Color Correction for Multi-view Images Combined with PCA and ICA

FENG SHAO, GANGYI JIANG* and MEI YU Faculty of Information Science and Engineering Ningbo University 315211 Ningbo CHINA shaofeng@nbu.edu.cn_jianggangyi@126.com_yumei@nbu.edu.cn

Abstract: - Color information is very important in setting the style of images, but always influenced by internal and external imaging factors. For this reason, it is necessary to eliminate color inconsistency between different views in multi-view imaging. Up to now, many color transfer methods have been proposed mainly for vivid color appearance, in which the mean, variance, or covariance information are used to transfer color. In this paper, by taking the advantage of the principal component analysis (PCA) and independent component analysis (ICA), a general color correction framework is proposed. We first analyze the correction property for target image and source image. Then we perform global PCA color correction if global correction property satisfied. Otherwise, we separate the independent sources. Finally, we combine the ICA correction results to get the final corrected image. The Experimental results show that well subjective and objective performances can be achieved with the proposed method.

Key-Words: - Multi-view image, color correction, principal component analysis, independent component analysis, dominant color extraction, CIEDE2000.

1 Introduction

Recently, multi-view video processing is attracting more and more attention for its unique advantages, which allows the user freely controlling the viewpoint position of any dynamic real-word scene in real time^[1]. Research activities on multi-view video processing include ATTEST/3DTV projects in Europe^[2], free viewpoint TV at Nagoya University in Japan^[3], 3D environment reconstruction in CMU^[4], 3DTV system in MERL^[5], realistic broadcasting system in Korea^[6], and so on.

A common problem in multi-view video is that different camera sensors acquire different color response to an imaged object^[7]. This problem occurs because physical factors during the imaging process introduce a variation that differs for each camera: in addition, it is practically impossible to capture an object under perfectly constant lighting conditions at different spatial position within an imaging environment. This variation degrades the performance of multi-view video processes such as multi-view video coding or virtual viewpoint rendering; in addition, the involved nature of correction routines means that the correction step is often ignored. Therefore, color inconsistency is an urgent problem in multi-view imaging.

Many color transfer methods have also been proposed. Reinhard et al. presented a pioneering work that transferred color statistic from one image to another for coping color characteristic using the mean and standard deviation^[8]. Using these values, the color of each pixel is transferred. If images are dissimilar from the point of view of color composition, user needs to manually set and match color pixels between the two images. Wang et al. presented an effective algorithm for image sequence color transfer^[9]. The mean and variance used for image transfer in an image sequence were interpolated to produce in-between color transfers, and a color variation curve was used to control the interpolation across color statistics values for an inbetween image sequence. Another example of transferring the color can be found in [10]. Their method applied the color mapping by transforms any N-dimensional probability density function into another one, and reduced the grain artifact through a post-processing algorithm by preserving the gradient field of the original image. A common feature of color transfer algorithms is that they all borrow colors from a given reference image.

Principal component analysis has been an important and useful mathematical tool in color technology for over fifty years, which is widely used in image understanding, feature extraction, data reconstruction^[11]. PCA considers image elements as random variable with Gaussian distribution and minimized second-order statistics. However, for non-Gaussian distribution, largest variance will not correspond to PCA basis vectors. ICA minimizes both second-order and higher-order dependencies in the input data and attempts to find the basis along which the data are statistically independent^[12].

In this paper, a general color correction framework combined with PCA and ICA is proposed. The rest of paper is organized as follows: first, PCA model and ICA model are described in Section II. The general color correction framework combined with PCA and ICA is described in Section III. The objective performance evaluation is given in Section IV. The experimental results are shown in Section V. Finally, the conclusions are given and future work is suggested.

2 PCA and ICA Model Description

PCA finds a set of the most representative projection vectors such that the projected images retain the most information about original images. The definition of PCA is presented as follows:

Definition 1 Given an *s*-dimensional vector representation of image, PCA tends to find a *t*-dimensional subspace whose basis vectors correspond to the maximum variance direction in the original image space. This new subspace is normally lower dimensional (t << s). If the image elements are considered as random variable, the PCA basis vectors are defined as eigenvectors of the covariance matrix. The projection matrix is composed of *t* eigenvectors corresponding to *t* largest eigenvalues.

The most important characteristic of PCA are dimension reduction and cross correlation reduction. The PCA matrix is the one containing the eigenvectors of the respective covariance matrix as its columns sorted by the eigenvalues in a descending fashion $\mathbf{V}_{PCA}=\mathbf{D}^{-1/2}\mathbf{E}^T$, where $\mathbf{EDE}^T=E[\mathbf{xx}^T]$ is the eigensystem of the covariance matrix.

ICA resolve the fundamental problem concerning estimating the independent unknown source s_i only through the observed data x and acquiring the mixing matrix **A** in the condition that those independent source are nongaussianly distributed. The general definition of ICA is as follows:

Definition 2 Given a random vector **x**, ICA tends to find a linear transform **x**=As so that the components s_i are as independent as possible, in the sense of maximizing some function $F(s_1,...,s_m)$ that measures independence.

Denote \mathbf{x} an *n*-dimensional random variable, *s* an *m*-dimension transform, then the linear model of ICA could be written as

$$\mathbf{x} = \mathbf{A}\mathbf{s} = \sum_{i=1}^{m} a_i s_i \tag{1}$$

There are two quite standard preprocessing steps in ICA. First, the mean of the data is usually subtracted to center the data on the original. The second step is to whiten the data. This means that we transform the data so that the components are uncorrelated and have unit variance.

$$\mathbf{z} = \mathbf{V}\mathbf{x}$$
 so that $E\{\mathbf{z}\mathbf{z}^T\} = \mathbf{I}$ (2)

where V is the whitening matrix and z the whitened data. If V is any solution then WV is also a whitening matrix for any orthogonal W, as

$$E\{\mathbf{W}\mathbf{z}\mathbf{z}^{T}\mathbf{W}^{T}\} = \mathbf{W}E\{\mathbf{z}\mathbf{z}^{T}\}\mathbf{W}^{T} = \mathbf{W}\mathbf{W}^{T} = \mathbf{I}$$
(3)

The purpose of ICA is find a transform \mathbf{W} which minimizes the statistical dependencies between the estimated sources

$$\hat{\mathbf{s}} = \mathbf{W}\mathbf{z} = \mathbf{W}\mathbf{V}_{PCA}\mathbf{x} \tag{4}$$

Fast-ICA is a fast computational method for ICA, which makes the projection $\mathbf{w}^T \mathbf{x}$ have the maximal nongaussianity through finding a unit vector w representing the projective direction^[13]. In essence, each iteration of the algorithm consists of updating each row \mathbf{w}_i^T of **W** by

$$\mathbf{w}_i \coloneqq E\{\mathbf{z}g(\mathbf{w}_i^T \mathbf{z})\} - E\{g'(\mathbf{w}_i^T \mathbf{z})\}\mathbf{w}_i$$
(5)

followed by orthonormalization of the matrix through $\mathbf{W}:=(\mathbf{W}\mathbf{W}^T)^{-1/2}\mathbf{W}^T$. The iteration is continued until convergence. Here $g(\cdot)$ is a certain nonlinear function and $g'(\cdot)$ is its derivative.

3 Color Correction Framework Combined with PCA and ICA

Fig.1 shows the general color correction framework for multi-view images in the paper. In this method, both reference image and uncorrected input image are captured through multiple cameras. First, global PCA correction matrix is obtained. Then, by analyzing matrix property, we determine whether global correction is satisfied or not, if it is satisfied, global PCA correction is performed; otherwise, dominant color extraction and ICA correction are performed. Finally, color correction is achieved for input image.

We proposed a content-adaptive color correction method^[14]. If two images are completely consistent, the global correction matrix is just identity matrix. If the elements in matrix are convergent in diagonal line, we regard global correction is satisfied. Therefore, the distance between global correction matrix $\mathbf{M} = \{m_{ij}, 0 < i, j \leq 3\}$ and identity matrix $\mathbf{I} = \{i_{ij}, 0 < i, j \leq 3\}$ is defined as



Fig.1: A general color correction framework for multi-view image

$$D_{m,i} = \left\| \mathbf{M} - \mathbf{I} \right\| = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} \left| m_{i,j} - i_{i,j} \right|$$
(6)

If $D_{m,i} \leq T_1$, correction is regarded to be not necessary. If $T_1 \leq D_{m,i} \leq T_2$, global correction is performed. Otherwise, if $D_{m,i} \geq T_2$, image segmentation and color correction between corresponding matching regions are performed. The thresholds are experientially selected, T_1 =0.01, T_2 =0.2 in this paper.

If global correction is satisfied, global PCA correction matrix is enough to describe color change between reference image and input image. While for the reverse case that global correction is not satisfied, the color change between reference image and input image is comparatively complicated, ICA correction is performed to minimizes both second-order and higher-order dependencies.

Assume that the reference image I_{ref} and the input image I_{inp} both contain homogeneous color distribution, \mathbf{x}_{ref} and \mathbf{x}_{inp} are color vectors for reference image and input image, $\boldsymbol{\mu}_{ref}$ and $\boldsymbol{\mu}_{inp}$ are mean vectors for reference image and input image. Global PCA correction matrix is obtained as

$$\mathbf{M}_{PCA} = (\mathbf{V}_{ref})^{-1} (\mathbf{V}_{inp}) \tag{7}$$

Here \mathbf{V}_{ref} and \mathbf{V}_{inp} are PCA matrix for reference image and input image, respectively. Then global PCA correction is performed for \mathbf{x}_{inp} to get the corrected color vector \mathbf{x}_{cor} as

$$\mathbf{x}_{cor} = \mathbf{M}_{PCA}(\mathbf{x}_{inp} - \boldsymbol{\mu}_{inp}) + \boldsymbol{\mu}_{ref}$$
(8)

If global correction is not satisfied, dominant color extraction is first performed. To extract dominant color, image segmentation which is regard as 'color naming' in this paper, is first performed by using first color naming and fuzzy color naming^[15].

$$P_{xy}^{i} = \frac{\sum_{(x',y')\in N} P_{x'y'}^{i} w((I(x,y),I(x',y')))}{\sum_{(x',y')\in N} w((I(x,y),I(x',y'))}$$
(9a)

$$w((I(x, y), I(x', y')) = 1 - \frac{1}{1 + e^{-0.5(Dist(I(x, y), I(x', y'))) - T_3)}}$$
(9b)

Here I(x, y) is the color value of pixel (x, y). *Dist*(I(x, y), I(x', y')) is Euclidean distance between I(x, y) and I(x', y'), N is the set of 8-neightbour pixels of pixel (x, y), and w is a weighting function that goes to 1 as the color difference between I(x, y) and I(x', y') becomes small, and goes to 0 if becomes large. T_3 is a threshold value.

Each pixel (x,y) is described by one of the 11 basic color categories (BCCs) and the corresponding probabilities P_{xy}^{i} belonging to the *i*-th BCC. The percentage that the *i*-th BCC occupying in the whole 11 BCCs can be described by

$$p_i = \sum_{x,y} P_{xy}^i / \left(\sum_i \sum_{x,y} P_{xy}^i \right)$$
(10)

Then $\{p_i\}$ is sorted in descending order. If the accumulative percentage $\left(\sum_{i=1}^{M} p_i\right) < T_4$, the colors with highest p_i are regarded as dominant colors. Here, M is the dominant color number, T_4 is a threshold controlling dominant color number and $T_4 = 0.9$ in this paper.

The *i*-th dominant color mean is

$$\mu_i = \sum_{x,y} P_{xy}^i \cdot I_i(x,y) \bigg/ \bigg(\sum_{x,y} P_{xy}^i \bigg)$$
(11)

For the color vector \mathbf{x}_{ref} and \mathbf{x}_{inp} in *i*-th dominant color regions, we can get the independent vectors \mathbf{s}_{ref} and \mathbf{s}_{inp} as

$$\mathbf{s}_{inp} = \mathbf{W}_{inp} \mathbf{V}_{inp} (\mathbf{x}_{inp} - \boldsymbol{\mu}_{inp})$$
(12a)

$$\mathbf{s}_{ref} = \mathbf{W}_{ref} \mathbf{V}_{ref} (\mathbf{x}_{ref} - \boldsymbol{\mu}_{ref})$$
(12b)

Then mapping relation can be established in independent component space. The ICA correction matrix can be described as

$$\mathbf{M}_{ICA} = (\mathbf{W}_{ref} \mathbf{V}_{ref})^{-1} (\mathbf{W}_{inp} \mathbf{V}_{inp})$$
(13)

Assume a set of homogeneous dominant colors $r_{ref}^{1}, \ldots, r_{ref}^{M}$ in the input image and a set of homogeneous dominant colors $r_{inp}^{1}, \ldots, r_{inp}^{M}$ in the reference image are given. Then dominant scheme are incorporated to blend the results

$$\mathbf{x}_{cor} == \sum_{i=1}^{M} p_i [\mathbf{M}_i (\mathbf{x}_{inp} - \boldsymbol{\mu}_{inp}^i) + \boldsymbol{\mu}_{ref}^i] / \sum_{i=1}^{M} p_i \qquad (14)$$

4 Objective Performance Evaluations

In order to objectively evaluate the performance of the color correction method, the color differences between reference image and color corrected image are calculated. The recently developed CIEDE2000 color difference is used^[16]. The CIEDE2000 color difference formula is based on the CIELAB color space. Given a pair of color values in CIELAB space L_{1}^{*} , a_{1}^{*} , b_{1}^{*} and L_{2}^{*} , a_{2}^{*} , b_{2}^{*} , we denote the CIEDE2000 color difference between them as

$$\Delta E_{00}(L_1^*, a_1^*, b_1^*; L_2^*, a_2^*, b_2^*) = \Delta E_{00}$$
(15)

The CIEDE2000 color difference ΔE_{00} formula is defined as

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T \left(\frac{\Delta C'}{k_C S_C}\right) \left(\frac{\Delta H'}{k_H S_H}\right)} \quad (16)$$

The detail definition of the equation is described in appendix.

5 Experimental Results and Analyses

The experiments are performed on two application, including color dimension reduction, and subjective, objective performance evaluation. In the proposed method, multi-view test sequences, 'golf1' and 'flamenco1', provided by KDDI Corp, and 'Uli', provided by HHI, are used as test sets. The size of 'golf1' and 'flamenco1' is 320×240, and the images are taken by a horizontal parallel camera configuration with eight viewpoints and 200mm camera interval^[17]. The size of 'Uli' is 1024×768, with eight viewpoints and 20cm camera distance^[18].

The principal components or independent sources are assumed to be linear combinations of the original signals. In a same way, the original signals are linear combinations of the principal components or the independent sources obtained by PCA or ICA scheme respectively. We can partly reconstruct the signals after discarding some principal components or independent source. Figs.2-3(a) and (b) shows two standard images of Tanaka, Peppers, and the reconstruction images of PCA and ICA scheme. As clearly shown in Figs.2-3, for PCA scheme, the restoration image only using the first component contains the most texture information, nevertheless the color information is lost, while the phenomenon is not evident for ICA.



Fig.2 (a): Tanaka and the PCA restoration images



Fig.2 (b): the ICA restoration images



Fig.3 (a): Peppers and the PCA restoration images



Fig.3 (b): the ICA restoration images

Figs.4-5(a) and (b) show the reference image and input image of 'golf1' and 'Uli', for which global correction property is satisfied. Figs.4-5(c) show the corrected image of global PCA correction method. The color appearance between reference image and color corrected image is very similar.

Fig.6(a) and (b) show the reference image and input image of 'flamenco1'. Fig.6(c) shows the corrected image obtained with global ICA correction method. Because global correction property is not satisfied, the color distribution in corrected image has significant distortion, while the corrected image obtained with the proposed ICA correction method in Fig.6(d) can eliminate the distortion completely.





Fig.4 (c): result of global Fig.5 (a): reference image PCA correction

of 'Uli'



Fig.5 (a): input image of 'Uli'





Fig.6 (a): reference image of 'flamenco1'





Fig.6 (b): input image

Fig.6 (c): result of global PCA correction

Fig.5 (a): result of dominant ICA correction

In order to objectively evaluate color correction performance, we calculate the color difference between reference image and input image, and compare with the color difference between the reference image and the corrected image. For the color corrected image, there is no original signal for evaluation methods, therefore, the average CIELAB values are first calculated, then the color difference between the average CIELAB values of reference image and corrected image are calculated. Fig.7(a)-(c) show the color difference comparison results before correction and after correction. From the figure, it is noted that color correction can achieve smaller color difference compared with before correction results.



Fig.7 (a): Color difference comparison of 'golf2'









Fig.7 c): Color difference comparison of 'flamencol'

5 Conclusions

Color correction for multi-view image is an important issue in multi-view video systems. In this paper, a general color correction framework combined with PCA and ICA is proposed. PCA have the advantage in dimension reduction and cross correlation reduction and ICA could incarnate the superiority in data statistical independent. The experimental results show that PCA scheme is suit to global color correction, and ICA scheme can achieve well effect for complicated color change.

In future work, we will do future researches on how to better combine ICA with PCA to generate better results. And effective virtual viewpoint rendering or multi-view video coding for following processing.

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Gangyi Jiang is correspondence author.

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Appendix

Given two CIELAB color values $\{L_i^*, a_i^*, b_i^*\}_{i=1}^2$ and parametric weighting factor K_L , K_C and K_H , the process of computation of the color difference is summarized the following equations, grouped as three main steps.

1. Calculate C'_i , h'_i .

$$C_{i,ab}^{*} = \sqrt{(a_{i}^{*})^{2} + (b_{i}^{*})^{2}}$$

$$\overline{C}_{ab}^{*} = \frac{C_{1,ab}^{*} + C_{2,ab}^{*}}{2}$$

$$G = 0.5 \left(1 - \sqrt{\frac{\overline{C}_{ab}^{* 7}}{\overline{C}_{ab}^{* 7} + 25^{7}}} \right)$$

$$a_{i}^{'} = (1+G)a_{i}^{*}$$

$$C_{i}^{'} = \sqrt{(a_{i}^{'})^{2} + (b_{i}^{'})^{2}}$$

$$h_{i}^{'} = \begin{cases} 0 \qquad b_{i}^{*} = a_{i}^{'} = 0 \\ \tan^{-1}(b_{i}^{*}, a_{i}^{'}) \qquad \text{otherwise} \end{cases}$$

2. Calculate $\Delta L'$, $\Delta C'$, $\Delta H'$:

$$\Delta L' = L_2^* - L_1^*$$
$$\Delta C' = C_2' - C_1'$$

$$\Delta h' = \begin{cases} 0 & C_1'C_2' = 0 \\ h_2' - h_1' & C_1'C_2' \neq 0; |h_2' - h_2'| \le 180^\circ \\ (h_2' - h_1') - 360 & C_1'C_2' \neq 0; |h_2' - h_2'| > 180^\circ \\ (h_2' - h_1') + 360 & C_1'C_2' \neq 0; |h_2' - h_2'| < -180^\circ \end{cases}$$
$$\Delta H' = 2\sqrt{C_1'C_2'} \sin\left(\frac{\Delta h'}{2}\right)$$

3. Calculate CIEDE2000 Color Difference ΔE_{00} :

$$\begin{split} \overline{L}' &= (L_1^* + L_2^*) / 2 \\ \overline{C}' &= (C_1^* + C_2^*) / 2 \\ \\ \overline{h}' &= \begin{cases} \frac{h_1' + h_2'}{2} & |h_1' - h_2'| \leq 180^\circ; C_1'C_2' \neq 0 \\ \frac{h_1' + h_2' + 360^\circ}{2} & |h_1' - h_2'| > 180^\circ; |h_1' + h_2'| < 360^\circ; C_1'C_2' \neq 0 \\ \frac{h_1' + h_2' - 360^\circ}{2} & |h_1' - h_2'| > 180^\circ; |h_1' + h_2'| \geq 360^\circ; C_1'C_2' \neq 0 \\ \frac{h_1' + h_2' - C_1'C_2' = 0 \\ T &= 1 - 0.17 \cos(\overline{h}' - 30^\circ) + 0.24 \cos(2\overline{h}') \\ &+ 0.32 \cos(3\overline{h}' + 6^\circ) - 0.20 \cos(4\overline{h}' - 63^\circ) \\ \Delta \theta &= 30 \exp\left\{ -\left[\frac{\overline{h}' - 275^\circ}{25}\right]^2 \right\} \\ \\ R_c &= 2\sqrt{\frac{\overline{C''}}{\overline{C''} + 25^7}} \\ S_L &= 1 + \frac{0.015(\overline{L}' - 50)^2}{\sqrt{20 + (\overline{L}' - 50)^2}} \\ S_C &= 1 + 0.045\overline{C'}T \\ S_H &= 1 + 0.015\overline{C'}T \end{split}$$

$$R_T = -\sin(2\Delta\theta)R_C$$

$$\Delta E_{00} = \sqrt{\left(\frac{\Delta L'}{k_L S_L}\right)^2 + \left(\frac{\Delta C'}{k_C S_C}\right)^2 \left(\frac{\Delta H'}{k_H S_H}\right)^2 + R_T \left(\frac{\Delta C'}{k_C S_C}\right) \left(\frac{\Delta H'}{k_H S_H}\right)}$$