On the sound production of the timpani

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Abstract: - The acoustic features of kettledrums through modal analysis and acoustic radiation measure were investigated. Modal analysis exciting the system by a hammer and a shaker on two different kettledrums was studied. Through this analysis 15 vibration modes were found. Their mappings resulted very clear and defined. Acoustic radiation using two different parameters was measured. The first parameter, p/v, has been already used to measure acoustic radiation of the soundboard of a piano (Giordano), and of a harpsichord (Tronchin et al). The second, $p \cdot v$, is a new parameter called *intensity of acoustic radiation*. This parameter resulted more linked to the frequency response function than p/v.

Keywords: - Timpani, Kettledrum, Modal analysis, Intensity, Acoustic radiation.

1. INTRODUCTION
Kettledrum is the only orchestra drum that can produce defined pitch notes. For this reason many physicists investigated its acoustic features coming to interesting results, that however can be improved. In this work, studies on modal analysis and on the measure of acoustic radiation of kettledrum using different techniques were performed. Two different kettledrums were analysed, by means of two different ways of excitation of the system (hammer and shaker). The acoustic radiation ($p \cdot v$) was measured and compared to (p/v) used in studies on the soundboard of the piano and of the harpsichord. In such a way, the generation of sound of musical instruments with vibrating surfaces (as the membrane or the soundboard) was better measured.

2. TIMPANI PHYSICS
Timpani are the most important instruments among orchestra drums, because they produce definite notes. Real or ideal membrane is similar to a string. The string frequency and the membrane frequency are both directly proportional to the square tension and inversely proportional respectively to the string's length and the membrane's diameter. The basic difference between a string and a membrane is that the string's partials are harmonic, while the membrane's partials are not harmonic. Membrane's surface static zones, called nodes are not punctiform, as string's nodes. Membrane's nodes are circular lines concentric to the circumference and straight lines that correspond to the diameters of the membrane itself. Each partial composing the sound of the instrument corresponds to a specific membrane vibration mode. The number and the type of nodes they are made of conventionally define vibration modes. For example is called (0,1) the first vibration mode of an ideal membrane, characterized by a circular node correspondent to the circumference itself with no diametrical node. Some physicists investigated how could kettledrum produce a defined pitch note. Lord Rayleigh, A. Benade and T. D. Rossing, in particular, found some hints to answer this question. In 1877 Lord Rayleigh in his treatise *The Theory of Sound*, explains the important limiting effect of the kettle on the movement of air in touch with the inferior surface of the membrane [1]. He was one of the first who studied how the influence of air can modify membrane vibration. Rayleigh noticed that the main sound of kettledrum corresponds to the second partial: (1,1) mode. Thanks to the experiences he made with a 25 inches (65 cm) kettledrum, Rayleigh found three following partials respectively far one to the other of a fifth (1,5 frequency ratio), a major seventh (1,89 f.r.), and an imperfect octave (about 2 f.r.) and he thought them linked to the (2,1), (3,1) and (1,2) modes. Arthur H. Benade in 1973 found the first ten components of the sound using a 25 inches kettledrum tuned on the Do note (130.8 Hz) and he found them in harmonic ratio with a missing fundamental an octave below the audible sound [2]. Rossing (in several studies from 1982 to 1998) and the staff of the Northern Illinois University investigated vibration modes of the kettledrums using modern instrumentations. They discovered that all the vibration modes

with only diametral nodes stay in harmonic ratio one to the other and that (1,1), (2,1), (3,1), (4,1), (5,1) vibration modes stay respectively in a 1, 1,5, 2, 2,44 (about 2,5) and 2,90 (about 3) ratio with the fundamental mode (1,1) [3]. According to Benade's results these modes are in harmonic ratio to the missing fundamental an octave below the pitch of audible sound. Rossing found the agents, which make the sound of kettledrum harmonic. The main agent is the effect of air loading: the ideal membrane waves in an ideal vacuum, while the real membrane vibrates in a sea of air. The air mass moved by membrane swinging get the frequencies of the vibration modes lower. This effect is stronger to the lowest frequencies and influences (1,1) mode in particular. The kettle affects the circular nodes getting their vibration frequencies higher and makes the frequencies of the other modes lower. The effect is mostly related to (0,1) mode. This rise of frequency is due to the alternative compression and decompression of the air in the kettle, made possible by the membrane movement in circular modes. The difference between the pressure inside and outside the kettle behaves as a force that tries to bring the system to its primary conditions. Rossing called this effect air loading. The other two agents known as interaction of vibration modes of air in the kettle and the bending stiffness of the membrane, play a secondary role compared to the effect of the weight of the air mass, and contribute to the "fine tuning" of the membrane frequencies. The first agent originates from the air in the kettle: this quantity of air has its own resonance, which potentially can interact with the modes of the membrane. Rossing found a great difference between membrane modes frequencies and the respective weight of air modes frequencies. As a consequence kettle air vibrations have a very low interaction with membrane vibrations, although they influence the gradations of kettledrum sound. The other secondary effect is due to the "strength" of the membrane that raises the frequencies of the superior partial. This effect can be neglected. Richard S. Christian e Arnold Tubis found a mathematical technique to calculate the effects of air loading on timpani membrane vibrations [4].

3. MODAL ANALYSIS

This work is based on the experience made on two timpani with different features. The first is a Plexiglas Adam 25 inches kettledrum with a Remo Mylar skin with a central reinforce, tuned to 166 Hz. The second one is a 25 inches Ludwig kettledrum with a Mylar skin with no central reinforces tuned to 140 Hz.

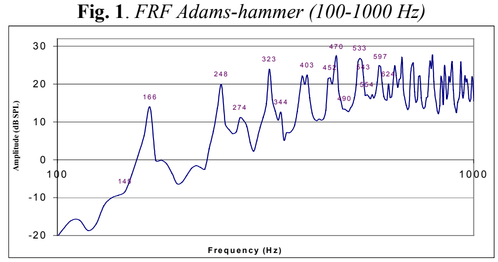
3.1 Experimental setup

The membrane has been excited by percussive impulse in 213 different measuring points fixed on a squared mash sided 4 cm. All the measurements were executed on the same points. Waveforms have been recorded in the PC and post-processed. The investigative methodology and the instrumentation used are different for the two kettledrums.

3.1.1 Adams kettledrum

The 213 fixed point were excited by hammer, and the vibration of the structure was measured by means of an accelerometer in the "normal" point, where "normal" is the point usually hit by the performer (10 cm far from the edge of the skin). The average on ten consecutive inputs for each point was considered, to eliminate any measuring mistake. Modal analysis has been conducted with an inverse method: normally only one point is excited, and the responses on all the other fixed points are recorded, but with reciprocity the position could be inverted. The acoustic features of the system useful to the modal analysis are described by the frequency response function obtained from the ratio between the respond measured by the accelerometer and the excitation induced by the hammer or the shaker in the frequency domain. 213 frequency response functions (FRF) were found. The FRF of the system is the average of all the FRFs. Each peak of resonance of the graphic corresponds to a proper vibration mode of the membrane. The spectrum of frequency response (amplitude and phase) of each FRF measured gives a snapshot of the mode shape.

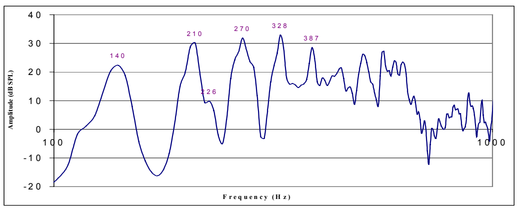
Fig. 1. *FRF Adams-hammer (100-1000 Hz)*



3.1.2 Ludwig kettledrum

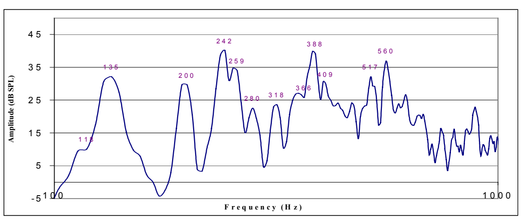
In this case the vibration of the structure was measured moving the accelerometer from one to the other of the 213 fixed points, exciting the system through an electrodynamic shaker placed in the “normal” point. The type of stimulation used was a swept sine wave of 10 seconds ranging from 20 Hz to 20 kHz with an exponential modulation. In this manner the Impulse Response between the excitation point and the accelerometer was measured. Finally, the FRF of each point was calculated applying the FFT to the IRs and then averaged.

Fig. 2. *FRF Ludwig-normal point.*



Afterwards, the same method applied to the Adams kettledrum to obtain the mappings of the vibration modes was used.

Fig. 3. *FRF Ludwig-shaker in the middle.*



3.2 Results of the modal analysis

3.2.1 Adams kettledrum

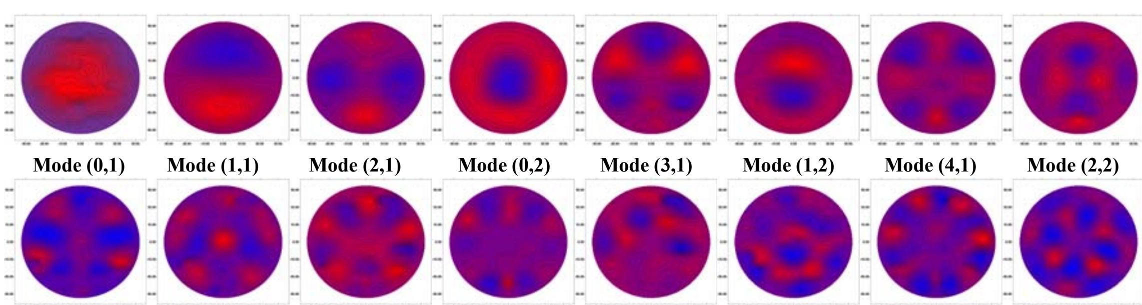
15 vibration modes in the range of frequencies going from 140 Hz to 540Hz. were studied.
The scheme below summarizes the frequencies found corresponding to the vibration mode.

Table 1. *Frequencies and measured ratios and comparison to Rossing’s results. In grey: frequencies that help the harmonicity.*

Modes	Frequencies	Ratios	Ratios found by Rossing		
m,n	$f_{m,n}(Hz)$	$f_{m,n}/f_{(1,1)}$	measured	calculated	
(0,1)	145	0.87	0.83	0.81	0.80
(1,1)	166	1	1.00	1.00	1.00
(2,1)	248	1.495	1.50	1.50	1.52
(0,2)	274	1.65	1.65	1.65	1.68
(3,1)	323	1.95	1.98	1.97	2.00
(1,2)	344	2.07	-	2.00	2.27
(4,1)	403	2.43	2.45	2.44	2.48
(2,2)	452	2.72	-	2.86	2.74
(5,1)	470	2.83	2.91	2.91	2.94
(0,3)	490	2.95	2.87	2.71	2.97
(6,1)	533	3.22	3.37	-	3.40
(3,2)	543	3.27	3.27	-	3.29
(1,3)	554	3.34	3.38	-	-
(7,1)	597	3.59	-	-	-
(4,2)	624	3.76	-	-	-

The results are very similar to those expected. Circular and mixed modes almost perfectly correspond to those found by Rossing. To diametrical modes correspond frequencies lower than expected. The discrepancy between the results and theoretical results grows as much as the frequency of the mode is high. The (5,1) mode, not very harmonic in theory, is the less similar to Rossing’s measurements. A very small difference between the modes (2,1), (3,1), (4,1) was found. For the same reason the mode (5,1) and the mode (6,1) have frequencies lower than those of the modes (0,3) and (3,2), when normally is the contrary. Circular and mixed modes perfectly correspond to theory. Two similar mappings, corresponding to the mode (5,1), and freq. 506Hz and ratio 3,04, doesn’t correspond to any resonance peak even if it has the same features of the mode (5,1), but stays in harmonic ratio with the mode (1,1). It could be interesting investigating on the origins of this vibration mode.

Fig. 4. ***Mappings of Adams kettledrum’s vibrational modes:***



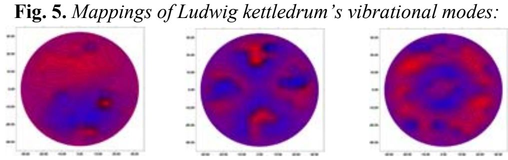
Mode (0,1) **Mode (1,1)** **Mode (2,1)** **Mode (0,2)** **Mode (3,1)** **Mode (1,2)** **Mode (4,1)** **Mode (2,2)**

Mode (5,1) **Mode (0,3)** Similar to mode (5,1) **Mode (6,1)** **Mode (3,2)** **Mode (1,3)** **Mode (7,1)** **Mode (4,2)**

3.2.2 Ludwig kettledrum

Only 3 vibration mode mappings, with a low definition, were measured. The experience could have been influenced by the mass loading and by the use of the shaker and of the accelerometer on the all surface of the membrane, not in just one point as in the modal analysis.

Fig. 5. *Mappings of Ludwig kettledrum's vibrational modes:*



Mode (1,1), 140 Hz **Mode (2,1)**, 210 Hz **Mode (0,2)**, 226 Hz

The same agent let the modal frequencies shifting some Hz higher, especially in the low frequency range. The frequency response function graphics are valid to 2.8 kHz. At 3 kHz, a peak probably correspondent to the resonance frequency of a very thin bar connecting the membrane to the shaker was found.

Comparing the FRF obtained exciting the system in the "normal" point to the FRF obtained exciting the middle of the membrane the second resulted having the more stimulated resonance peaks in correspondence to the circular and mixed vibration modes.

4. ACOUSTIC RADIATION

The second part of the investigations was dedicated to the measurement of acoustic radiation. The *efficiency* of acoustic radiation is the effectiveness of a vibrating surface to generate sound power. It is defined by the relationship:

$$\sigma = \frac{W}{\rho_0 c S \left\langle \overline{v_n^2} \right\rangle}$$

In which W is the sound power radiated by a surface with area S, and $\left\langle \overline{v_n^2} \right\rangle$ is the average of the surface normal velocity in the space [5]. Sound power is determined by the measurement of sound pressure, whereas the surface velocity is measured through the accelerometer. From this definition different measurement methods for the study of sound emission are obtained.

4.1 Researches on acoustic radiation

The studies on this argument have been conducted on the soundboards of the piano and of the harpsichord. K. Wogram, H. Suzuki and N. Giordano studied the soundboard of the piano using different methods for the measurements. In 1980 Wogram used the parameter F / v, defining F as the excitation force and v as the resulting velocity at the point of excitation [6]. He reported that it exhibits a maximum at a frequency near or below 1 kHz, and that it falls sharply at frequencies below 100 Hz, and above 1 kHz. He found that it falls by typically a factor of 10 as the frequency is varied from 1 to 5 kHz. In 1985, Suzuki used the method of the intensity of surface with the formula $I = \mathrm{Re}[p(\alpha / j\omega *)2]$ [7], where I is the average intensity in time, perpendicular to the vibrating surface, ω is the angular frequency, Re and $*$ are the real part and the complex conjugate of a complex number, p and α are the pressure and the normal acceleration in the measuring point and $j = (-1)^{1/2}$. His results imply that integrated sound intensity normalized by the input power is approximately constant from 200 Hz up to 5 kHz. In 1998, Giordano used the parameter p / v, where p is the sound pressure and v is the velocity of the soundboard [8].

In all the measured points p / v is largest at the 1kHz frequencies about, and it falls off below a few hundred Hz and above 5 kHz.

In 2001, L. Tronchin *et al.* applied the same parameter as Giordano's (p / v) to the studies on the soundboard of two Italian harpsichords of 17th century from Luigi Ferdinando Tagliavini's private collection [9]. In Giusti's harpsichord the maxima of acoustic radiation can be observed already before 1 kHz, and they continue up to about 2 kHz. The higher zone of emission is located at 1200-1300 Hz. In Mattia di Gand's instrument the maxima are at higher frequencies, between 1500 and 3000 Hz. The lowest values of acoustical radiation can be found at the lowest frequencies for both of the instruments, about 80 Hz. Suzuki's, Giordano's and Tronchin's results are homogeneous, but they contrast with Wogram's results on the average value of F / v which falls by a factor of 10 or more, going from 1 kHz to 5 kHz. Is important to notice that all of these studies have one result in common: the resonance frequencies didn't coincide with those of acoustical emission; on the contrary they were often in antithesis.

4.2 Acoustic radiation measurements

4.2.1 Experimental setup

The measurements were conducted the Adams kettledrum used in the modal analysis, but tuned on 145 Hz. We fixed the same points used for the modal analysis. The aim of the investigation is to calculate two different parameters of the acoustic radiation and compare one to the other: p / v, the parameter used by Giordano and Tronchin to $p \cdot v$, a new parameter that can be defined as *Intensity of Acoustic Radiation*, because is an average parameter between acoustic intensity and radiation. p represents sound pressure, and v the vibration velocity of the membrane. Pressure and velocity of both p/v and $p \cdot v$ have not been measured in the same point. During the measurements the shaker was positioned in the "normal" point and the accelerometer was moved from one point to the other. Sound pressure was measured locating a microphone about 25 cm from the membrane. During the measurements the IR was measured, and the FRF calculated by means of FFT of IR. Afterwards, p/v and $p \cdot v$ were calculated.

4.2.2 Results of the measurement of the acoustic radiation

The efficiency of acoustic radiation seems approximately constant from 270 Hz to 3,8 kHz, with a peak in this range at 1200 Hz. Rapid fluctuations of p/v in reason of the frequency variations were noticed. Even though kettledrum is completely different from piano and harpsichord the curve has the same features as those of Suzuki, Giordano and Tronchin. Observing $p \cdot v$ graphics different results were found. The maximum range of sound radiation intensity was between 140 Hz and 900 Hz, with a progressive small decrease of the amplitude value as much as the frequency grows.

Tall peaks and valleys characterize this zone. The strongest intensity in the frequency range of the mode (1,1), at 156 Hz, was found, but the amplitude of the modes (2,1) and (0,2), respectively at 226 Hz and at 247 Hz, was substantial, too. A sudden decrease of amplitudes after 900 Hz was noticed. From 900 Hz to 3000 Hz amplitude remained constant. Substantial peaks and valleys were recorded from 900 Hz to 1500 Hz, too. From 1500 Hz to 3000 Hz they became less and less evident. All the graphics obtained exciting the system through the shaker had a peak around 3000-3200 Hz, this couldn't be related to the sound properties of timpani.

This phenomenon could correspond to the resonance frequency of a very thin bar of connection between the membrane and the shaker. From 2,8 kHz data didn't seem to be more reliable. This didn't influence our work. The most important results of this investigation derived from the comparison between the FRFs and the parameters p/v and $p \cdot v$. About p/v: frequencies with great radiation efficiency did not correspond to the resonance frequencies of the frequency response (to the vibration modes). They were in antithesis to them. The curve corresponding to p/v (in green) was in phase opposition to that of the FRF curve (in blue), according to previous studies.

About $p \cdot v$: from the comparison between the graphic of the FRF (in blue) and the graphic of the intensity of the acoustic radiation $p \cdot v$ (in red), very interesting results were drawn.

The graphics had a very similar curve and resonance frequencies corresponded perfectly to sound emission frequencies.

Fig. 6. *FRF* - p/v

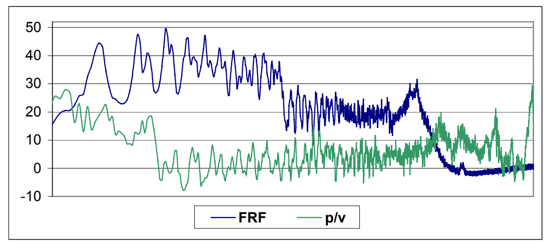
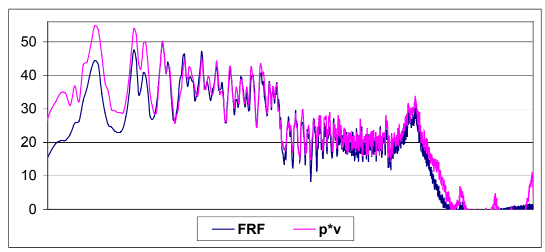


Fig. 7. *FRF* - $p \cdot v$



5. Conclusions

Using the hammer to excite the membrane modal analysis gave good results: the mappings of the 15 individuated vibration modes were very clear and defined. Measured ratios agreed with the theoretical ones. A greater responds for the circular and mixed vibration modes were found. To diametral modes correspond frequencies slightly lower than the aimed ones. Finally, kettledrum's acoustic radiation was calculated in two different ways: The first as a ratio between acoustic pressure and the vibration velocity of the membrane (p/v). This one is the method used by N. Giordano and Tronchin. The second as a product between sound pressure and vibration velocity of the membrane ($p \cdot v$), this is a new parameter that could de called *intensity of acoustic radiation*.

Comparing the FRF graphic and the p/v one, the resonance frequencies were often in opposition to those of acoustic emission, according to previous studies. Applying $p \cdot v$ parameter, the resonance frequencies perfectly corresponded to those of sound emission and the curves of the two graphics were very similar. The $(p \cdot v)$ parameter is mainly related to frequency response function and for this reason it was preferred compared to p/v. It is a medium parameter between acoustic intensity and acoustic radiation so that it is suitable to define the proper characteristics of the generation of sound of the musical instruments with vibrating soundboards. This parameter can be used to qualify and define the directivity of musical instruments, which is important for architectural acoustic, as well as for auralisation purposes.

6. ACKNOWLEDGMENTS

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