Designing a Reduced-Order Fuzzy Observer

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Abstract. In this paper, a method for designing reduced-order fuzzy observers for systems expressed in the Takagi-Sugeno fuzzy model is proposed. Also, for the stability condition, by use of pole location, the control signal is derived which will make the system's states stable. Furthermore, the Independence condition, which expresses the independence of the fuzzy observer design and the fuzzy controller is studied. Also, for better understanding, the simulation results for the inverted pendulum is shown and discussed.

keywords: Takagi-Sugeno fuzzy model, Reduced-order observers, Fuzzy controller.

1 Introduction

Fuzzy Logic introduces one of the best methods for exploiting qualitative information in designing a system's controller and in the last decade, it has achieved a special place in controlling industrial and laboratory processes. Since the designing technique of a fuzzy logic controller is based on the expert person's knowledge, there's not any formal method for designing the fuzzy controller and thus, the fuzzy controller's operation is highly dependent on the expert's experiences.

In order to construct a fuzzy model for the system, the expert's personal experiences are used as a non-linear function. Based on the system's fuzzy model, different controllers can be designed including controllers supported by simple ideas such as using state feedback for making the fuzzy system stable [1].

In this paper, the fuzzy model is Takagi-Sugeno in which the dynamics of the system is expressed by linear state space equations in different regions of the work space; also, the system is controlled by using the state feedback for each of the local dynamics and thus the significance of designing the observer will become apparent [2], [3].

Since some of the states do not require estimation and are directly measurable, it is possible to use reduced-order observers for estimating other states which causes less computation and thus higher performance. The main goal of this research is to design such observers. In Subsection 2.1, the system's fuzzy model and the control strategy is discussed. The designing method of reduced-order observer for the fuzzy model of the system is introduced in 2.2. In 2.3, the simulation results for a sample case is shown and finally in Section 3, we arrive at the conclusions and suggestions for future research directions.

2 The Design of Reduced-Order Fuzzy Observer

2.1 System's Fuzzy Model and the Control Strategy

The T-S fuzzy continuous-time dynamics model is expressed by **IF-THEN** rules; and in each rule the input-output relation is introduced in by linear state space equations. The ith rule of the fuzzy model is in this form :

Plant Rule i :
If $z_1(t)$ is F_{i1} and ... and $z_g(t)$ is F_{ig} Then

$$\dot{X}(t) = A_i.X(t) + B_i.u(t)$$

$$Y_i(t) = C_i.X(t)(1 \leq i \leq r) \qquad (1)$$

where F_{ig}s for $(1 \leq j \leq g)$ are fuzzy sets, $X(t) \in R^n$ is the state vector, $u(t) \in R^m$ indicates the input vector, $y_i(t)$ is the output vector;also we have $A_i \in R^{n*n}$, $B_i \in R^{n*m}$, $C_i \in R^{p*n}$; besides, r is the number of **IF-THEN** rules and $Z_k(t)$ for $1 \leq k \leq g$ are some measurable variables of the system. By using Singleton fuzzifier, Mamdani inference mechanism and the center-average defuzzifier, the final fuzzy system's state will be :

$$X(t) = \frac{\sum_{i=1}^{r} w_i(z(t))[A_i.X(t) + B_i.u(t)]}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} \mu_i(z(t))(A_i.X(t) + B_i.u(t))$$

where

$$w_i(z(t)) = \prod_{j=1}^{g} F_{ij}(z_j(t))$$

and also,

$$\mu_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$$

for which $F_{ij}(z_j(t))$ shows the truth value of $z_j(t)$'s membership in the F_{ij} fuzzy set. In this paper, it is assumed that :

$$w_i(z(t)) \geq 0, 1 \leq \forall i \leq r; \forall t : \sum_{i=1}^{r} w_i(z(t)) > 0$$

Therefore, we have :

$$\mu_i(z(t)) \geq 0, 1 \leq i \leq r; \sum_{i=1}^{r} \mu_i(z(t)) = 1$$

Thus the fuzzy system's state vector can be shown in equation 2.

$$\dot{X}(t) = \sum_{i=1}^{r} \mu_i.A_i.X(t) + \sum_{i=1}^{r} \mu_i.B_i.u(t) \tag{2}$$

and by a similar argument, the T-S fuzzy system's output will be defined as in equation 3.

$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t)).C.X(t)}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} \mu_i(z(t)).C.X(t) \tag{3}$$

Definition 1. If all of the pairs (A_i, B_i) are controllable, then the fuzzy system (1) is locally controllable.

If the fuzzy system is controllable, the state feedback control rule can be used as follows :

If $z_1(t)$ is F_{i1} and ... and $z_g(t)$ is F_{ig} Then

$$u(t) = -K_i.X(t) \tag{4}$$

And by considering the same fuzzy system's definitions, the final fuzzy controller's output will be :

$$u(t) = -\sum_{i=1}^{r} \mu_i.K_i.X(t) \tag{5}$$

By substituting equation 5 in 2, and also having $\sum_{j=1}^{r} \mu_j = 1$, we conclude that :

$$\dot{X} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j (A_i - B_i.K_j) X(t)$$

The stability condition for the above system depends on the existence of a positive matrix P for which [1]:

$$(A_i - B_i.K_i)^T p + p(A_i - B_i.K_i) < 0, 1 \le \forall i \le r$$

and

$$(\frac{A_i - B_i.K_j + A_j - B_j.K_i}{2})^T.p + p.(\frac{A_i - B_i.K_j + A_j - B_j.K_i}{2}) < 0, for i < j \le r \tag{6}$$

For finding a solution satisfying these conditions and finding p, we can use MATLAB's LMI toolbox.

2.2 Designing the Reduced-Order Fuzzy Observer

Several methods has been proposed for designing full-order observers [1], [4], [5], [6]. In this part, the method for designing reduced-order fuzzy observer is introduced and also the convergence condition for the designed observer is studied. If we assume that the measure matrix C is the same in all fuzzy system's rules[1], by using the identity transformation $\bar{X} = PX$, each rule in the Takagi-Sugeno fuzzy model can be written like this :

If $z_1(t)$ is F_{i1} and ... and $z_g(t)$ is F_{ig} Then
$\dot{\bar{X}} = P.A_i.P^{-1}.X + P.B_i.u$
$\bar{Y} = C.P^{-1}.\bar{X} = C.Q.\bar{X} = \begin{bmatrix} I_{q,q} & 0 \end{bmatrix} \bar{X}$

OR :

$$\begin{bmatrix} \dot{\bar{X}}_1 \\ \dot{\bar{X}}_2 \end{bmatrix} = \begin{bmatrix} \bar{A}_{11,i} & \bar{A}_{12,i} \\ \bar{A}_{21,i} & \bar{A}_{22,i} \end{bmatrix} \begin{bmatrix} \bar{X}_{1,i} \\ \bar{X}_{2,i} \end{bmatrix} + \begin{bmatrix} \bar{B}_{1,i} \\ \bar{B}_{2,i} \end{bmatrix} u$$

$$\bar{Y}_i = \begin{bmatrix} I_{q,q} & 0 \end{bmatrix} \bar{X} = \bar{X}_1 \tag{7}$$

In the above equations, q is the order of matrix C and $P = \begin{bmatrix} C & R \end{bmatrix}^T$ in which R is selected so that P will become nonsingular(regular). This transformation has the property that divides the system's variables into two groups; due to this transformation, we see that $\bar{X}_1$ indicates the measurable values for each rule and $\bar{X}_2$ shows the system's states which should be estimated. According to equation 2, equation 7 can be rewritten as :

$$\dot{\bar{X}}_1 = \sum_{i=1}^{r} \mu_i(\bar{A}_{11,i}.\bar{X}_1 + \bar{A}_{12,i}.\bar{X}_2 + \bar{B}_{1,i}.u)$$

$$\dot{\bar{X}}_2 = \sum_{i=1}^{r} \mu_i(\bar{A}_{21,i}.\bar{X}_1 + \bar{A}_{22,i}.\bar{X}_2 + \bar{B}_{2,i}.u) \tag{8}$$

We define the ith rule of the reduced-order fuzzy observer as :

Reduced-order observer rule i :
If $z_1(t)$ is F_{i1} and ... $z_g(t)$ is F_{ig} Then

$$\begin{bmatrix} \hat{\bar{X}}_1 \\ \hat{\bar{X}}_2 \end{bmatrix} = \begin{bmatrix} \bar{X}_1 \\ Z_i + L_i.Y_i \end{bmatrix}$$

with :

$$\dot{Z}_i = (\bar{A}_{22,i} - L_i.\bar{A}_{12,i})\hat{\bar{X}}_{2,i} + (\bar{B}_{2,i} - L_i.\bar{B}_{1,i}).u + (\bar{A}_{21,i} - L_i.\bar{A}_{11,i}).Y_i \tag{9}$$

[1] This assumption can be true without loss of generality.

where :

$$\begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = P^{-1} \begin{bmatrix} \hat{\bar{X}}_1 \\ \hat{\bar{X}}_2 \end{bmatrix} = \begin{bmatrix} Q_1 \ Q_2 \end{bmatrix} \begin{bmatrix} \hat{\bar{X}}_1 \\ \hat{\bar{X}}_2 \end{bmatrix}$$

and like equation 3, using the production inference and the center-average defuzzifier, we have :

$$\begin{bmatrix} \hat{\bar{X}}_1 \\ \hat{\bar{X}}_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{r} \mu_i.Q_1.\bar{X}_1 \\ \sum_{i=1}^{r} \mu_i.Q_2.\bar{X}_2 \end{bmatrix} \tag{10}$$

By the definition of error as $\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} - \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \end{bmatrix}$, E_1 will be zero and due to equations 1, 9 and the error definition ,the dynamics equation E_2 will be in the following form :

$$\dot{E}_2 = \sum_{i=1}^{r} \mu_i(\bar{A}_{22,i} - L_i.\bar{A}_{12,i}).E_2 \tag{11}$$

Proof :

$$E_2 = X_2 - \hat{X}_2$$

According to the identity transformation ($\begin{bmatrix} \bar{X}_1 \ \bar{X}_2 \end{bmatrix}^T = \begin{bmatrix} P_1 \ P_2 \end{bmatrix} \begin{bmatrix} X_1 \ X_2 \end{bmatrix}^T$), we multiply both sides of the above equation by P_2 and then compute the derivatives.

$$\dot{\bar{E}}_2 = \dot{\bar{X}}_2 - \dot{\hat{\bar{X}}}_2$$

By substituting equations 8 and 9 in the last equation, and simplification, equation 11 is derived.
Regarding equation 11, it can be seen that if all of the eigenvalues of the matrix $(\bar{A}_{22,i} - L_i.\bar{A}_{12,i})$ has negative real part, the error will approach to zero. Since estimated values are used in the feedback state, we have :

Control rule i : If $z_1(t)$ is F_{i1} and $\ldots$ and $z_g(t)$ is F_{ig} Then $u(t) = -K_i.\hat{X}$

and thus the final output of the fuzzy controller will be (Based on equation 5) :

$$u(t) = -\sum_{i=1}^{r} \mu_i.K_i.\hat{X}(t) \tag{12}$$

By substituting equation 11 in the dynamics equation of the Takagi-Sugeno system (equation 2), we have :

$$\dot{X}(t) = \sum_{i=1}^{r} \mu_i.A_i.X(t) - \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j.B_i.K_j.\hat{X}(t) \tag{13}$$

By having the definition $E = X - \hat{X}$ and equation 12, the closed-loop system's equation, will be as equations 14 and 15.

$$\dot{X}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(A_i - B_i.K_i)X(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j.B_i.K_j.E(t) \tag{14}$$

$$\dot{E}_2 = \sum_{i=1}^{r}\mu_i(\bar{A}_{22,i} - L_i.\bar{A}_{12,i})E_2 \tag{15}$$

By assuming two available positive functions, one of them related to the system states and the other one to the error value; by derivation and substituting the paths of equation 14, it can be shown that if all of the eigenvalues of matrix $(\bar{A}_{22,i} - L_i.\bar{A}_{12,i})$ have negative real part, and also the controller stability condition (equation 6) holds, then the derivative of the negative functions are computable; and we can also conclude the independence of the controller's design and the reduced-order fuzzy observer, designed by the proposed method [1].

2.3 Experimental Results

Consider the inverted pendulum system [7] (see figure 1). The fuzzy system is expressed by the following two rules :

If x_1 is about 0 Then

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ -0.1765 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} X$$

If x_1 is about $\pm\frac{\pi}{2}$ Then

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 11.0073 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ -0.1763 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} X$$

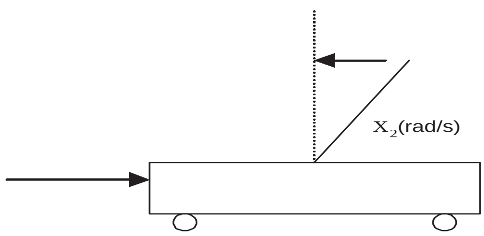

Fig. 1. The inverted pendulum system

The membership function used in the above rules is in the following form :

$$\mu_1(x_1) = \frac{1 - \frac{1}{1+exp(-7(x_1 - \frac{\pi}{4}))}}{1 + exp(-7(x_1 + \frac{\pi}{4}))}$$

$$\mu_2(x_1) = 1 - \mu_1(x_1)$$

Since the above state space equations are well formed for designing the reduced-order observer, there's no need for changing their forms ($P = I_{2*2}$). we locate the reduced-order observer's pole at (-20) and then by equation 8, we design the reduced-order observer :

If x_1 is about 0 Then

$\hat{x}_1 =$ Y
$\hat{x}_2 = z + 20Y$
$\dot{z} = -20z - 382.7059Y - 0.1765u$

If x_1 is about $\pm\frac{\pi}{2}$ Then

$\hat{x}_1 = Y$
$\hat{x}_2 = z + 20Y$
$\dot{z} = -20z - 388.9927Y - 0.163u$

We design the controller such that the closed-loop system's poles of the first rule will be located at $\begin{bmatrix} -4.09 & -4.21 \end{bmatrix}$ and for the second rule at $\begin{bmatrix} -3.25 & -3.37 \end{bmatrix}$. To achieve this, the k_1 and k_2 values should be set to :

$$k_1 = \begin{bmatrix} -195.9697 & -47.1288 \end{bmatrix}; k_2 = \begin{bmatrix} -124.8741 & -37.6446 \end{bmatrix}$$

By this setting, the closed-loop stability condition will be preserved. When the initial state of the system is $\begin{bmatrix} 20(deg.) & 0 \end{bmatrix}$, the states' values, the estimation error for state x_2 and the control signal is shown in figure 2. It is observed that, the unstable system with the initial state $\begin{bmatrix} 20(deg.) & 0 \end{bmatrix}$, will become stable by this controller (The systems' states will converge to zero), which corresponds to the controller's impact on the system; and also, it can be seen that that the estimation error for state x_2 will become zero very soon, in the specified time interval, which shows the high precision of the observer. In figure 3 the above diagrams is illustrated when the initial state is $\begin{bmatrix} -50(deg.) & 0 \end{bmatrix}$.

3 Conclusion

In this paper, we proposed a designing method for a reduced-order fuzz observer in a T-S fuzzy system; also, the stability condition for the observer was studied. Using the state feedback, a controller was designed for the system. By considering the stability condition of the whole system, we observe that if the

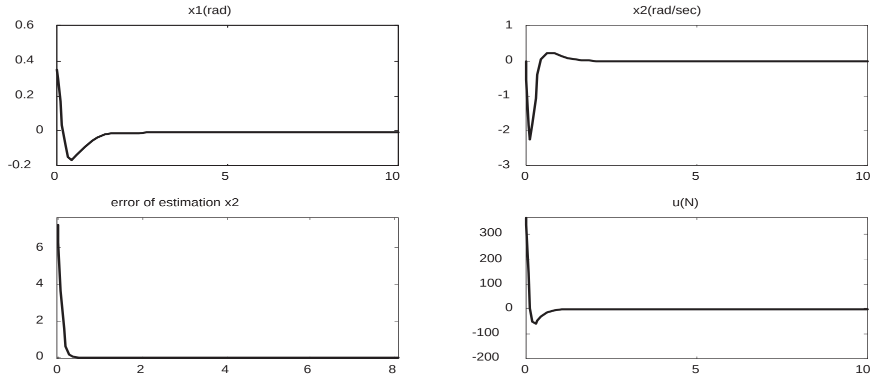

Fig. 2. The system's states, the estimation error of state x_2 and the control signal - The initial state of the system is $\left[\, 20(deg.)\ 0 \,\right]$.

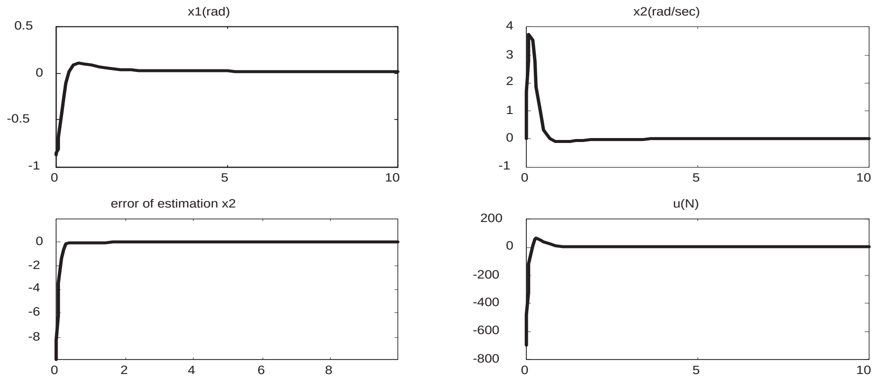

Fig. 3. The system's states, the estimation error of state x_2 and the control signal - The initial state of the system is $\left[\, -50(deg.)\ 0 \,\right]$.

Takagi-Sugeno dynamics model is a correct estimation for the non-linear system, the proposed method for designing the observer and the controller can be an effective approach in controlling complex non-linear processes; thus, there can be serveral further research directions in this context, giving more attention to the relevant issues such as acquiring Takagi-Sugeno fuzzy model for a non-linear process [8], designing tracker systems, designing functional observers and further study of problems involving optimized control.

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