting conclusions regarding the induction motor parameters influences on the dynamic regime behaviour of the analyzed driving systems can be emphasized with the help of the programs detailed before:

- the stator resistance decrease increases very little the duration of the currents transient process and decreases the system stability, respectively;

- the rotor resistance decrease causes the increase of the stabilization time and the stability decrease, respectively;

- when the value of the stator inductance increases the transient process duration increases;

- the rotor inductance value increase also involves the increase of the transient process duration;

- the main inductance decrease determines a faster stabilization of the process (stability increase);

- the inertia moment value increase leads to the increase of the currents stabilization time, and to the stability decrease.

Moreover, when the inverter with pre-established commutation moments is used, it is also noticed that the maximum values of the motor phase currents are modified.

As one can observe, these experimental conclusions confirm the conclusions obtained with the new numerical method for analysis.

References:

[1] R. Belmans, L. D'Hondt, Analysis of the Audible Noise of Three Phase Squirrelcage Induction Motors Supplied by Inverters, *IEEE Transaction on Industry Applications*, Volume IA-23, September-October 1997, pp. 842-847.

[2] A. Campeanu, I. Vlad, A. Ionescu, S. Enache, Establisement of Electrical Machines Optimum Constructive Dimensions, *ELECTROMOTION 2003*, Marrakesh, Maroc, vol. 1, p. 187-193, INSPEC (Material Identity Number F171-2004-001, Inspec classification codes B8310a.c. machines, Optimisation techniques).

[3] A. Campeanu, A. Ionescu, I. Vlad, S. Enache, Modelling and Simulation of Magnetic Saturation in Induction Motors, *International Electric Machines and Drives Conference IEMDC '07*, CD version, AF-010162, IEEE Catalog Number: 07EX1597C, ISBN: 1-4244-0743-5.

[4] A. Campeanu, I. Vlad, S. Enache, Aspects Regarding Optimal Design of High Power Electrical Machines, Braşov, *The 5th International Conference on Optimization of Electric and Electronic Equipment, OPTIM '2002*, INSPEC (Material Identity Number D115-2002-002, Inspec classification codes B8310 a.c. machines).

[5] A. Campeanu, M. Radulescu, I. Vlad, S. Enache, Influence of Cross-Saturation for Modeling and Simulation of the Dynamic Processes in the Induction Motors, *IEEE Region 8, EUROCON2007 - The International Conference on "Computer as a Tool"*, Warsaw, Poland, 9-12 septembrie 2007, IEEE Catalog Number: 07EX1617C, ISBN:1-4244-0813-X, Library of Congress: 2006937182, p. 1830.

[6] Y. P. Chiu, F. Cheng, C. Ting, Algebraic Methods for Optimizing EPQ Model with Rework and Scrap, *WSEAS TRANSACTIONS on SYSTEMS*, Issue 11, Volume 6, November 2007, pp. 1319-1325.

[7] B. Delemontey, B. Jacquot, C. Iung, B. De Fornel, J. Bavard, Stability Analysis and

Stabilisation of an Induction Motor Drive with Input Filter, *6th European Conference on Power Electronics and Applications*, Sevilla, Spain, 1995.

[8] S. Enache, I. Vlad, M.A. Enache, Aspects Regarding Dynamic Regimes of Induction Motors Control by the Stator Flux, *Power Engineering, Energy and Drives, 2007, POWERENG 2007*, Setubal, Portugal, International Volume, Issue, 12-14 Aprilie 2007, ISBN 978-1-4244-0895-5, IEEE Xplore, DOI: 10.1109/ POWERENG. 2007. 4380110, pp. 152-155.

[9] S. Enache, R. Prejbeanu, A. Campeanu, I. Vlad, Aspects Regarding Simulation of the Saturated Induction Motors Control by the Voltage Inverter Commanded in Current, *IEEE Region 8, EUROCON2007 - The International Conference on "Computer as a Tool", Warsaw*, Poland, 9-12 septembrie 2007, IEEE Catalog Number: 07EX1617C, ISBN:1-4244-0813-X, Library of Congress: 2006937182, p. 1826.

[10] S. Enache, A. Campeanu, I. Vlad, M.A. Enache, A New Method for Induction Motor Stability Analysis when Supplying at Variable Frequency, *Proceedings of the 7-th WSEAS International Conference on SYSTEMS THEORY AND SCIENTIFIC COMPUTATION (ISTASC'07)*, Vouliagmeny, Athens, Greece, August 24-26, 2007, ISSN: 1790-5115, ISBN: 978-960-8457-98-0, pp. 117-120.

[11] M.A. Enache, S. Enache, M. Dobriceanu, Considerations Regarding Determination of the Induction Motors Parameters, *International Electric Machines and Drives Conference IEMDC '07*, CD version, AF-003093, IEEE Catalog Number: 07EX1597C, ISBN: 1-4244-0743-5

[12] M.A. Enache M.A., S. Enache, M. Dobriceanu, Influences of Induction Motor Parameters on Stability in Case of Operation at Variable Frequency, *WSEAS International Conference on SYSTEMS THEORY AND SCIENTIFIC COMPUTATION*, Vouliagmeny, Athens, Greece, August 24-26, 2007, pp. 113-116.

[13] M. Hasni, O. Touhami, R. Ibtiouen, M. Fadel, S. Caux, Modelling and Parameter Identification of a Synchronous Machine by Standstill Tests, *WSEAS TRANSACTIONS on CIRCUITS and SYSTEMS*, Issue 9, Volume 6, September 2007, pp. 553-560.

[14] J. L. Lin, L. G. Shiau, On Stability and Performance of Induction Motor Speed Drives with Sliping Mode Current Control, *Asian Journal of Control*, Vol. 2, No. 2, pp. 122-131.

[15] W. Liu, H. Tang, W. Fang, Analysis of Power Transfer Ability and Stability for Voltage Source Inductive Coupled Power Transfer Systems, *WSEAS TRANSACTIONS on CIRCUITS and SYSTEMS*, Issue 7, Volume 6, July 2007, pp. 519-525

[16] T. Lipo, Stability Analysis of a Rectifier-inverter Induction Motor Drive, *IEEE Transaction on Industry Applications*, 88, 1, 1993, pp. 57-68.

[17] L. Qing, D. Xuegong, L. Xiong, Y. Gang, A Dynamic Weighing System Based on System Model, *WSEAS TRANSACTIONS on SYSTEMS*, Issue 8, Volume 6, August 2007, pp. 1244-1250.

[18] H. Mosskull, Stabilization of an Induction Motor Drive with Resonant Input Filter, *11th European Conference on Power Electronics and Applications*, Dresden, Germany, 2005, p. 102.

[19] N. Vinatoru, *Teoria sistemelor*, Ed. Universitaria, Craiova, 1996.

[20] *** , *Matlab Reference Guide*, The Math Works, Inc., Natick, Massachusetts, 2002.