# A Low-Power Scheduling Tool for System On a Chip Designs 

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#### Abstract

As semiconductor technology scales down, integration on a chip becomes higher and concerns complex algorithm implementation. Those algorithms concern plenty of applications in many fields. Thus, adequate scheduling techniques that cope with such a variety of applications are required. The method presented in this paper addresses that concern and takes advantage of both data flow and control flow approaches. Knowing that such a Controlled Data Flow Graph (CDFG) scheduling is not polynomial, an efficient heuristic-based approach is then needed. In addition, because time and resources are user-constraints that gain a particular attention, our heuristic-based method targets a minimal number of cycles. More, it detects exclusive operations of the same type that can be scheduled in the same control step and share the same resource. Because the power dissipation is a crucial problem for SOC designs, our tool automatically introduces additional constraints so that the switching power dissipation at a high design level is reduced.


Key-Words: - CDFG; scheduling; heuristic; user constraints; variety of domain applications; low power design, efficient cycle number, high design level, SOC designs

## 1 Introduction

The speed, area and power consumption are the main parameters that a VLSI system designer has to focus on during the design process (e.g. [32], [33]). The scheduling is a synthesis task conducted at a high design level and aims to achieve some objective in terms of area, speed and / or power consumption. In [3], the scheduling is performed only by considering time constraints, while in [4], [5] and [6] a low power-based scheduling is addressed. Because actual VLSI systems may include both data flow and control flow operations, appropriate scheduling techniques are required. Indeed, previous works ([7], [8], [9], [10], [11]) addressed only intensive data flow circuits (control structures are not included) such as DSPs, in which the parallelism of operations is the main goal, which is a severe limitation. Other earlier works ([12], [13], [14], [15]) that use control flow-based algorithms avoided such a limitation by determining which operations are exclusive and can be scheduled in the same control step while sharing the same resource. However, because the operation ordering is not changed, some of these techniques suffer from
the fact that the parallelism in the schedule is limited. More enhanced methods ([16], [17], [18], [19]) addressed both the data flow and the control flow-based applications. Interesting ILP basedmethods ([6] and [29]) were developed but they are inefficient for large VLSI systems since they are not polynomial and suffer in terms of running time [2]. Thus, appropriate heuristic methods such as those based on genetic algorithms ([20], [21]) are more suitable for larger scale designs in order to output near-optimal solutions in a reasonable CPU time. In [30], a game theory-based approach is presented. It gives interesting results with the expense of CPU time due to its time complexity.
Our proposed tool is a set of techniques that efficiently deal with arbitrary circuits. It performs a global scheduling for SOC designs, and assumes that the operations are first flattened. Indeed, independent schedulings previously performed on those systems do not yield good results with respect to those obtained with a global scheduling. The computational complexity for each technique implemented in our tool is first studied: an exact algorithm is then developed for each polynomial
task, else a new efficient colored graph -based algorithm ([22], [23], [24]) is used (we will show that those non polynomial tasks are NP complete problems). Our tool allows the system designer to describe his system behaviour in a C-like code, which is less tedious than Controlled Data Flow Graphs (CDFGs) for complex circuits. The huge number of Data Flow Graphs (DFGs) contained in the CDFG is reduced thanks to an exact algorithm. Then, for each DFG, a first scheduling technique subject to the operation dependencies is performed with an exact algorithm, which leads to an optimal number of control steps. A Second scheduling technique follows then and addresses user constraints satisfaction. This latter problem is NPcomplete and was solved using a new efficient colored graph technique. Notice that these two scheduling steps are the main advantages of the data flow-based techniques since the amount of the parallelism in the schedule is increased. Because the power dissipation becomes a crucial problem for current designs (SOC designs), our tool allows the user to introduce additional constraints so that the tasks of the circuit are performed with a lower power budget. Obviously, further power reduction should be tackled at lower design levels. Finally, the results of the DFGs scheduling are synthesized and grouped into a single scheduling report in order to detect the exclusive operations. Such operations can be scheduled in the same control step and can share the same resource, which is the advantage of control flow-based techniques. The paper is organized as follows: The next section shows how a C-like code is transformed into a set of traces (DFGs). Section 3 then describes our technique for reducing the number of DFGs. In section 4, our scheduling technique is detailed, and in section 5 , we present our technique that deals with the reduction of the switching power dissipation at a high design level. The results are described in section 6. Finally, section 7 concludes the paper.

## 2 Algorithmic Transformation

Our CAD tool deals with the scheduling of operations written in a C-like code. The first task our tool performs is the translation of a C-like code (Fig.2.1) into a set of traces. This latter format will allow the determination of the different Data Flow Graphs (DFG) contained in the CDFG (Fig.2.2). Then, the scheduling task is performed on each obtained DFG (also termed trace) thanks to an appropriate process that guarantees a polynomial time for scheduling the CDFG while targeting an advantageous number of cycles subject to resource and time constraints. The results obtained from the
different DFGs are synthesized, then reported in a single file that contains the global scheduling, namely the CDFG scheduling. Finally, a reduction of the switching power dissipation ( P ) is performed in order to allow the user to obtain the desired tradeoff between the circuit performance and the switching power dissipation at a high design level (as shown in the flow depicted in Fig.2.3).

```
\(\mathrm{x}=1\);
\(y=x+1\);
if(A)
then \(\{\mathrm{z}=\mathrm{x}+\mathrm{y}+4\);
    \(\mathrm{t}=\mathrm{x}+\mathrm{y}-6\);
    \}
else \(\mathrm{t}=8+\mathrm{y}\);
endif
if(B)
then \(\mathrm{z}=\mathrm{x}+\mathrm{y}-\mathrm{t}\);
else if(C)
    then \(\{\mathrm{z}=\mathrm{y}-\mathrm{x}+9\);
            \(\mathrm{t}=\mathrm{z}+\mathrm{y}-3\);
            \}
    else \(\mathrm{z}=\mathrm{x}-5\);
    endif
endif
\(\mathrm{y}=\mathrm{x}+\mathrm{z}-\mathrm{t}\);
```

Fig. 2.1 A simple algorithm in a C-like code.
First, the different data graphs are extracted (notice that inner control structures are allowed) according to the control structures of the algorithm. For instance, the data graphs corresponding to the algorithm in Fig.2.1 are those depicted in Fig. 2.2. In general, the data graph number is equal to ${ }_{i=1}^{N} n_{i}$ where N is the number of the outer control $i=1$
structures and $n_{i}$ is the number of the paths contained in control structure i. We can see in the example given in Fig.2.1, that there are two outer control structures (A and B) and six paths ( $2 * 3$ ):
trace 1
True condition: A and B

```
    x=1;
    y=x+1;
    z=y+x+4;
    t=x+y-6;
    z=x+y-t;
    y=x+z-t;
    trace 2
True condition: A and not(B) and C
            x=1;
            y=x+1;
            z=x+y+4
            t=x+y-6;
            z=y-x+9;
            t=z+y-3;
            y=x+z-t;
```

trace 3
True condition: $A$ and $\operatorname{not}(B)$ and $\operatorname{not}(C)$

$$
\begin{gathered}
\mathrm{x}=1 ; \\
\mathrm{y}=\mathrm{x}+1 \\
\mathrm{z}=\mathrm{x}+\mathrm{y}+4 \\
\mathrm{t}=\mathrm{x}+\mathrm{y}-6 \\
\mathrm{z}=\mathrm{x}-5 \\
\mathrm{y}=\mathrm{x}+\mathrm{z}-\mathrm{t}
\end{gathered}
$$

trace 4
True condition: not(A) and B

$$
\begin{gathered}
\mathrm{x}=1 \\
\mathrm{y}=\mathrm{x}+1 \\
\mathrm{t}=8+\mathrm{y} \\
\mathrm{z}=\mathrm{x}+\mathrm{y}-\mathrm{t} \\
\mathrm{y}=\mathrm{x}+\mathrm{z}-\mathrm{t}
\end{gathered}
$$

trace 5
True condition: $\operatorname{not}(A)$ and $\operatorname{not}(B)$ and $C$

$$
\begin{gathered}
\mathrm{x}=1 ; \\
\mathrm{y}=\mathrm{x}+1 \\
\mathrm{t}=8+\mathrm{y} ; \\
\mathrm{z}=\mathrm{y}-\mathrm{x}+9 \\
\mathrm{t}=\mathrm{z}+\mathrm{y}-3 ; \\
\mathrm{y}=\mathrm{x}+\mathrm{z}-\mathrm{t}
\end{gathered}
$$

trace 6
True condition: $\operatorname{not}(\mathrm{A})$ and $\operatorname{not}(\mathrm{B})$ and $\operatorname{not}(\mathrm{C})$

$$
\begin{gathered}
\mathrm{x}=1 ; \\
\mathrm{y}=\mathrm{x}+1 ; \\
\mathrm{t}=8+\mathrm{y} ; \\
\mathrm{z}=\mathrm{x}-5 ; \\
\mathrm{y}=\mathrm{x}+\mathrm{z}-\mathrm{t} ;
\end{gathered}
$$

Fig. 2.2 The six data graphs of the algorithm in Fig.2.1.

```
Algorithmic_transformation();
    Input: a C-like code
    Output: a CDFG
Collapsing_and_Cut_Operations(); // determination of the
                    number of data graphs
                        // allowing a polynomial
                scheduling
    . Output: a set of DFGs (traces)
Power= \(+\infty\);
while(Power > P_desired \& \& Power reduction feasible)
do \{for each trace
    do \{Perform the 1rst step of the scheduling subject to the
        operation dependencies
        Perform the \(2^{\text {nd }}\) step of the scheduling subject to the
        user constraints
        \}
    end
    Synthesize the results, and then report the global
    scheduling into a single file;
    Power= computed switching power dissipation();
    \}
End
```

Fig.2.3 Overview of our low power scheduling tool

Let $\mathrm{M}_{1}=\prod_{i=1}^{N} n_{i}$. The overall scheduling algorithm (A) will be:
for $\mathrm{i}=1$ to $\mathrm{M}_{1}$
do schedule data graph $i$;
end for
Group the results into a single report;
The time complexity of this algorithm is $\mathrm{T}_{\mathrm{c}}=\mathrm{O}\left(\mathrm{c} * \mathrm{M}_{1}\right), \mathrm{c}=\max \left[\mathrm{O}\left(\mathrm{c}_{\mathrm{i}}\right)\right] ; \mathrm{i}=1,2, \ldots, \mathrm{M}_{1}$. This clearly shows that the time complexity is not polynomial. Notice that in case we have only two paths in each of the N outer control structures, the time complexity would be $\mathrm{O}\left(\mathrm{c} * 2^{\mathrm{N}}\right)$. Thus, even if $\mathrm{O}(\mathrm{c})$ is time-polynomial, $\mathrm{T}_{\mathrm{c}}$ will not be polynomial. Thus, an heuristic-based method is required to solve this problem. It is obvious that the time complexity of the heuristic method should be polynomial. In addition, the solution must be closer to the exact one. The rest of the paper is devoted to this challenge.

## 3 Huge Data Graphs Number Reduction

### 3.1 Conditional branches collapsing

Due to the large number of data graphs inferred by N control structures, $\mathrm{M}_{1}$ value has to be reduced such that that algorithm (A) mentioned previously can be solved in a polynomial time. We recall that our tool deals with a CDFG scheduling subject to user-constraints (time and resource constraints). Let us consider the example depicted in Fig.3.1. As we can see, there are two traces in the CDFG depicted in that figure. Notice that there is a user-constraint concerning operations 2 and 6 of the CDFG, and that there is no constraint in the two branches of the structure controlled with logical variable A. Thus, the two paths $[1,2,3,4,6]$ and $[1,2,5,6]$ will be scheduled in the same way, namely with the whole if-then-else contained in one state because the userconstraint is independent from the operations inside the if branches. Thus, the 2 conditional branches can be collapsed into a super-node for scheduling purposes, resulting in a single (instead of two) data flow (Fig.3.2). Notice also that two or more operations scheduled in the same step are not simultaneously performed in case they are not subject to the same true condition (the scheduling and control steps have different meanings).
In general, such a way for collapsing conditional branches that do not contain constraints may considerably reduce the number of traces (from $\mathrm{M}_{1}$ to $\mathrm{M}_{2}$ ), resulting in a new scheduling algorithm (B) which is similar to the previously described one
(algorithm A) with a prospective smaller loop (from 1 to $\mathrm{M}_{2}$ instead of 1 to $\mathrm{M}_{1}$ ). Notice that $1 \leq \mathrm{M}_{2} \leq \mathrm{M}_{1}$ and that $\mathrm{M}_{2}=1\left(=\mathrm{M}_{1}\right)$ occurs in case all the control structures are collapsed (in case all the conditional branches contain at least one user-constraint).


Fig. 3.1 A simple CDFG


Fig. 3.2 FSM corresponding to the CDFG in Fig. 3.1.

### 3.2 Control partitioning

Because $\mathrm{M}_{2}$ may still be high, the time complexity of algorithm (B) may still be not polynomial. In order to cope with this problem, we have to use another strategy such that the CDFG scheduling can be feasible in a reasonable CPU time. This can be achieved with an appropriate number of cut insertions such that the quality of the solution (number of cycles) does not suffer. Indeed, as we will see after, the number of cycles gets higher as the number of the cut insertions increases. Such a cut (control step insertions) transforms a
combinational control into sequential ones. To review this in details, let us consider the example depicted in Fig.3.3.
In case an if-then-else structure contains one or more user constraints, the paths containing such a structure cannot be scheduled in the same way, which prevents the collapse of these conditional branches. Indeed, in Fig.3.3.a, assuming that operation 4 is both independent from operations 8 and 10 , it is possible to schedule operations 4 and 10 in the same cycle while this is not possible for operations 4 and 8 (because of the user constraint). Since some conditional branches cannot be collapsed and since the number of the data graphs is still high, the CDFG scheduling is not feasible in a reasonable CPU time. In order to cope with that problem, and in case conditional branch collapsings are not sufficient, an additional strategy based on a control partitioning should be used.
Unfortunately, the cycle number gets higher as the cut number increases. Indeed, assuming that all the operations are independent, operations 3 and 10 in Fig.3.3.a can be scheduled in the same cycle but the CDFG scheduling operates on $8(2 * 2 * 2)$ data graphs. On the other hand, in Fig.3.3.b, operation 10 can be scheduled into the $2^{\text {nd }}$ cycle with a scheduling operating on only $6(2 * 2+2)$ traces thanks to a cut insertion. Finally, operation 10 in Fig.3.3.c is scheduled into the $3^{\text {rd }}$ cycle with a scheduling operating on $6(2+2+2)$ data flows thanks to 2 cut insertions. In this last case, the combinational control is totally transformed into a sequential one. This shows that:

- the cut insertion increases the number of cycles and can decrease the number of data flows
- an arbitrary cut insertion may not decrease the number of data flows while the number of cycles increases (Fig.3.3.b and Fig.3.3.c)

The challenge is then to find appropriate cut insertions such that the CDFG scheduling can be feasible in a reasonable CPU time while not decreasing the solution quality (number of cycles).
Our algorithm that performs cut insertions has a time complexity which is $\mathrm{O}\left(\mathrm{N}^{2}{ }_{\mathrm{cs}}\right)$ where $\mathrm{N}_{\mathrm{cs}}$ is the number of the control structures. This algorithm is exact and recursive. Our proposed algorithm is shown below:

N_data_graphs= \#data graphs fixed by the user
Control_Partitioning(S) // initially, S includes all the CDFG operations

## \{

Calculate $\mathrm{M}_{3}$, the data graph number included in S ; if $\left(\mathrm{M}_{3}>\mathrm{N}_{2}\right.$ data_paths and at least one cut insertion is feasible) then $\{$ Determine the best cut insertion (the one which best decreases the data flow number); // this is merely done with a dichotomy- way Partition the interested set $\mathrm{S}_{\mathrm{j}}$ into 2 ones $\mathrm{S}_{\mathrm{j} 1}$ and $\mathrm{S}_{\mathrm{j} 2}$ (the interested set is the one in which the cut insertion has been done, the last operation of $\mathrm{S}_{\mathrm{j} 1}$ is the one on which a cut occurred, the $1^{\text {st }}$ operation of $\mathrm{S}_{\mathrm{j} 2}$ is the one which follows the interested operation in set S);
$S=S \cup S_{j 1} \cup S_{\mathrm{j} 2}$;
Remove $S_{j}$ from $S$;
Control_Partitioning(S);
\}
endif
if( $\mathrm{M}_{3}>\mathrm{N}$ _data_paths)
then report("Infeasible: all the conditional branches are sequential");
else the operations in the same set $\mathrm{S}_{\mathrm{j}}$ are $\mathbf{1}^{\text {st }}$ scheduled in the same cycle (there is a direct relationship between the obtained sets and the cycle numbers);
endif
Now, the CDFG scheduling algorithm is given as follows:

## for $\mathrm{i}=1$ to $\mathrm{M}_{3}$

do schedule data graph $i$ according to the partial results obtained with Control_Partitioning() procedure;
end for
Synthesize the results then report the global scheduling into a single report;

In the next section, we will discuss the scheduling of each data graph i.

## 4 Data Graph Scheduling Technique

### 4.1 Operation Dependencies

The Control_Partitioning() procedure has given a partial scheduling. We recall that operations included in different sets are scheduled into different cycles. We now deal with the operations that belong to the same subset $\mathrm{S}_{\mathrm{j}}$. In each subset, the operations are again scheduled according to:

- the dependencies ( 2 dependent operations are scheduled into 2 different new cycles)
- the operation type (until now, our tool assumes that operations of type addition are performed within a single cycle while operations of type multiply require 2 cycles)

For each subset $S_{j}$, the operations are again scheduled subject to the two above considerations while updating the number of cycles of the subsets that follow the interested one.
In [25], [26] and [27], the addressed problem is the switching power dissipation in CMOS circuits. In addition to many other parameters, this kind of power dissipation also depends on the gate levels. We determine the gate level in a polynomial time $\left(\mathrm{O}\left(\mathrm{N}^{2}\right)\right.$ where N is the number of the gates) as follows:


Fig.3.3 A control partitioning
$\operatorname{Level}(\mathrm{G})=1$ if all the inputs of gate $G$ are primary inputs
$\operatorname{Level}(\mathrm{G})=1+\mathrm{k} ; \mathrm{k}$ is the maximum among the levels of the gates that feed $G$

In order to determine operation dependencies, the procedure that determines the gate levels has been transformed. Notice that there is a direct relationship between operations and gates, and between the initial scheduling steps and the gate levels. Finally, even if 2 operations belonging to the same subset are independent, they cannot be scheduled in the same cycle in case there exists a user-constraint preventing such a scheduling. The next subsection will deal with this problem.

### 4.2 Scheduling subject to User Constraints

Let us assume that many operations of the same type (e.g addition) are scheduled in the same control step. Those operations can be then performed simultaneously, but with the expense of using the same number of resources (adders), which would require a great amount of silicon area. In order to overcome that problem, the designer can introduce some constraints so that some operations will be scheduled in different control steps even they are independent. In this way, the same resources would be used for different operations of the same type when they are not scheduled in the same control step.
Let us suppose the operations in Fig. 4.1 be all independent. Without the users-constraints $a, b$ and $c$, the 6 operations can be scheduled in a single cycle. However, user-constraint a prevents us to schedule operations 1 and 5 in the same cycle. Similarly, the user-constraint $b$ (c) prevents us to schedule operations 2 (3) and 4 (6) in the same cycle (in the remaining of this section we assume that the constraints concern independent operations which are first scheduled in the same cycle since the user constraints would be already satisfied else). One of some solutions consists to insert cuts after operations 1, 2 and 3 . Then, the following scheduling results:
Cycle i: 1 ;
Cycle i+1: 2;
Cycle i+2: 3;
Cycle i+3: 4; 5; 6;
On the other hand, inserting a cut after operation 3 will yield the following scheduling:
Cycle i: $1 ; 2 ; 3$;
Cycle i+1: 4; 5; 6;
This clearly shows that an appropriate cut-insertion strategy leads to a better scheduling while satisfying


Fig. 4.1 Operations subject to user-constraints.

(a) user constraints


Fig.4.2 A graph transformation
all the user constraints (in the last example, a good cut insertion achieved $50 \%$ reduction in the cycle number). Unfortunately, the problem being addressed is not a time-polynomial but rather a combinational one.
Many previous works have solved this problem using the interval graph-based technique. Let us consider the example depicted in Fig.4.2. The best solution such a technique can yield is obtained by inserting a cut after operation 2 (the cut is assigned to constraints $a$ and $b$ ) and another after operation 5 (the cut is assigned to constraint $c$ ).
We then have the following scheduling:
Cycle i: $\quad 1 ; 2$;
Cycle i+1: 3; 4; 5;
Cycle i+2: 6;
The cut insertion is done using the following strategy:

```
for each cut ct \({ }_{i}\)
do \(\left\{\right.\) define \(I_{i}=\max \left[\min \left(\right.\right.\) constraint \(_{j} ; j=1,2, \ldots\), number of
    constraints assigned to cut \(\mathrm{ct}_{\mathrm{i}}\) )];
    insert cut \(\mathrm{ct}_{\mathrm{i}}\) at operation \(\mathrm{I}_{\mathrm{i}}\);
\}
end for
```

For the example in Fig.4.2, we obtain:
$\mathrm{I}_{1}=\max [\min (1,3), \min (2,4)]=2$ because cut $_{1}$ is assigned to constraints $a=[1,3]$ and $b=[2,4]$
$\mathrm{I}_{2}=\max [\min (5,6)]=5$ because cut ${ }_{2}$ is assigned to constraint $\mathrm{c}=[5,6]$
Because our procedure concerning the userconstraint satisfaction operates on independent operations (the dependence constraints are already solved), we have used another technique that gives better solutions.
More precisely, the problem that is being addressed can be stated as follows:
Find the minimal value of cycles while satisfying all the user-constraints, which problem is similar to the graph colouring one:

Find function $\boldsymbol{f}$ minimizing $\boldsymbol{K}(1 \leq K \leq|V|)$ such that.
$f: V \rightarrow\{1,2, \ldots ., K\}$
$f\left(v_{i}\right) \neq f\left(v_{j}\right) \forall\left(v_{i}, v_{j}\right) \in E$

The last problem is known as NP-complete and until now one conjecture that there exists no polynomial algorithm solving it [28]. Our problem under consideration can be polynomially transformed into the graph colouring one. Because we have previously developed a new efficient technique solving all the NP-complete problems ([22], [23], [24]), we can efficiently solve the problem being addressed by transforming it into the graph colouring one as follows:
// Construct graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$
for each operation $O_{i}$
do create a node $\mathrm{v}_{\mathrm{i}} \in \mathrm{V}$
end for
$\left(\mathrm{v}_{\mathrm{i}}, \mathrm{v}_{\mathrm{j}}\right) \in \mathrm{E}$ iff $\mathrm{O}_{\mathrm{i}}$ and $\mathrm{O}_{\mathrm{j}}$ are subject to the same constraint Colour_G();
Transform the results into cycles; // there are as many cycles as // colours;

Let us now briefly describe our graph colouring technique.
For the graph colouring problem, the number of colours increases as $|\mathrm{E}|$ grows. This is clearly depicted in Fig.4.3 : Fig.4.3.a shows that the number of colours remains unchanged although $|\mathrm{V}|$ has been increased. On the other hand, the number of colours gets higher in Fig.4.3.b even if $|\mathrm{V}|$ holds the same value. Thus, additional nodes do not affect the colour number in case the set of edges remains
unchanged even the number of the new nodes is a large value. On the other hand, adding a few numbers of edges can increase the colour number, considerably. This means that the graph colouring problem is strongly dependent on the edges of the interested instance. However, the node degree order is also a parameter that affects the solution of the problem under consideration. For this purpose, let us sort the nodes of the graph instance that is depicted in Fig.4.4 according to the increased number of their degree. Giving a graph $G=(V, E)$, we define the degree of a node $i$ as the number of edges $(\mathrm{i}, \mathrm{j}) \in \mathrm{E} ; \mathrm{i} \neq \mathrm{j}$. The result of this node sorting is shown in Tablel.

| Node | v1 | v4 | v5 | v6 | v3 | v2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Degree | 0 | 1 | 1 | 1 | 2 | 3 |

Table 1. A node sorting according to the increased number of the node degrees.

Let us colour the graph instance of Fig.4.4 using the greedy algorithm while maintaining the node degree order shown in Table 1. We obtain three colours ( $\mathrm{R}, \mathrm{G}, \mathrm{B}$ ). This result is given in Table 2.

| Node | v1 | v4 | v5 | v6 | v3 | v2 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Colour | R | R | R | R | G | B |

Table 2. A node colouring according to the increased number of the node degrees.

Considering the same instance that is depicted in Fig.4.4, let us now colour the nodes using the greedy algorithm according to the decreased number of their degree. The colour number is smaller (2 instead of 3) as shown in Table 3.

a. Adding nodes while maintaining the same edges does not increase the number of colours.

b. Adding edges to graph $G$ could increase the colour number even if the number of nodes remains the same.

Fig.4.3. Variation of the colour number vs. the graph complexity

| Node | v2 | v3 | v4 | v5 | v6 | v1 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Colour | R | G | R | G | G | G |

Table 3. A node colouring according to the decreased number of the node degrees.

The above results show that the sorting key (increased or decreased degrees) affects the colour number. However, it would be wrong to claim that a node colouring according to the decreased number of their degree yields fewer colours. The instance that is depicted in Fig.4.6 contradicts such a statement.
Let the sorting key be the decreased node degrees.
The result is shown in Table 4.
The node colouring according to the previous sorting key will yield 2 colours as shown in Table 5. However, keeping the same node degree order and reordering only nodes v6 and v2 (that have the same degree), the obtained result is $\mathbf{3}$ colours as shown in Table 6.
Finally, notice that the minimum number of colours (2) can be obtained for the last instance when the increased node degree is considered. This is shown in Table 7.
We have just seen that the node degree order affects the colour number. More, we have seen that neither the increased node degree nor the decreased one will always yield the best result. Hence, the only way to guarantee the exact result is to consider all the node degree orders then to perform the graph colouring over them. Given an undirected graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$, the number of such node degree orders is : $\mathrm{A}^{\mathrm{N}}{ }_{\mathrm{N}}=\mathrm{N}!/(\mathrm{N}-\mathrm{N})!=\mathrm{N}!; \mathrm{N}=|\mathrm{V}|$.
Performing N ! graph colourings for the same instance is computationally infeasible for a large value of N . However, a single graph colouring can yield poor results. Therefore, a heuristic method that overcomes these two drawbacks can be the one that avoids either N ! graph colourings or a single one. Thus, considering $\mathrm{m}(1 \leq \mathrm{m} \leq \mathrm{N}$ !) node degree orders is a good tradeoff to make a large solution space exploration while maintaining a reasonable CPU time. However, the challenge consists in selecting m node degree orders among N ! ones while targeting good solutions. The details of this technique are described in [22]. We merely say that our heuristic-based method groups the graph nodes into k classes $(\mathrm{k} \ll \mathrm{N})$. Then, instead of performing N ! node degree orders, only k ! ones are considered : the node degree orders are performed over classes, regardless the order of the nodes belonging to the same class.
Because the node assignment over the classes is not unique, we also need the best one. Our node
assignment is the solution of the following optimisation problem:

$$
\min \sum_{i=1}^{k} d i f f_{i}
$$

such that

$$
\begin{equation*}
\text { i) } \operatorname{diff}_{\mathrm{i}}=\mathrm{d}_{\mathrm{Mi}}-\mathrm{d}_{\mathrm{mi}} ; \mathrm{i}=1,2, \ldots, \mathrm{k} \tag{1}
\end{equation*}
$$

$$
\text { ii) } \sum_{i=1}^{k}\left|C l{ }_{i}\right|=N \text {; }
$$

where $\mathrm{d}_{\mathrm{mi}}$ and $\mathrm{d}_{\mathrm{Mi}}$ are the least and the greatest degree of the nodes included in class $\mathrm{Cl}_{\mathrm{i}}$, respectively. Notice that the motivation of this node assignment is detailed in [22].
Thanks to our algorithm which has just been presented, the graph corresponding to the constraints depicted in Fig. 4.2.a is the one shown in Fig.4.2.b(notice that the operations are independent)

For Fig.4.2, the best solution obtained with our graph colouring technique can be only two colours:

- colour 1: for operations 1,2 and 5
- colour 2: for operations 3, 4 and 6

Then, the following scheduling results:
Cycle i: $1 ; 2 ; 5$;
Cycle i+1: 3; 4; 6;


Fig.4.4. A graph instance.


Fig.4.5. Another gravh


Fig.4.6. Another graph instance.

Notice that there is no operation reordering procedure (it is an implicit process thanks to our graph colouring-based strategy) and that the best solution given with the interval graph- based technique is 3 (because the latter technique does not allow operations reordering). Finally, it is obvious that for a large set of user-constraints, our graph colouringbased technique will considerably reduce the cycle number with respect to the graph interval-based one.

| Node | v6 | v4 | v3 | v2 | v1 | v7 | v5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Degree | 2 | 2 | 2 | 2 | 2 | 1 | 1 |

Table 4. A node sorting according to the decreased number of the node degrees.

| Node | v6 | v4 | v3 | v2 | v1 | v7 | v5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Colour | R | R | R | G | G | G | G |

Table 5. A node colouring of the graph instance of Fig. 4.6.

| Node | v2 | v4 | v3 | v6 | v1 | v7 | v5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Colour | R | G | R | G | B | G | R |

Table 6. Another node colouring of the graph instance of Fig. 4.6.

| Node | v5 | v7 | v1 | v2 | v3 | v4 | v6 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Colour | R | R | R | R | G | G | G |

Table 7. A node colouring according to the increased number of the node degrees.

The node assignment is better explained by considering the graph instance that is depicted in Fig.4.5. The node degrees are given in Table 8 while Table 9 gives the value of $\left(\operatorname{diff}_{1}+\operatorname{diff}_{2}+\operatorname{diff}_{3}\right)$ for each node assignment.

| Node | v1 | v2 | v3 | v4 | v5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Degree | 1 | 2 | 2 | 2 | 3 |

Table 8. Node degrees.

| Cl1 <br> \#nodes diff <br> $\mathbf{1}$ |  | Cl2 <br> \#nodes diff <br> 2 | Cl3 <br> \#nodes diff <br> 3 | diff $_{\mathbf{1}}+$ <br> diff $_{2}+$ <br> diff $_{3}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 1 | 0 | 3 | 1 | 1 |
| 1 | 0 | 2 | 0 | 2 | 1 | 1 |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{3}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| 2 | 1 | 1 | 0 | 2 | 1 | 2 |
| 2 | 1 | 2 | 0 | 1 | 0 | 1 |
| 3 | 1 | 1 | 0 | 1 | 0 | 1 |

Table 9. A node partitioning
Assuming $\mathrm{k}=3$, the third node partitioning (bold entry in Table 9) is the best since (4.1) is minimal (it is equal to 0 ): $\mathrm{Cl}_{1}=\{\mathrm{v} 1\}, \mathrm{Cl}_{2}=\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\}$ and $\mathrm{Cl}_{3}=\{\mathrm{v} 5\}$.
The graph instance that is depicted in Fig. 3.5 will be handled according to the $(\mathrm{k}!=3!=6)$ following node degree orders:
$1^{\text {st }}$ node degree order:
$\mathrm{Cl}_{1}, \mathrm{Cl}_{2}, \mathrm{Cl}_{3} \rightarrow\{\mathrm{v} 1\},\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\},\{\mathrm{v} 5\}$
$2^{\text {nd }}$ node degree order:
$\mathrm{Cl}_{2}, \mathrm{Cl}_{1}, \mathrm{Cl}_{3} \rightarrow\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\},\{\mathrm{v} 1\},\{\mathrm{v} 5\}$
$3^{\text {rd }}$ node degree order:
$\mathrm{Cl}_{3}, \mathrm{Cl}_{2}, \mathrm{Cl}_{1} \rightarrow\{\mathrm{v} 5\},\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\},\{\mathrm{v} 1\}$
$4^{\text {th }}$ node degree order :
$\mathrm{Cl}_{1}, \mathrm{Cl}_{3}, \mathrm{Cl}_{2} \rightarrow\{\mathrm{v} 1\},\{\mathrm{v} 5\},\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\}$
$5^{\text {th }}$ node degree order :
$\mathrm{Cl}_{3}, \mathrm{Cl}_{1}, \mathrm{Cl}_{2} \rightarrow\{\mathrm{v} 5\},\{\mathrm{v} 1\},\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\}$
$6^{\text {th }}$ node degree order :
$\mathrm{Cl}_{2}, \mathrm{Cl}_{3}, \mathrm{Cl}_{1} \rightarrow\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\},\{\mathrm{v} 5\},\{\mathrm{v} 1\}$
Thus, our heuristic-based method performs like the exact method with the difference that $\{\mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4\}$ is assumed as a "node" (the node degree orders concerning v 2 , v 3 and v 4 are not considered). However, in order to better improve the quality of the solution, we perform two graph colourings (from the left to the right and from the right to the left in the list of the nodes) for each node degree order. Let us consider for instance the $1^{\text {st }}$ node order $\left(\mathrm{Cl}_{1}, \mathrm{Cl}_{2}, \mathrm{Cl}_{3}\right)$. The two following node orders will be considered:
-left to right: v1,v2, v3,v4,v5 (notice that this node order is different from the right to the left order in $\mathrm{Cl}_{3}$, $\mathrm{Cl}_{2}, \mathrm{Cl}_{1}$ which is $\left.\mathrm{v} 1, \mathrm{v} 4, \mathrm{v} 3, \mathrm{v} 2, \mathrm{v} 5\right)$
-right to left: v5, v4, v3, v2, v1 (notice that this node order is different from the left to the right order in $\mathrm{Cl}_{3}$, $\mathrm{Cl}_{2}, \mathrm{Cl}_{1}$ which is $\left.\mathrm{v} 5, \mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4, \mathrm{v} 1\right)$.
Therefore, k !*2 graph colourings are performed for a graph instance whose nodes are partitioned into k classes. For the graph instance that is depicted in Fig.4.5, the best solution has been 2 colours, which is also the exact one.
Finally, notice that our graph colouring technique:
i) includes both the ascending node degree and the descending node degree strategies. For the graph instance depicted in Fig.4.5, these two strategies represent the following node degree orders:

- left to right in $\mathrm{Cl}_{1}, \mathrm{Cl}_{2}, \mathrm{Cl}_{3}: \mathrm{v} 1, \mathrm{v} 2, \mathrm{v} 3, \mathrm{v} 4$, v5
- right to left in $\mathrm{Cl}_{1}, \mathrm{Cl}_{2}, \mathrm{Cl}_{3}: \mathrm{v} 5, \mathrm{v} 4, \mathrm{v} 3, \mathrm{v} 2, \mathrm{v} 1$
ii) yields the exact solution in case $\mathrm{k}=|\mathrm{V}|$
iii) yields the exact solution in case the solution is independent on the ordering of the nodes belonging to the same class


## 5 Reduction of the Switching Power Dissipation at a High Design Level

In current designs (SOC designs), the power dissipation is a crucial problem (limited life of a battery, reliability problem due to a temperature increase ...). Thus, we need low power designs to cope with this problem. The main components of the power dissipation are the switching activity, the short circuits and the leakage power.
The power dissipation due to short circuits can be minimized using appropriate technologies (e.g. CMOS) with techniques reducing the current due to short circuits. This current can be reduced by shortening the time transitions of the signals that feed the transistors (transistors N et P will conduct simultaneously in a very short time). The leakage power is reduced by assigning high threshold voltages to some transistors in a dual-threshold and a dual- $\mathrm{V}_{\mathrm{DD}}$ based-design.
In fact, the reduction of the power dissipation is tackled at each design level. Reducing the switching power can be done at earlier stages of the design. We will see hereafter how our tool gives the user the ability to cope with the reduction of the switching power dissipation at a high design level.
We recall that two any operations subject to some constraint are scheduled in different steps. Because the switching power is as high as the number of the operations that are simultaneously performed is high, additional constraints are introduced such that the switching power is reduced. However, the introduction of constraints would be tedious and the user has no idea on how to introduce them in order to obtain less switching power dissipation, which needs an automated task.
The starting point of our heuristic for reducing the switching power dissipation is the results provided by the scheduling process. Indeed, many operations scheduled in the same control step will dissipate some amount of power. Thus, our heuristic introduces constraints for the operations scheduled in such control steps. Then, the operations are again scheduled subject to those new constraints. The process iterates while the switching dissipation remains higher than the desired one $\mathrm{P}_{\text {desired }}$. Notice that we need a minimal number of control steps while satisfying the power dissipation constraint. Our heuristic is detailed in Fig.5.1.

```
// Reduction of the Switching Power Dissipation at the Register
/ Transfer Level
\(p=+\infty\);
while \(\left(\mathrm{P}>\mathrm{P}_{\text {desired }}\right)\)
do \{ schedule_traces(); // use our previously described tool
    \(\mathrm{P}=-\infty\);
    for each trace 1
    do \{use SPOT ([25], [26], [27]) to determine POWER, the
            switching power dissipated per cycle;
            if(POWER > P)
            then \(\mathrm{P}=\mathrm{POWER}\);
            endif
                for \(\mathrm{j}=1\) to nb _cycles \({ }_{\mathrm{i}}\)
                // nb_cycles \({ }_{i}\) is the number of cycles in trace \(i\) with
                // the current scheduling
            do if \(n b i_{j}>\) avg \(n b i\)
                \(/ / n b_{-} i_{j}\) is the number of operations of trace i
                // in cycle j of the current scheduling
                // avg_nb_i is the average number of the
                // operations per cycle in the next
                    // scheduling
                then \(\left\{\mathrm{k}_{\mathrm{ij}}=\left\lfloor\right.\right.\) nb_ \(\mathrm{i}_{\mathrm{j}} /\) avg_nb_i \(\rfloor\);
                            construct \(\mathrm{Ng}_{\mathrm{ij}}=\left\lceil\mathrm{nb}_{-} \mathrm{i}_{\mathrm{j}} / \mathrm{k}_{\mathrm{ij}}\right\rceil\) graphs \(\mathrm{G}_{1}, \quad \mathrm{G}_{2}\),
                        ..., \(\mathrm{G}_{\mathrm{Ngij}}\) such that:
                        \(.\left|\mathrm{V}_{\mathrm{k}}\right|=\mathrm{k}_{\mathrm{ij}} ;\left|\mathrm{E}_{\mathrm{k}}\right|=\mathrm{k}_{\mathrm{ij}} *\left(\mathrm{k}_{\mathrm{ij}}-1\right) / 2\);
                        \(\mathrm{k}=1,2, \ldots, \mathrm{Ng}_{\mathrm{ij}}\)
                                \(\mathrm{n}=\mathrm{Ng}_{\mathrm{ij}}{ }^{*} \mathrm{k}_{\mathrm{ij}}\);
                                if \(\left(\mathrm{n}<\mathrm{nb}_{-} \mathrm{i}_{\mathrm{j}}\right)\)
                                then \(\{\) construct an additional graph
                                    \(\mathrm{G}_{\mathrm{a}}=\left(\mathrm{V}_{\mathrm{a}}, \mathrm{E}_{\mathrm{a}}\right)\) such that:
                                    . \(\left|\mathrm{V}_{\mathrm{a}}\right|=\mathrm{nb} \mathrm{i}_{\mathrm{j}}-\mathrm{n}=\mathrm{n}_{\mathrm{a} i \mathrm{ij}}\);
                                    . \(\left|\mathrm{E}_{\mathrm{a}}\right|=\mathrm{n}_{\mathrm{a}-\mathrm{ij}} *\left(\mathrm{n}_{\mathrm{a}-\mathrm{ij}}-1\right) / 2\)
                                    \}
                                endif
                                for each operation \(I_{1}\) in cycle \(j\) of the current
                        scheduling, there exists 1 and
                                    only 1 node \(\mathrm{V}_{1}\) in \(\quad{\underset{u}{\mathrm{~N}}}_{\mathrm{Ng}}^{V_{k}}{ }_{k} \quad \cup \quad V_{a}\)
                                    for each graph \(\mathrm{G}_{\mathrm{k}}=\left(\mathrm{V}_{\mathrm{k}}, \mathrm{E}_{\mathrm{k}}\right) \quad\left(1 \leq \mathrm{k} \leq \mathrm{Ng}_{\mathrm{ij}}\right)\)
                                    do for each edge \(e_{l m} \in E_{k}\)
                                    do generate \(a\) constraint between
                                    operations \(\mathrm{I}_{1}\) and \(\mathrm{I}_{\mathrm{m}}\);
                    end
                            for each edge \(\mathrm{e}_{\mathrm{lm}} \in \mathrm{E}_{\mathrm{a}} / /\) if \(\mathrm{G}_{\mathrm{a}}\) exists
                            do generate \(a\) constraint between
                                    operations \(I_{1}\) and \(I_{m}\);
                                    end
                                    end
                        \}
                endif
            end
            \}
        end
    \}
end
```

Fig.5.1. Algorithm balancing the throughput and the power dissipation of a circuit

Notice that in each iteration of the previous algorithm, the number of constraints is equal to:

$$
\begin{aligned}
& \left.\sum_{i=1}^{\text {Unacs }} \sum_{j=1}^{\text {tec }} \sum_{i=1}\left[k_{i j} *\left(k_{i j}-1\right) / 2\right] * N g_{i j}+n_{\alpha-i j} *\left(n_{a-i j}-1\right) / 2\right] \\
& =\frac{1}{2} \sum_{i=1}^{\text {\#naces }} \sum_{j=1}^{i c}\left[\left[k_{i j}{ }^{*}\left(k_{i j}-1\right)\right] * N g_{i_{j}}+n_{a_{-i}}{ }^{*}\left(n_{a_{-i}}-1\right)\right]
\end{aligned}
$$

where $\mathrm{Ng}_{\mathrm{ij}}$ is the number of the graphs constructed for the $\mathrm{j}^{\text {th }}$ cycle of the $\mathrm{i}^{\text {th }}$ trace, $\mathrm{k}_{\mathrm{ij}}$ is the number of nodes in each of the $\mathrm{Ng}_{\mathrm{ij}}$ graphs, $\mathrm{n}_{\mathrm{a}_{\mathrm{i}} \mathrm{ij}}$ is the number of nodes in the prospective additional graph for the $\mathrm{j}^{\text {th }}$ cycle of the $\mathrm{i}^{\text {th }}$ trace, \#c_i is the number of cycles in the current scheduling of trace $i$ such that there are more than $a v g_{-} n b \_i$ operations in each of them.
As it is shown in the last equation, the number of constraints can be very high. So, an automated task is required to cope with the problem of their determination.
Notice that operation scheduling can be performed together with binding. Thus, we have developed a procedure that assigns resources to operations as follows:

- in each cycle, the sum of areas of the selected resources $\left(\mathrm{S}_{\mathrm{a}}\right)$ that perform operations scheduled in the same cycle must be less than a fixed area value $A$
- assignment priority is given for the fastest resources whenever it is possible (area constraints met)
- the power reduction is then performed by algorithm in Fig.5.1

Example: Let us explain the reduction technique with the following example. Let us assume that 26 ( $n b \_i_{j}$ ) operations are scheduled in some control step $j$ of some trace i of the CDFG. Let P be the calculated switching power dissipation of the circuit that represents the CDFG at the register transfer level. Let us assume that $\mathrm{P}>\mathrm{P}_{\text {desired }}$ and that the average number of operations per cycle in the next scheduling is 8 (avg_nb_i). We have to schedule the 26 operations in different cycles (not in the same cycle as they are currently). Thus, we have to introduce power constraints such that to reduce $P$. These constraints are built as follows:

$$
\mathrm{k}=\left\lfloor\text { nb_ } \mathrm{i}_{\mathrm{j}} / \text { avg_nb_i }\right\rfloor=\lfloor 26 / 8\rfloor=4
$$

$$
\text { - construct } \mathrm{Ng}=\lceil 26 / 4\rceil=6 \text { graphs } \mathrm{G}_{1}, \mathrm{G}_{2}, \ldots, \mathrm{G}_{6}
$$

$$
\left|\mathrm{V}_{\mathrm{k}}\right|=\mathrm{k}=4 ;\left|\mathrm{E}_{\mathrm{k}}\right|=\left|\mathrm{V}_{\mathrm{k}}\right| *\left(\left|\mathrm{~V}_{\mathrm{k}}\right|-1\right) / 2=6 ; \mathrm{k}=1,2, \ldots, 6
$$

- construct an additional graph $\mathrm{G}_{\mathrm{a}}=\left(\mathrm{V}_{\mathrm{a}}, \mathrm{E}_{\mathrm{a}}\right)$

$$
\left|\mathrm{V}_{\mathrm{a}}\right|=\mathrm{nb} \mathrm{i}_{\mathrm{j}}-\mathrm{Ng} *\left|\mathrm{~V}_{\mathrm{k}}\right|=26-6 * 4=2
$$

- for each operation $\mathrm{O}_{1_{6}}(1 \leq 1 \leq 26)$ there is one and only one node $\mathrm{v}_{1}$ in $\bigcup_{k=1} V_{k}$
- generate a constraint between operations $I_{1}$ and $I_{m}$ if $\mathrm{e}_{\mathrm{lm}} \in \mathrm{E}_{\mathrm{k}} ; \mathrm{k}=1,2, \ldots, 6$ (for each of the 24 operations there is a node belonging to $\mathrm{V}_{\mathrm{k}} ; \mathrm{k}=1,2, \ldots 6$ (see Fig.5.1)
- generate a constraint between operations $I_{1}$ and $I_{m}$ if $\mathrm{e}_{\mathrm{lm}} \in \mathrm{E}_{\mathrm{a}}$

$\mathrm{O}_{\mathrm{i} 1} \mathrm{O}_{\mathrm{i} 2} \mathrm{O}_{\mathrm{i} 3} \mathrm{O}_{\mathrm{i} 4} / /$ there are 3 constraints $: \mathrm{O}_{\mathrm{i} 1}-\mathrm{O}_{\mathrm{i} 2}, \mathrm{O}_{\mathrm{i} 1}-\mathrm{O}_{\mathrm{i} 3}, \mathrm{O}_{\mathrm{i} 1}-\mathrm{O}_{\mathrm{i} 4}$ $\mathrm{O}_{\mathrm{i} 2} \mathrm{O}_{\mathrm{i} 1} \mathrm{O}_{\mathrm{i} 3} \mathrm{O}_{\mathrm{i} 4}$ $\mathrm{O}_{\mathrm{i} 3} \mathrm{O}_{\mathrm{i} 1} \mathrm{O}_{\mathrm{i} 2} \mathrm{O}_{\mathrm{i} 4}$ $\mathrm{O}_{\mathrm{i} 4} \quad \mathrm{O}_{\mathrm{i} 1} \mathrm{O}_{\mathrm{i} 2} \mathrm{O}_{\mathrm{i} 3}$ $\mathrm{O}_{\mathrm{i} 5} \mathrm{O}_{\mathrm{i} 6} \mathrm{O}_{\mathrm{i} 7} \mathrm{O}_{\mathrm{i} 8}$
$\mathrm{O}_{\mathrm{i} 24} \mathrm{O}_{\mathrm{i} 21} \mathrm{O}_{\mathrm{i} 22} \mathrm{O}_{\mathrm{i} 23}$
$\mathrm{O}_{\mathrm{i} 25} \mathrm{O}_{\mathrm{i} 26}$
(c) The new constraints added in the constraint
- we have to schedule at most 8 (avg_nb_i) operations per cycle
- the control step j is replaced with $=\left|\mathrm{V}_{\mathrm{k}}\right|=4$ new cycles (notice that there is a trade off between the circuit performance and the switching power dissipation).

The scheduling is then performed as depicted in Fig.2.3 and accordingly to the algorithm which is previously described.

## 6 Results

Our tool first transforms a C-like code description into a set of DFGs. Then, scheduling techniques are performed on each DFG, followed by a result synthesis then a report of the global scheduling. In case the number of DFGs is huge, techniques for reducing the number of DGS are required such that the CDGF scheduling can be possible (section 3). The first reducing technique of that number aims to collapse an if-then-else branch into a super-node in case that branch does not include operations subject to constraints. Because the time complexity of this technique is $\mathrm{O}\left(\mathrm{N}_{\mathrm{cs}}{ }^{2}\right)$, where $\mathrm{N}_{\mathrm{cs}}$ is the number of the control structures, we have developed an exact reducing technique that detects and collapses all the conditional branches including constraint-free operations.
The results given in Table 10 show an interesting CPU time (measured on an Intel i386-based processor running at 863 MHz ) for reducing a large number of DFGs into a single one (in order to consider the worst case, we have assumed that no conditional branch includes operations subject to constraints). Notice that although our machine is not able to deal with values greater than $1.7^{*} 10^{308}$, our tool is able to process such instances (e.g. $2^{100000}$ and $2^{500000}$ DFGs in Table 10). Table 10 shows interesting CPU times (that include both the C-like code translation into a set of traces and control structure collapsings). Unfortunately, most of designs are subject to user constraints, which prevent to collapse some conditional branches. Thus, a control partitioning-based technique (subsection 3.2) is required. The challenge is to find optimal cuts such that the amount of parallelism is not decreased. Notice that the best cut insertion is then needed. For a complex CDFG, an adequate reducing technique has to be used.
Using the procedure Control_Partitioning() described in subsection 3.2, the results that represent the cut insertions are shown in Table 11. This table shows the efficiency of our control partitioning procedure. Indeed, a minimal number of cuts achieved a great reduction of the number of DFGs. For example,

- only 1 cut achieved a reduction of $1.980704 * 10^{28}$ DFGs (ip\#) into $2.81475^{*} 10^{14}$ ones (op\#) when the number of DFGs defined by the user is $9.90352 * 10^{27}$. Notice that the ratio op\# / ip\# is very low
- only 1008 cuts achieved a reduction of $8.98846 * 10^{307}$ DFGs into 2526 ones (the ratio is very low) when the required number of DFGs is 5000 Notice that:
- entries 1, 5, 9 and 13 in Table 11 represent the case in which the required number of DFGs is greater than or equals to the initial number (the number of cuts is then 0 )
- entries 4, 8, 12 and 16 in Table 11 represent the case in which the required number of DFGs is less than the minimal one (the number of cuts is then maximal and equals to $93,1022,99999$ and 499999, respectively). For such cases, the obtained numbers are equal to the minimal ones and still be greater than the required ones
- entries 3, 7, 11 and 15 in Table 11 represent the case in which the required number of DFGs equals to the minimal one. In this case, the number of cuts is less (because $i^{*} 2^{2}=i *(2+2), 0$ cut achieves the same number of DFGs than 1 cut for each couple of i disconnected couples of control structures) than or equal to $\mathrm{N}-1, \mathrm{~N}$ is the number of control structures
- entries 2, 6, 10 and 14 in Table 11 represent the case in which the required number of DFGs is greater than the minimal one and less than the initial one. The results in that table show that a minimal number of cuts ( $1,1008,97956$ and 497956) achieves very low ratios (obtained \#paths / initial \#paths): $1.42 * 10^{-14}$, $2.81 * 10^{-305}, 2^{-98977}$ and $2^{-499981}$, respectively while the obtained number of DFGs is less than or equal to the required one
- the CPU times represent the C-like code transformation and the control partitioning. For huge values of the initial numbers of DFGs (e.g. $8.98846^{*} 10^{307}$ ), the CPU times equal to 0 second. Only 5867 s (nearly 1 h 38 mn ) achieved the processing of a very large number of initial numbers of DFGs ( $2^{500000}$ ). In addition, notice that for entries $4,8,12$ and 16, the CPU times still be low even for a huge number of initial DFGs (e.g. $2^{500000}$ ) because in case the required number of the DFGs is less than the minimal one, our tool delivers a message indicating that impossibility. For the last case, the CPU time actually represents the C-like code transformation and the computation of the minimal numbers of the DFGs and the cuts. Finally, we have to notice that our tool does not fail for values that are greater than $1.7 * 10^{308}$ (the greatest value that can be stored in our machine) as it is shown in Table 11 for the numbers $2^{100000}$ and $2^{500000}$ that are both greater than $1.7 * 10^{308}$.

| Initial \#DFGs | Obtained \#DFGs $\left(^{*}\right)$ | CPU Time (s) |
| :---: | :---: | :---: |
| $1.980704^{*} 10^{28}$ | 1 | 0 |
| $8.98846^{*} 10^{307}$ | 1 | 0 |
| $2^{100000}$ | 1 | 12 |
| $2^{500000}$ | 1 | 239 |

(*) in order
collapsed.
Table 10. A conditional branch collapsing

| Entry \# | \# Control Structures | Initial \#paths | Required \# paths | Obtained \# paths | \#cuts | CPU Time <br> (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 94 | $1.980704 * 10^{28}$ | $2^{94}$ | $2^{94}$ | 0 | 0 |
| 2 | 94 | $1.980704 * 10^{28}$ | $9.90352 * 10^{27}$ | $2.81475 * 10^{14}$ | 1 | 0 |
| 3 | 94 | $1.980704 * 10^{28}$ | 188 | 188 | 92 | 0 |
| 4 | 94 | $1.980704 * 10^{28}$ | 187 | 188 | 93 | 0 |
| 5 | 1023 | $8.98846 * 10^{307}$ | $9.0 * 10^{307}$ | $8.988466 * 10^{307}$ | 0 | 0 |
| 6 | 1023 | $8.98846 * 10^{307}$ | $5.0 * 10^{3}$ | 2526 | 1008 | 0 |
| 7 | 1023 | $8.98846 * 10^{307}$ | 2046 | 2046 | 1020 | 0 |
| 8 | 1023 | $8.98846 * 10^{307}$ | 2045 | 2046 | 1022 | 0 |
| 9 | 100000 | $2^{100000}$ | $2^{100000}$ | $2^{100000}$ | 0 | 13 |
| 10 | 100000 | $2^{100000}$ | $1.0 * 10^{308}$ | $8.988466 * 10^{307}$ | 97956 | 3809 |
| 11 | 100000 | $2^{100000}$ | $2.0 * 10^{5}$ | $2.0 * 10^{5}$ | 99997 | 3845 |
| 12 | 100000 | $2^{100000}$ | $15.0 * 10^{4}$ | $2.0 * 10^{5}$ | 99999 | 13 |
| 13 | 500000 | $2^{500000}$ | $2^{500000}$ | $2^{500000}$ | 0 | 81 |
| 14 | 500000 | $2^{500000}$ | $1.69 * 10^{308}$ | $8.988466 * 10^{307}$ | 497956 | 5256 |
| 15 | 500000 | $2^{500000}$ | $1.0 * 10^{6}$ | $1.0 * 10^{6}$ | 499997 | 5867 |
| 16 | 500000 | $2^{500000}$ | $0.5 * 10^{6}$ | $1.0 * 10^{6}$ | 499999 | 74 |

Table 11. A control partitioning

| $\begin{gathered} \hline \text { DIMACS } \\ \text { benchmark } \\ \text { instance } \\ \hline \end{gathered}$ | Exact solution | $\begin{gathered} \text { \#colours } \\ (\text { \#classes }=1) \end{gathered}$ | \% error | CPU Time <br> (s) | (\#colours, \#classes) | \% error | $\begin{gathered} \text { CPU } \\ \text { Time (s) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1. anna | 11 | 11 | 0 | 0 | $(11,1)$ | 0 | 0 |
| 2. david | 11 | 11 | 0 | 0 | $(11,1)$ | 0 | 0 |
| 3. myciel3 | 4 | 4 | 0 | 0 | $(4,1)$ | 0 | 0 |
| 4. myciel4 | 5 | 5 | 0 | 0 | $(5,1)$ | 0 | 0 |
| 5. myciel5 | 6 | 6 | 0 | 0 | $(6,1)$ | 0 | 0 |
| 6. myciel6 | 7 | 7 | 0 | 0 | $(7,1)$ | 0 | 0 |
| 7. huck | 11 | 11 | 0 | 0 | $(11,1)$ | 0 | 0 |
| 8. zeroin.i. 3 | 30 | 30 | 0 | 0 | $(30,1)$ | 0 | 0 |
| 9. miles 750 | 31 | 31 | 0 | 0 | $(31,1)$ | 0 | 0 |
| 10. miles500 | 20 | 20 | 0 | 0 | $(20,1)$ | 0 | 0 |
| 11. miles 1500 | 73 | 73 | 0 | 4 | $(73,1)$ | 0 | 4 |
| 12. miles 1000 | 42 | 42 | 0 | 1 | $(42,1)$ | 0 | 1 |
| 13. games 120 | 9 | 9 | 0 | 0 | $(9,1)$ | 0 | 0 |
| 14. jean | 10 | 10 | 0 | 0 | $(10,1)$ | 0 | 0 |
| 15. myciel7 | 8 | 9 | 12.5 | 0 | $(8,3)$ | 0 | 0 |
| 16. queen5 5 | 5 | 7 | 40 | 0 | $(5,7)$ | 0 | 2 |
| 17. miles250 | 8 | 9 | 12.5 | 0 | $(8,3)$ | 0 | 0 |
| 18. queen6 6 | 7 | 10 | 42.86 | 0 | $(8,8)$ | 14.29 | 49 |
| 19. queen8_8 | 9 | 14 | 55.56 | 0 | $(11,8)$ | 22.22 | 1858 |
| \%error average |  |  | 8.60 |  |  | 1.92 |  |

Table12. Experimental results for the graph colouring problem

| \#paths | \#cuts | 「average \#cyclesך <br> (no user constraint) | 「average \#cycles7 <br> $(22$ user constraints) | CPU Time <br> $(\mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: |
| 16384 | 0 | 7 | 11 | 12671 |
| 256 | 1 | 8 | 12 | 0 |
| 152 | 2 | 9 | 12 | 0 |
| 48 | 3 | 10 | 0 | 0 |

Table13. A CDFG scheduling

Although the graph colouring problem is NPcomplete, the exact solution is known for some DIMACS benchmarks. The results in Table 12 show the efficiency of our graph colouring technique. Indeed, using only one class, the CPU time does not exceed 4 s for allowing our technique to yield the exact solution for the 14 first instances given in Table 12. For the $15^{\text {th }}, 16^{\text {th }}$ and $17^{\text {th }}$ instance, the exact solution is obtained with $\mathrm{k}=3$ ( k is the number of classes), $\mathrm{k}=7$ and $\mathrm{k}=3$, respectively. A closer solution to the exact one is obtained for the $18^{\text {th }}$ instance (\#colours $=8, \mathrm{k}=8$ ) and the $19^{\text {th }}$ one (\#colours=11, $\mathrm{k}=8$ ). It is also shown that $30^{\prime} 58^{\prime \prime}$ allowed a closer solution (11) to the exact one (9) than that obtained (14) with one class instead with 8 ones. Finally, this table shows that the average of the error percentage decreases from 8.60 to only 1.92 when more than one class is used for the node assignment.
The results in Table 13 represent the 2 steps of the CDFG scheduling (data dependencies and user constraints satisfaction):

- the $1^{\text {st }}$ entry of Table 10 shows that instructions included in 16384 DFGs are scheduled in 7 cycles (11 cycles) when there is no partitioning and no user constraint ( 22 user constraints). Notice that the $1^{\text {st }}$ step of the scheduling requires 7 cycles
- noticing that a cut increases the number of cycles but in the same time can satisfy some user constraints (whose satisfaction requires extra cycles), the average numbers of the cycles after the two scheduling steps are nearly the same (11 and 12)
As expected from the results of our graph colouring technique, Table 13 shows that:
- 22 user constraints yielded only 4 extra cycles, which shows that our constraint satisfaction procedure is efficient
- the average number of the cycles is determined from the obtained cycle numbers resulted from the different DFGs schedulings
- the CPU time includes the C-like code transformation, the cut partitioning and the 2 steps of the scheduling. We recall that in order to obtain very close results to the exact ones, different graph colourings of the same graph instance are processed ([22], [23], [24]) with the expense of the CPU time (but within a fixed time duration: to the best of our knowledge, it is not advantageous to obtain very low CPU times with the expense of poor results which will implement an integrated circuit whose life time will be some years).

| Bench. Circuit | [29] |  |  | [30] |  |  | Our Method <br> With binding ; Without binding |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Lat. } \\ \text { (cycles) } \end{gathered}$ | $\begin{gathered} \text { Pwr } \\ (\mathrm{mW}) \end{gathered}$ | $\begin{gathered} \text { Run } \\ \text { Time } \\ \text { (s) } \\ \hline \hline \end{gathered}$ | $\begin{gathered} \text { Lat. } \\ \text { (cycles) } \end{gathered}$ | $\begin{gathered} \text { Pwr } \\ (\mathrm{mW}) \end{gathered}$ | Run Time <br> (s) | $\begin{gathered} \text { Lat. } \\ \text { (cycles) } \end{gathered}$ | $\begin{aligned} & \text { MaximalPower } \\ & (\mathrm{mW}) \end{aligned}$ | Average Power (mW) | Run Time (s) |
| Diff. eqn | 4 | 22.9 | 35 | 5 | 16.0 | 37 | 4;4 | 25.00; 25.00 | 20.00;20.00 | 0;20 |
| Diff. eqn |  |  |  |  |  |  | 5;5 | $20.00 ; 25.00$ | $16.00 ; 16.00$ | 0; 20 |
| Diff. eqn |  |  |  |  |  |  | 7;8 | 14.29; 14.01 | 11.43; 11.39 | 0; 20 |
| FIR | 10 | 61.7 | 221 | 10 | 50.4 | 165 | 9;9 | 13.06; 13.06 | 10.44 ; 10.44 | 0;91 |
| FIR |  |  |  |  |  |  | 10;11 | 11.75 ; 11.58 | 9.40; 9.38 | 0;91 |
| FIR |  |  |  |  |  |  | 13;12 | 9.04; 9.26 | 7.23;7.36 | 0;91 |
| IIR | 8 | 12.8 | 100 | 9 | 11.2 | 96 | 8;8 | 15.31; 15.31 | $12.25 ; 12.25$ | 0;74 |
| IIR |  |  |  |  |  |  | 9;9 | 13.61; 13.61 | 10.89; 10.89 | 0; 74 |
| IIR |  |  |  |  |  |  | 11;12 | 11.14;11.06 | 8.91; 8.88 | 0;74 |
| Lattice | 9 | 66.3 | 207 | 10 | 55.9 | 161 | 8;9 | 80.63; 8.47 | 64.50;64.47 | 0;153 |
| Lattice |  |  |  |  |  |  | 10; 10 | 65.75; 65.75 | $51.60 ; 51.60$ | 0;153 |
| Lattice |  |  |  |  |  |  | 19;18 | 33.95;34.06 | 27.16; 27.41 | 0;153 |
| Ellip | 17 | 204.0 | 259 | 19 | 191.3 | 204 | 16;16 | 137.19; 137.19 | 109.75; 109.75 | 0;188 |
| Ellip |  |  |  |  |  |  | 20; 21 | 119.20; 119.04 | 87.80 ; 87.74 | 0;188 |
| Ellip |  |  |  |  |  |  | 24;24 | 91.46;91.46 | 73.17 ; 73.17 | 0;188 |
| WAVE | 26 | 201.2 | 427 | 27 | 179.2 | 332 | 24;25 | 304.38; 304.03 | 243.50; 243.46 | 194;326 |
| WAVE |  |  |  |  |  |  | 25;26 | 292.20; 191.85 | 233.76; 233.73 | 216;326 |
| WAVE |  |  |  |  |  |  | 29;28 | 251.90; 252.06 | 201.52; 201.57 | 265;326 |
| NC filter | 27 | 324.3 | 501 | 27 | 286.7 | 377 | 25;26 | 371.80; 371.65 | 297.44; 297.32 | 196;364 |
| NC filter |  |  |  |  |  |  | 29;31 | 320.52; 320.17 | 256.41; 256.13 | 211;364 |
| NC filter |  |  |  |  |  |  | 33;32 | 281.67;282.09 | 225.33;225.39 | 256;364 |

Table14. Experimental results for scheduling

| Entry <br> (\#traces= <br> $\mathbf{5 0}$ ) | \# power constraints | \# cycles | Average switching <br> power (mW) | Maximal switching <br> power (mW) | User-fixed <br> switching power <br> (mW) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 28 | 11.870 | 20.000 | 15.0 |
| 2 | 15400 | 56 | 8.239 | 11.000 | 10.0 |
| 3 | $15400+7000=22400$ | 68 | 5.402 | 8.500 | 7.0 |
| 4 | $22400+6800=29200$ | 90 | 4.533 | 6.500 | 5.0 |
| 5 | $29200+94500=123700$ | 634 | 0.4999 | 0.500 | 0.5 |

Table15. Trade off between the throughput and the switching power dissipation of a circuit

With the advent of new silicon technologies, SOC designs are possible. However, the reduction of the power dissipation is required at different levels of abstraction. Unfortunately, one cannot improve both the power consumption and the throughput of a VLSI system, which needs appropriate techniques to deal with. Our tool allows the designer to introduce constraints in order to achieve a good balancing of those two parameters. Table 14 shows our results and those of two earlier methods. It is shown that:

- in general, the game-theoretic scheduling and binding method [30] gives better results than the ILP-based scheduling \& LP-based binding one [29]
- our values of the power dissipation are closer to those of [30] than to those of [29] (notice however, that for fir instance, our value of the power dissipation is very far from that obtained with the 2 other methods and that the fir instance we dealt with come from [31])
- our method is faster than the two other ones (the 1 rst one is an ILP-based method, while the $2^{\text {nd }}$ is a game theory-based method whose time complexity is $\mathrm{O}\left(\mathrm{N}^{\mathrm{S}} * \mathrm{~N}^{*} \mathrm{~S}\right)$ for an N player game with S strategies for each player. In contrast, our method performs as shown in Fig.2.3.)
- our method is more flexible than the two first ones since it outputs different pairs of the power dissipation and the throughput for the same circuit, which allows the designer to select the best trade-off

Table 15 better shows the variation of the power dissipation by considering different numbers of constraints. The first entry of that table shows that operations of the VLSI system are scheduled in 28 cycles in case the circuit is subject to no powerconstraint and that the average (maximal) switching power dissipation given by SPOT ([25], [26], [27]) is $11.87 \mathrm{~mW}(20.0 \mathrm{~mW})$ for a user-fixed value of 15 mW . Starting with this first result, the user can iteratively introduce power-constraints in order to obtain the desired trade off between the circuit performance and the power dissipation. Notice also
that the number of the power constraints can be high (e.g. one need 123700 power constraints to achieve a switching power dissipation of 0.4999 mW for a circuit performing its operations in 634 cycles versus a dissipation of 11.870 mW for operations subject to no power constraint and scheduled in 28 cycles). This clearly shows that an automatic task is required to generate appropriate power constraints so that the desired switching power dissipation at the register transfer level is achieved.

## 7 Conclusion

In this paper, we have shown that there are two major scheduling approaches: the data flow based and the control flow based ones. The former deals with the data flow dominated circuits (such as DSP applications) while the second deals with the control flow dominated ones (e.g. controllers). We have shown that our method deals with both types of circuits and have also shown that our tool deals with huge numbers of data flows, which is useful for current designs. Also, our method is able to yield good scheduling results in interesting CPU times thanks to either exact techniques or near optimal ones when the time complexity is not polynomial. Notice that the non-polynomial part of the target problem has been tackled by a graph colouring technique but can also be solved by another appropriate technique such as genetic or evolutionary algorithms that feature a simple parallelisation for an efficient solution space exploration.
Finally, because the power dissipation is a critical problem for current circuits, in particular for SOC ones, our tool is able to automatically introduce power constraints so that the desired trade off between the throughput and the switching power dissipation of a circuit (at the register transfer level) is obtained.

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