Simple fuzzy adaptive control for a class of nonlinear plants

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Abstract: A fuzzy adaptive control algorithm is presented in the paper. Its hallmarks are simplicity and global stability, i.e. boundedness of all the signals in the system. The control can be successfully applied to nonlinear plants that are predominantly of the first order. Thus, this approach covers quite broad class of plants that are often encountered in process industries. The proposed algorithm was tested on a simulated hydraulic plant. The test plant consisted of three water tanks. The performance of the proposed algorithm is compared to the performance of the robust adaptive control.

Key-Words: model reference adaptive control, fuzzy control, Takagi-Sugeno model, three-tank plant

1 Introduction

When designing control for poorly known and non-linear systems, there are many approaches to accomplish the aim. One possibility is to use classical adaptive controllers. The drawback of this approach is that estimated parameters change all the time and poor quality of control is achieved if the controlled system is highly nonlinear. If some sort of robust controller is used, the consequences are similar. It is also possible to apply nonlinear control and the results are usually quite good if the knowledge of system dynamics is sufficient.

When this is not the case, probably the most straightforward solution is to use nonlinear adaptive control. Most of the “classical” nonlinear adaptive control algorithms [4] demand fairly good knowledge of mathematics from the designer and are thus avoided by practicing engineers.

In recent years, a lot of effort has been put to neuro-fuzzy identification of complex plants, which cannot be easily theoretically modelled. Based on neuro-fuzzy presentation of the plant model numerous neuro-fuzzy adaptive control approaches appeared in the literature [3, 5, 6, 8].

The proposed algorithm (direct fuzzy model reference adaptive control – DFMRAC) tends to preserve the advantages of adaptive and nonlinear control while still assuring the robustness of the system. It greatly resembles the classical MRAC of the first-order plant. In fact, it can be obtained by fuzzification of control gains and the inclusion of e_1-modification [2] into the adaptive law. The main advantage of the approach is the simplicity of the control algorithm. The latter is obtained by presuming that the plant is predominantly of the first order. Even though that this seems as an unrealistic assumption, quite broad class of nonlinear plants is included. In our opinion, such plants occur quite often in process industries.

The paper is organised as follows. Section 2 presents DFMRAC algorithm, section 3 describes the plant which was used for simulation study. The results of the comparison between classical robust MRAC and DFMRAC are presented in Section 4. The conclusions are stated in Section 5.

2 The DFMRAC algorithm

Our main motivation was to find a simple solution for adaptive control of nonlinear systems. As mentioned before, DFMRAC can be viewed as an extension of MRAC to nonlinear plants. To simplify the algorithm and equations first order plants are supposed in the paper. Even when this assumption is violated good results are still obtained if the dominant part in system dynamics is of the first order.

2.1 MRAC of LTI plants

If the plant is linear time invariant system of the first order, it can be described by the transfer function

$$G_p(s) = \frac{b}{s+a} \qquad (1)$$

where a and b are unknown constants. By choosing reference model

$$G_m(s) = \frac{b_m}{s+a_m} \qquad (2)$$

a control law

$$u = fw - qy_p \qquad (3)$$

follows to achieve design objective. Classical solution to find the correct values for control parameters f and q is to estimate them by the following adaptive law:

$$\begin{aligned} \dot{f} &= -\gamma_f \operatorname{sgn}(b)\varepsilon w \\ \dot{q} &= \gamma_q \operatorname{sgn}(b)\varepsilon y_p \end{aligned} \qquad (4)$$

where ε is tracking error, defined by

$$\varepsilon = y_p - y_m \tag{5}$$

and γ_f and γ_g are arbitrary positive constants.

Usually the system is not linear and linear approximation is only used to simplify the analysis. The consequence is that stability is obtained in small operation region where the process can be described sufficiently well by the linear model.

2.2 DFMRAC of nonlinear plants

The proposed fuzzy adaptive control system assumes the fuzzification of feedforward gain f and feedback gain q. The choice of fuzzification variables depends on the process behaviour and is similar problem to that of structural identification in case of Takagi-Sugeno (TS) model [7]. The fuzzified gains are described by means of fuzzy numbers **f** and **q**.

$$\begin{aligned} \mathbf{f}^T &= [f_1 \quad f_2 \quad \cdots \quad f_k] \\ \mathbf{q}^T &= [q_1 \quad q_2 \quad \cdots \quad q_k] \end{aligned} \tag{6}$$

where k stands for the number of fuzzy rules.

We assume that the process under investigation can be modelled by the TS fuzzy model of the form [7]:

$$\begin{aligned} &\mathbf{R}^i : \textbf{if } u \textbf{ is } \mathbf{A}_{i_a} \textbf{ and } y_p \textbf{ is } \mathbf{B}_{i_b} \textbf{ then } \dot{y}_p = -a_i y_p + b_i u \\ &i_a = 1,\ldots,n_a;\; i_b = 1,\ldots,n_b;\; i = 1,\cdots,k \end{aligned} \tag{7}$$

where u and y_p are input variables of the fuzzy system, $\dot{y}_p$ is the output variable, $\mathbf{A}_{i_a}$, $\mathbf{B}_{i_b}$ are fuzzy membership functions. The number of membership functions for the first and the second input variable defines the number of rules $k = n_a \times n_b$. The membership functions have to cover the whole operating area of the closed-loop system. In Eq. (7) it is assumed that u and y_p are the so-called antecedent variables that give information about the current operating point of the system. In general, other variables can be used instead. Since the choice does not influence the proposed approach, they will be gathered in vector $\boldsymbol{\varphi}$. The output of TS model is then given by the following equation

$$\dot{y}_p = \sum_{i=1}^{k} \left(\beta_i^0(\boldsymbol{\varphi})(-a_i y_p + b_i u)\right) \Big/ \sum_{i=1}^{k} \beta_i^0(\boldsymbol{\varphi}) \tag{8}$$

The degree of fulfilment $\beta_i^0(\boldsymbol{\varphi})$ is obtained using T-norm [7]. The degrees of fulfilment for the whole set of rules can be written as

$$\boldsymbol{\beta}^0 = \left[\beta_1^0 \quad \beta_2^0 \quad \ldots \quad \beta_k^0\right]^T \tag{9}$$

and given in normalized form as

$$\boldsymbol{\beta} = \boldsymbol{\beta}^0 \Big/ \sum_{i=1}^{k} \beta_i^0 \tag{10}$$

Due to the Eq. (8) and Eq. (10) the plant can be modelled in fuzzy form as

$$\dot{y}_p = -(\boldsymbol{\beta}^T \mathbf{a}) y_p + (\boldsymbol{\beta}^T \mathbf{b}) u \tag{11}$$

where **a** and **b** stand for fuzzified parameters of the plant which have constant elements

$$\begin{aligned} \mathbf{a}^T &= [a_1 \quad a_2 \quad \cdots \quad a_k] \\ \mathbf{b}^T &= [b_1 \quad b_2 \quad \cdots \quad b_k] \end{aligned} \tag{12}$$

After choosing a certain Lyapunov function and following similar procedure to that of deriving adaptive and control laws for MRAC of LTI systems a following control law is obtained

$$u = (\boldsymbol{\beta}^T \mathbf{f}) w - (\boldsymbol{\beta}^T \mathbf{q}) y_p \tag{13}$$

It resembles classical control law (3). The direct extension of the classical adaptive law (4) for fuzzified gains **f** and **q** would have the following form

$$\begin{aligned} \dot{\mathbf{f}} &= -\gamma_f b_{\text{sgn}} \varepsilon w \boldsymbol{\beta} \\ \dot{\mathbf{q}} &= \gamma_q b_{\text{sgn}} \varepsilon y_p \boldsymbol{\beta} \end{aligned} \tag{14}$$

where b_{sgn} is +1 if b_i's are positive and –1 if b_i's are negative. (Note: To achieve global stability all b_i's have to be of the same sign).

It is not possible to prove the global stability of the adaptive law (14) combined with the control law (13). For this reason an extra term is added to the adaptive law and a new adaptive law is obtained

$$\begin{aligned} \dot{\mathbf{f}} &= -\gamma_f b_{\text{sgn}} \varepsilon w \boldsymbol{\beta} - \gamma_f \nu_0 |\varepsilon| \operatorname{diag}(\boldsymbol{\beta}) \mathbf{f} \\ \dot{\mathbf{q}} &= \gamma_q b_{\text{sgn}} \varepsilon y_p \boldsymbol{\beta} - \gamma_q \nu_0 |\varepsilon| \operatorname{diag}(\boldsymbol{\beta}) \mathbf{q} \end{aligned} \tag{15}$$

The appearance of the extra term in Eq. (15) also follows by the Lyapunov functions based design of the adaptive control. This extra term is equivalent to leakage or e_1-modification in robust adaptive control of LTI systems [2]. The global stability of the over-all system can be proven if adaptive law (15) is used [1].

2.3 Adding parasitic dynamics and disturbances to the plant model

The nonlinear plant model of the plant (11) will be made even more complex by including parasitic dynamics and unmeasured disturbance

$$\dot{y}_p = -(\boldsymbol{\beta}^T \mathbf{a}) y_p + (\boldsymbol{\beta}^T \mathbf{b}) u - \Delta_y(p) y_p + \Delta_u(p) u + d \tag{11}$$

where p is a differential operator d/dt, while $\Delta_u(p)$ and $\Delta_y(p)$ are stable linear operators in the time domain and d is disturbance.

To robustly stabilise the plant (11) the proposed DFMRAC controller is used. It comprises of Eq. (14), modified Eq. (15)

$$\begin{aligned} \dot{\mathbf{f}} &= -\gamma_f b_{\text{sgn}} \varepsilon w \boldsymbol{\beta} - \gamma_f \nu_0 |\varepsilon m| \operatorname{diag}(\boldsymbol{\beta}) \mathbf{f} \\ \dot{\mathbf{q}} &= \gamma_q b_{\text{sgn}} \varepsilon y_p \boldsymbol{\beta} - \gamma_q \nu_0 |\varepsilon m| \operatorname{diag}(\boldsymbol{\beta}) \mathbf{q} \end{aligned} \tag{16}$$

and

$$\begin{aligned} \varepsilon &= y_p - y_m - G_m(p)(\varepsilon n_s^2) \\ m^2 &= 1 + n_s^2 = 1 + w^2 + y_p^2 + m_s \\ \dot{m}_s &= -\delta_0 m_s + u^2 + y_p^2,\; m_s(0) = 0 \end{aligned} \tag{17}$$

where δ_0 is a positive constant chosen by the designer.
The following theorem can be proven [1]:
The model reference adaptive control system, described by (14), (16), and (17), is globally stable, i.e. all the signals in the system are bounded and the tracking error e has the following properties:

- $e \in L_\infty$, *and*
- $e \in S(\Delta_2^2 + \bar{d}^2 + \nu_0^2)$

if the following conditions are satisfied:

- $\frac{c}{\alpha_0^2}\Delta_\infty^2 + \frac{c}{\alpha_0^2} + c\Delta_\infty^2 < 1$,
- $c\Delta_2^2 + c\nu_0^2 < \delta_0$,
- $\Delta_u(s)$, $\Delta_y(s)$ *and* $G_m(s)$ *are analytic in* $\mathrm{Re}[s] \geq -\frac{\delta_0}{2}$,
- *reference signal is continuous, and*
- β *is a function of continuous signals*

where

- $\Delta_\infty = \max\left(\|\Delta_u(s)\|_{\infty\delta_0}, \|\Delta_y(s)\|_{\infty\delta_0} + \bar{f}_y\right)$,
- $\Delta_2 = \max\left(\|\Delta_u(s)\|_{2\delta_0}, \|\Delta_y(s)\|_{2\delta_0}, \bar{f}_y, \bar{f}_w\right)$,
- $\bar{d} = \sup_t |d(t)|$
- α_0 *is an arbitrary constant such that* $\alpha_0 > a_m$,
- *c are constants that depend on different system parameters.*

It should be noted that $\bar{f}_w$ and $\bar{f}_y$ denote the upper bounds of the fuzzy modelling error (see [1] for details). For the definition of extended L_{pe} norms ($\|\cdot\|_{\infty\delta_0}$ and $\|\cdot\|_{2\delta_0}$) see [2].

3 Description of the simulation plant

The proposed algorithm was tested on a simulation plant. The simulation test plant consisted of three water tanks. The schematic representation of the plant is given in Figure 1. The control objective was to maintain the water level in the third tank by changing the inflow into the first tank.

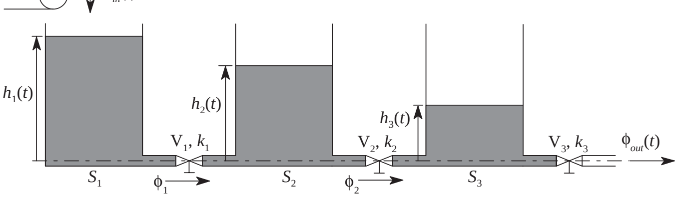

Fig. 1. Schematic representation of the plant

The mass conservation equations for the three tanks are:

$$
\begin{aligned}
S_1\dot{h}_1 &= \phi_{in} - k_1\,\mathrm{sgn}(h_1 - h_2)\sqrt{|h_1 - h_2|} \\
S_2\dot{h}_2 &= k_1\,\mathrm{sgn}(h_1 - h_2)\sqrt{|h_1 - h_2|} - k_2\,\mathrm{sgn}(h_2 - h_3)\sqrt{|h_2 - h_3|} \\
S_3\dot{h}_3 &= k_2\,\mathrm{sgn}(h_2 - h_3)\sqrt{|h_2 - h_3|} - k_3\,\mathrm{sgn}(h_3)\sqrt{|h_3|}
\end{aligned} \tag{18}
$$

where ϕ_{in} is the volume inflow into the first tank, h_1, h_2 and h_3 are the water levels in three tanks, S_1, S_2 and S_3 are areas of the tanks cross-sections, and k_1, k_2 and k_3 are coefficients of the valves. The parameters were chosen as follows:

$$
\begin{aligned}
S_1 = S_2 = S_3 &= 2\cdot 10^{-2}\ \mathrm{m}^2 \\
k_1 = k_2 = k_3 &= 2\cdot 10^{-4}\ \mathrm{m}^{5/2}\mathrm{s}^{-1}
\end{aligned} \tag{19}
$$

The nominal value of inflow ϕ_{in} was set to $8\cdot 10^{-5}\ \mathrm{m}^3\,\mathrm{s}^{-1}$, resulting in steady-state values 0.48 m, 0.32 m, and 0.16 m for h_1, h_2 and h_3, respectively. In the following u and y_p denote deviation of ϕ and h_3, respectively, from the operating point.

4 Description of the experiment and simulation results

The proposed fuzzy model reference adaptive control algorithm was compared to classical model reference adaptive control via two experiments. Adaptive gains γ_f and γ_g and e_1-modification constant ν_0 were the same in both cases. A reference signal was chosen as a periodical piece-wise constant function which covered quite wide area around the operating point (±50% of the nominal value). There were 11 triangular fuzzy membership functions (fuzzification variable was y) used that were distributed evenly across the interval [-0.1, 0.1]. No prior knowledge of the estimated parameters was available to us, so the initial parameter estimates were 0 for all examples.
The first simulation experiment assumes that the tanks are high enough so that they never fill up. Figs. 2 and 3 show the results of the classical MRAC while Figs. 4 and 5 depict the results of DFMRAC. By comparing responses in Figs. 2 and 4 one can observe that every change of reference signal results in the sudden increase of the tracking error ε (up to 0.01). This is due to the fact that zero tracking of the reference model with relative degree 1 is not possible if the plant has relative degree 3. Otherwise, much better results are achieved when using DFMRAC since the differences in system dynamics when changing operating point almost do not influence the responses of the system. Also, the oscillations in parameter estimates are smaller in the case of fuzzy adaptive law what can be noticed in Figs. 3 and 5. On the other hand, much longer period is needed that the estimates reach quasi-equilibrium if fuzzy adaptive law is used compared to the time needed if classical adaptive law is used. This is due to the larger number of estimated parameters. The classical adaptation could serve as

initialisation of fuzzy parameters, i.e. all elements of vectors **f** and **q** could be set to the values obtained by classical adaptation of f and q.

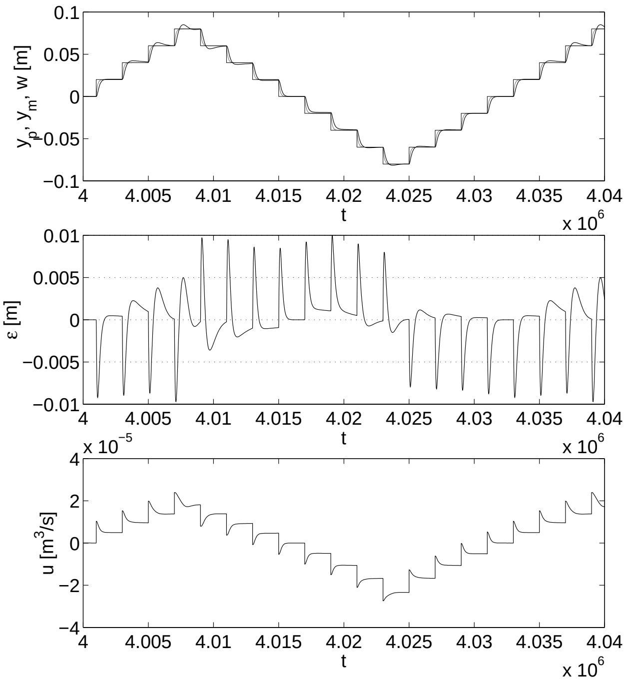

Fig. 2. The classical MRAC with e_1-modification – time plots of the reference signal and outputs of the plant and the reference model (upper figure), time plot of tracking error (middle figure), and time plot of the control signal (lower figure)

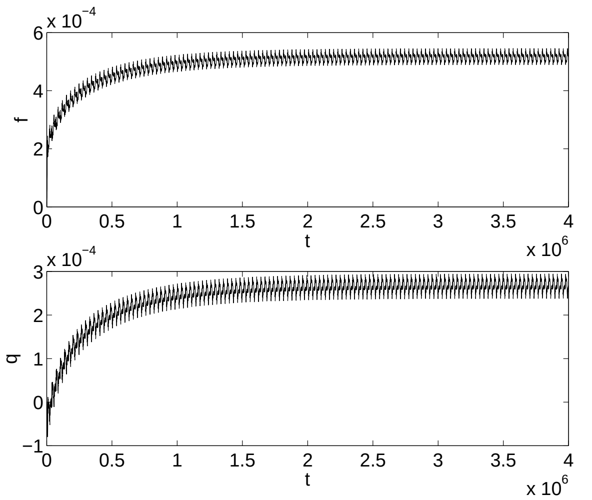

Fig. 3. The classical MRAC with e_1-modification – time plots of feedforward (upper figure) and feedback (lower figure) control gains

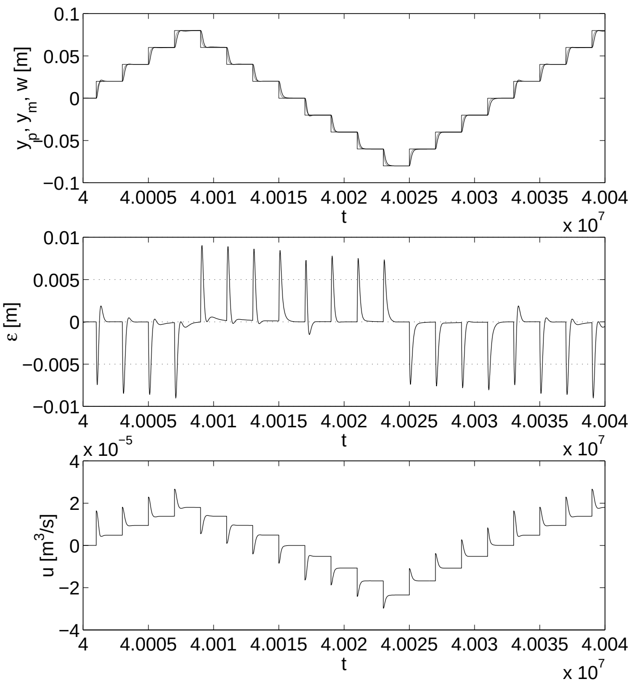

Fig. 4. The DFMRAC – time plots of the reference signal and outputs of the plant and the reference model (upper figure), time plot of tracking error (middle figure), and time plot of the control signal (lower figure)

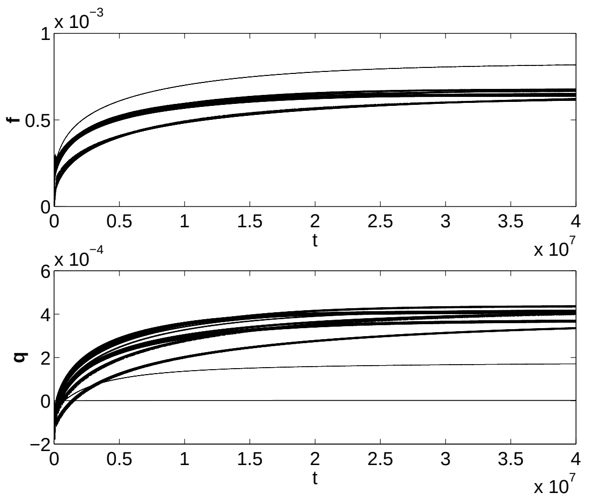

Fig. 5. The DFMRAC – time plots of feedforward (upper figure) and feedback (lower figure) control gains

The second experiment was conducted on the model where the tanks were 0.6 m high. The responses are shown in Figs. 6 and 7 for the classical MRAC and DFMRAC, respectively. When the water level in a tank reaches 0.6 m the security mechanism stops the water inflow and prevents spilling. This assumption introduced discontinuity into the system. A consequence was that meeting of control requirements was not possible. It is true that water level never exceeds 0.6 m in the third tank, but it does in the first tank when the desired level in the third tank reaches some point. There exists no control algorithm that could zero the tracking error when the reference signal is too high. The classical adaptive law responded to that disturbance by increasing the control parameters while fuzzy adaptive law increased only one parameter. When the system left that operating point, the behaviour of the DFMRAC system was good while classical MRAC had to retune the parameters to reach the normal values.

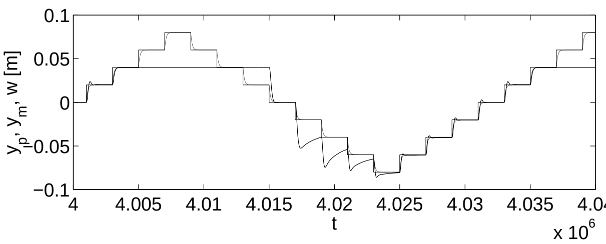

Fig. 6. Response of the classical MRAC with e_1-modification (the case with finite height of tanks)

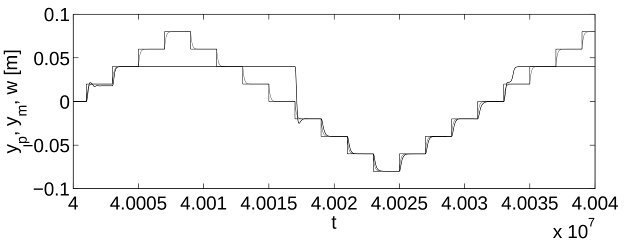

Fig. 7. Response of the DFMRAC (the case with finite height of tanks)

5 Conclusion

A fuzzy adaptive control algorithm was presented. It was compared to the classical MRAC on a simulation model of a three-tank plant. The advantage of the DFMRAC is that it is very simple to design it but it still offers the advantages of nonlinear and adaptive controllers. It was shown on the example that good results can be obtained if a plant of relative order 3 is treated as a first order plant. DFMRAC proves successful especially in the cases where disturbances are present in some region of the fuzzification space. In such cases only control parameters that belong to that region are affected by the disturbance. The drawback of the approach is long time of adaptation due to large number of estimated parameters. To speed it up, the classical adaptation can be used in the early phase.

References:

[1] S. Blažič, I. Škrjanc and D. Matko, Globally stable direct fuzzy model reference adaptive control, *Fuzzy Sets and Systems*, Vol. 139, 2003, pp. 3-33.

[2] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, Upper Saddle River, 1996.

[3] S. Jagannathan, F. L. Lewis and O. Pastravanu, Model reference adaptive control of nonlinear dynamical systems using multilayer neural networks, *Proc. of IEEE International Conference on Neural Networks*, New York, Vol. 7, 1994, 4766-4771.

[4] M. Krstić, I. Kanellakopoulos and P. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley and Sons, New York, 1995.

[5] K. S. Narendra and J. Balakrishnan, Adaptive Control Using Multiple Models, *IEEE Transactions on Automatic Control*, Vol. 42, No. 2, 1997, pp. 171-187.

[6] J. T. Spooner and K. M. Passino, Stable Adaptive Control Using Fuzzy Systems and Neural Networks, *IEEE Trans. on Fuzzy Syst.*, Vol. 4, No. 3, 1996, pp. 339-359.

[7] T. Takagi and M. Sugeno, Fuzzy identification of Systems and Its Applications to Modelling and Control, *IEEE Trans. on Syst., Man, and Cyber.*, Vol. 15, 1985, pp. 116-132.

[8] L. X. Wang, Stable adaptive fuzzy control of nonlinear systems, *IEEE Trans. Fuzzy Syst.*, Vol. 1, No. 2, 1993, pp. 146-155.