Sugeno Fuzzy Controller of Helium Evaporation

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Abstract: One special type of a dynamical system is studied and controlled. The system is approximately linear but its transfer function depends on the polarity of change of action value. Therefore two classical controllers should be used to control the system – one of them for positive change of action value and the other for negative one. But we must ensure a switching between the classical controllers. A Takagi- Sugeno fuzzy controller is useful for this situation. This fuzzy controller can perform a continuous switching between two classical controllers. Simulation results of the control process are presented.

Key-Words: Nonlinear systems, Sugeno fuzzy controller, simulation

1 Introduction
Various types of low-loss helium cryostats are used for NMR superconducting magnets. Helium evaporation is the fundamental quantity for the correct function of the cryostat. Significant disturbances in helium outflow are caused by pressure variation in the atmosphere or in the helium recovery system.

A pressure controller with an electromagnetic valve is used for controlling the pressure in the helium vessel. The helium vapour leaving the cryostat passes the gauge of an electronic flow meter with loss pressure drop and then it is throttled down to atmospheric pressure by the electromagnetic valve. A high stability sensor measures the actual absolute value of the helium vapour pressure in the vent line, supposed to be equal to the pressure in the helium vessel. The controller compares this value with the desired one. The resulting output voltage is used for controlling the opening of the valve.

2 Model of the plant
It is clear that if the valve opens the pressure is decreasing and vice versa. Our experiments with the helium cryostat verified that its dynamical properties (i.e. the decreasing or increasing of the pressure) are different for opening or closing of the electromagnetic valve. Thus the helium cryostat was identified using step function. For closing of the valve it was found that the plant is the linear (approximately) dynamic system with the transfer function.
The transfer function for the opening of the valve was identified as

\[
F(s) = \frac{3.052 \left( 801s^2 + 165s + 1 \right)}{\left( 140700s^3 + 12360s^2 + 249.9s + 1 \right)}. \tag{1}
\]

The transfer function for the opening of the valve was identified as

\[
F(s) = \frac{1.205 \left( 0.24s^2 + 0.16s + 1 \right)}{\left( 0.9738s^3 + 4.872s^2 + 4.554s + 1 \right)}. \tag{2}
\]

From the classical point of view two controllers should be used - one of them for the opening and the other for the closing of the electromagnetic valve. Furthermore, we had to ensure the continuous switching between them. In this situation it is advantageous to use a Takagi-Sugeno fuzzy controller. The Takagi-Sugeno fuzzy controller can perform a continuous switching between two or more classical controller.

### 3 Takagi-Sugeno Fuzzy Controller

From the transfer functions of the cryostat is clear that the classical PI controller will be sufficient for the control of the plant. The transfer function of the classical PI controller will be

\[
u(k) = K \left[ e(k) + \frac{1}{T_i} \sum_{i=0}^{k} e(i) \right] \tag{3}
\]

where \( u(k) \) is the action value, \( e(k) \) is a control error, \( K \) is a controller amplification factor and \( T_i \) is an integration time constant. It is useful to differentiate this equation for the controller realization to come to formula

\[
\Delta u(k) = K \left[ \Delta e(k) + \frac{1}{T_i} e(k) \right] = K_p \Delta e(k) + K_i e(k) \tag{4}
\]

where \( \Delta u(k) \) is a change of control value, \( \Delta e(k) \) is a change of error and \( K_p, K_i \) are the parameters of the classical PI controller. Of course we must add the summation at the output of the modified controller with accordance to equation

\[
u(k) = u(k-1) + \Delta u(k). \tag{5}
\]

The parameters \( K_p, K_i \) of the classical PI controller can be determined via classical design methods for opening and closing of the valve i.e. for plant transfer functions (1) and (2).

The Takagi-Sugeno fuzzy controller will be performed by a set of rules

\[
\text{if } (e(k) \text{ is } A) \text{ and } (\Delta e(k) \text{ is } B) \text{ then } \Delta u(k) = K_p \cdot e(k) + K_i \cdot \Delta e(k) \tag{6}
\]

where \( A, B \) are fuzzy sets in the universe of the control error \( e(k) \) and change of control error \( \Delta e(k) \) respectively. The parameters \( K_p, K_i \) are different for the opening and closing of the valve. For opening the control law is

\[
\Delta u_0(k) = K_{p_0} \Delta e(k) + K_{i_0} e(k) \tag{7}
\]

For closing the control law is

\[
\Delta u_c(k) = K_{p_c} \Delta e(k) + K_{i_c} e(k) \tag{8}
\]

The parameters \( K_p, K_i \) of two classical PI controllers for our two transfer functions are in the following table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( K_p )</th>
<th>( K_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening of the valve</td>
<td>0.2</td>
<td>0.25</td>
</tr>
<tr>
<td>Closing of the valve</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The universe of the control error is covered by the 6 fuzzy sets with the obvious names (NB= negative big, NM= negative medium, NS= negative small, PS= positive small, PM= positive medium, PB= positive big). The membership function of the control error (after scaling i.e. after normalisation) is depicted at the Fig.3.
From this picture one can see that there is no "zero" fuzzy set. The reason is the need of the continuous switching between two classical PI controllers. The membership function of the change control error is the same as for control error (after scaling of course). The rule base of the Takagi-Sugeno fuzzy controller consists from 36 rules. These rules of the Takagi-Sugeno fuzzy controller are in the following table.

Table 2. Rule base of the Takagi-Sugeno fuzzy controller

<table>
<thead>
<tr>
<th>$e'/\Delta e$</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>NM</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>NS</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
<td>open</td>
</tr>
<tr>
<td>PS</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
</tr>
<tr>
<td>PM</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
</tr>
<tr>
<td>PB</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
<td>close</td>
</tr>
</tbody>
</table>

The crisp output value $\Delta u^*$ of the Takagi-Sugeno fuzzy controller is enumerated using defuzzification method as

$$\Delta u^* = \frac{\sum_{r=1}^{R} w_r \cdot (K_p \cdot \Delta e + K_i \cdot e)}{\sum_{r=1}^{R} w_r}$$  \hspace{1cm} (9)$$

where $R$ is the number of rules (36 rules in our case) and $w_r$ is the weight of the $r$-th rule i.e. the value of the antecedent of the $r$-th rule. The control surface of the Takagi-Sugeno fuzzy controller is depicted in the Fig.4.

![Fig.4. The control surface of the Takagi-Sugeno fuzzy controller](image)

4 Results

We have used the step function with different amplitude to verify the control process. The time response of the control process is shown in the Fig.5.

![Fig.5. Result of simulation](image)

Our simulation was performed for two different helium cryostats. One of them (cryostat with smaller helium vessel) had the transfer function (1) and (2). The other (with bigger helium vessel) had the transfer functions

$$F(s) = \frac{6.31 \left(219.84 s^2 + 18.61 s + 1\right)}{\left(9680.42 s^3 + 13852.86 s^2 + 509.97 s + 1\right)}$$

for closing of the valve and

$$F(s) = \frac{3.154 \left(0.5 s^2 + 3.56 s + 1\right)}{\left(15.15 s^3 + 20.52 s^2 + 8.527 s + 1\right)}$$

for opening of the valve. We had to use different parameters of the classical PI controllers. These parameters are in the Table.3.

Table 3. The parameters of PI controllers for bigger helium vessel

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$K_p$</th>
<th>$K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opening of the valve</td>
<td>0.1</td>
<td>0.25</td>
</tr>
<tr>
<td>Closing of the valve</td>
<td>0.1</td>
<td>0.015</td>
</tr>
</tbody>
</table>

The result of control process simulation was similar for both helium vessels (except time scaling in the Fig.5 of course).

5 Conclusion

We have experimented also with the Mamdani type of fuzzy controller, see [8]. From the comparisons it seems that the Takagi Sugeno
fuzzy controller has a small mean square error of control process. So it is advantageous to use the Takagi Sugeno fuzzy controller for this type of the plant.

This work has been supported by the Grant Agency of Czech Republic, grant No. 102/02/0782 "Research in control of smart robotic actuators” and by the scientific intentions No. CEZ: J22/260000013 “Processes Automation”.

References