Fuzzy model-based predictive control for a CSTR with multiple steady state: A simulation study and a comparison with other nonlinear MBPC control algorithms

IGOR ŠKRJANC, MARKO LEPETIČ, JOSE LUIS FIGUEROA*, SAŠO BLAŽIČ
Faculty of Electrical Engineering, University of Ljubljana, Slovenia
Tržaška 25, 1000 Ljubljana

* Universidad Nacional del Sur, Bahia Blanca, 8000, Argentina

Abstract In the paper a comparison of different nonlinear model-based predictive control algorithms is presented as a case study for a continuous stirred reactor. The focus is given to the fuzzy predictive control approach which is compared to Wiener based model predictive control and nonlinear model predictive control based on optimization. It has been shown that fuzzy predictive control law which is given in analytical form gives very promising results in comparison to other two approaches which are both based on optimization. All the proposed approaches are potentially interesting in the case of batch reactors, heat-exchangers, furnaces and all the processes with strong nonlinear dynamics.

Keywords: nonlinear predictive control, fuzzy identification, fuzzy-model predictive control.

1 Introduction

Model-based predictive control (MBPC) refers to a class of control algorithms that control the plant through the use of process model. The principle is based on the forecast of the output signal at each sampling instant. The forecast is done implicitly or explicitly based on the model of the controlled process. In the next step the control is selected which brings the predicted process output signal back to the reference signal in a way to minimize the difference between the reference and the output signal.

Although, the processes in the nature are inherently nonlinear, the majority of MBPC applications and algorithms up to date are based on linear models. The reason for that is in use of a linear model and a quadratic objective function where the nominal MBPC algorithms takes the form of a highly structured convex Quadratic Program (QP), for which the solution can be easily found.

In some highly nonlinear cases the use of nonlinear model-based predictive control (NMBPC) can be easily justified. By introducing the nonlinear model into predictive control problem, the complexity increase significantly. In literature [1], [5] a overview of different nonlinear predictive control approaches are discussed.

In this work we have compared three different predictive control algorithms: fuzzy model based nonlinear predictive control (FPFC) ([13], [14]) , Wiener based model predictive control (WMPC) ([10]) and nonlinear model predictive control based on optimization (NMPC) ([2], [12]). The main focus is given to fuzzy model based algorithm. First two approaches are based on models given in a form which approximate the proces nonlinear behavior and the last one is based on explicit mathematical model of the proces. Both, WMPC and NMPC are based on optimization which can be sometimes a bottleneck of the whole algorithm, because it could be very time consuming. The control law in the case of FPFC approach which is based on fuzzy model is given in analytical form. This makes the approach very easy for implementation also in programmable logic controllers.

The paper is organized as follows. Section 2 describes the continuous stirred-tank reactor, where the process description is given and steady-state analysis is realized and identification of the process in the fuzzy model form is obtained. Section 3 presents the control strategies. In section 4 simulation results and comparison of different nonlinear MBPC control algorithms are presented and discussed. Finally, in section 5 some concluding remarks are presented.

2 Continuous Stirred-Tank Reactor (CSTR)

2.1 Process Description

The simulated continuous stirred-tank reactor (CSTR) process consists of an irreversible, exothermic reaction, $A \rightarrow B$, in a constant volume reactor cooled by a single coolant stream, which can be modelled by the following equation ([11]),

Measured concentration	C_A	$0.1 mol/l$
Reactor temperature	T	$438.54K$
Coolant flow rate	q_c	$103.41 lmin^{-1}$
Process flow rate	q	$100 lmin^{-1}$
Feed concentration	C_{A0}	$1 mol/l$
Feed temperature	T_0	$350K$
Inlet temperature	T_{c0}	$350K$
CSTR volume	V	$100l$
Heat transfer	hA	$7 \times 10^5 cal\,\min^{-1} K^{-1}$
Reaction rate	$k0$	$7.2 \times 10^{10} \min^{-1}$
Activation energy	E/R	$1 \times 10^4 K$
Heat of reaction	ΔH	$-2 \times 10^5 cal/mol$
Liquid densities	ρ,ρ_c	$1 \times 10^3 g/l$
Specific heats	$C_p C_{pc}$	$1 cal g^{-1} K^{-1}$

Table 1: Nominal CSTR parameter values.

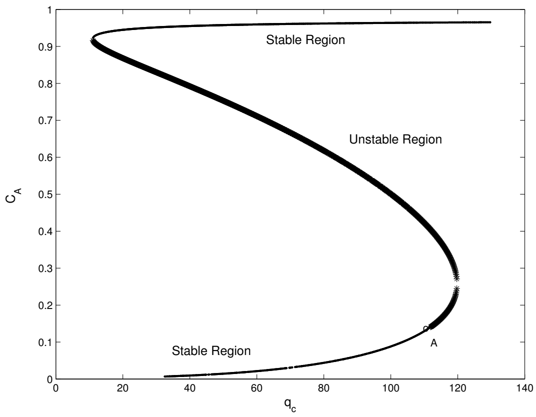

Figure 1: Stability analysis

$$\dot{C}_A = \frac{q}{V}\left[C_{A0} - C_A\right] - k_0 C_A \exp\left(\frac{-E}{RT}\right) \quad (1)$$

$$\dot{T} = \frac{q}{V}\left(T_0 - T\right) - \frac{k_0 \Delta H}{\rho C_p} C_A \exp\left(\frac{-E}{RT}\right) + \frac{\rho_c C_{pc}}{\rho C_p V} q_c \left[1 - \exp\left(-\frac{hA}{q_c \rho_c C_{pc}}\right)\right]\left(T_{c0} - T\right) \quad (2)$$

The measured concentration has a time delay d=0.5 min., then $C_{A_{meas}}(t) = C_A(t - t_d)$. The objective is to control the measured concentration of A, C_A, by manipulating the coolant flow rate q_c. This model is a modified version of the first tank of a two-tank CSTR example by ([4]). In the original model, the time delay was zero.

2.2 Steady-State Analysis

The state variables x_1 and x_2 stand for the dimensionless reactant concentration and the reactor temperature, respectively. The symbol q_c represents the coolant flow rate (manipulated variable) and the other symbols represent constant parameters whose values are defined in Table 1. The process dynamics is nonlinear due to the Arrhenius rate expression which describes the dependence of the reaction rate constant on the temperature (x_2). That is why the CSTR exhibits some operational and control problems. Fig. 1 shows the plot of the steady state values for x_1 versus the input q_c.

As shown in Fig. 1, the reactor presents multiplicity behavior with respect to the coolant flow rate. The CSTR modeled by equations (1-2) behaves as an open-loop unstable system if the concentration inside the reactor is between 0.14 and 0.92. In particular, the point A ($q_c \approx 111.85\ lmin^{-1}$) in Fig. 1 is a Hopf Bifurcation point. In our aplication, we are interested in the operation in the stable operative point given by $q_c = 103.41\ lmin^{-1}$ and $C_A = 0.1$.

2.3 Process Identification

Typical fuzzy model in [15] is given in the form of rules

$$\mathbf{R}_j: \textbf{ if } x_{p1} \textbf{ is } \mathbf{A}_{1,k_1} \textbf{ and } \ldots\ x_{pq} \textbf{ is } \mathbf{A}_{q,k_q} \textbf{ then } y = \phi_j(\mathbf{x}) \quad j = 1, \ldots, m \quad k_1 = 1, \ldots, f_1 \quad k_2 = 1, \ldots, f_2 \quad \ldots \quad k_q = 1, \ldots, f_q \quad (3)$$

The q-element vector $\mathbf{x}_p^T = [x_{p1}, ..., x_{pq}]$ denotes the input or variables in premise, and variable y is the output of the model. With each variable in premise x_{pi} $(i = 1, \ldots, q)$, f_i fuzzy sets $(\mathbf{A}_{i,1}, \ldots, \mathbf{A}_{i,f_i})$ are connected, and each fuzzy set $\mathbf{A}_{i,k_i}$ $(k_i = 1, \ldots, f_i)$ is associated with a real-valued function $\mu_{A_{i,k_i}}(x_{pi}) : \mathbb{R} \to [0,1]$, that produces membership grade of the variable x_{pi} with respect to the fuzzy set $\mathbf{A}_{i,k_i}$. To make the list of fuzzy rules complete, all possible variations of fuzzy sets are given in Eq. (3), yielding the number of fuzzy rules $m = f_1 \times f_2 \times \cdots \times f_q$. The variables x_{pi} are not the only inputs of the fuzzy system. Implicitly, the n-element vector $\mathbf{x}^T = [x_1, ..., x_n]$ also represents the input to the system. It is usually referred to as the consequence vector. The functions $\phi_j(\cdot)$ can be arbitrary smooth functions in general, although linear or affine functions are usually used.

The system in Eq. (3) can be described in closed form if the intersection of fuzzy sets is previously defined. The generalized form of the intersection is the so-called *triangular norm* (T-norm). In our case, the latter was chosen as algebraic product yielding the output of the fuzzy system

$$y = \frac{\sum_{k_1=1}^{f_1} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \ldots \mu_{A_{q,k_q}}(x_{pq})\ \phi_j(\mathbf{x})}{\sum_{k_1=1}^{f_1} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1})\ \ldots \mu_{A_{q,k_q}}(x_{pq})} \quad (4)$$

It has to be noted that a slight abuse of notation is used in Eq. (4) since j is not explicitly defined as running index. From Eq. (3) is evident that each j corresponds to the specific variation of indexes k_i, $i = 1, \ldots, q$.

To simplify Eq. (4), a partition of unity is considered where functions $\beta_j(\mathbf{x}_p)$ defined by

$$\beta_j(\mathbf{x}_p) = \frac{\mu_{A_{1,k_1}}(x_{p1}) \ \ldots \mu_{A_{q,k_q}}(x_{pq})}{\sum_{k_1=1}^{f_1} \cdots \sum_{k_q=1}^{f_q} \mu_{A_{1,k_1}}(x_{p1}) \ \ldots \mu_{A_{q,k_q}}(x_{pq})} \quad j = 1, \ldots, m \tag{5}$$

give information about the fulfilment of the respective fuzzy rule in the normalized form. It is obvious that $\sum_{j=1}^{m} \beta_j(\mathbf{x}_p) = 1$ irrespective of $\mathbf{x}_p$ as long as the denominator of $\beta_j(\mathbf{x}_p)$ is not equal to zero (that can be easily prevented by stretching the membership functions over the whole potential area of $\mathbf{x}_p$). Combining Eqs. (4) and (5) and changing summation over k_i by summation over j we arrive to the following equation:

$$y = \sum_{j=1}^{m} \beta_j(\mathbf{x}_p)\phi_j(\mathbf{x}) \tag{6}$$

From Eq. (6) it is evident that the output of a fuzzy system is a function of the premise vector $\mathbf{x}_p$ (q-dimensional) and the consequence vector $\mathbf{x}$ (n-dimensional). The dimension of the input space may be lower than $(q+n)$ since it is very usual to have the same variables present in vectors $\mathbf{x}_p$ and $\mathbf{x}$. Vector $\mathbf{z}$ (d-dimensional) comprises of the elements of $\mathbf{x}_p$ and those of $\mathbf{x}$ that are not present in $\mathbf{x}_p$.

Very often, the output value is defined as a linear combination of consequence states

$$\phi_j(\mathbf{x}) = \boldsymbol{\theta}_j^T \mathbf{x}, \quad j = 1, \ldots, m, \quad \boldsymbol{\theta}_j^T = [\theta_{j1}, \ldots, \theta_{jn}] \tag{7}$$

If Takagi-Sugeno model of the 0-th order is chosen, $\phi_j(\mathbf{x}) = \theta_{j0}$, and in the case of the first order model, the consequent is $\phi_j(\mathbf{x}) = \theta_{j0} + \boldsymbol{\theta}_j^T \mathbf{x}$. Both cases can be treated by the model (7) by adding 1 to the vector $\mathbf{x}$ and augmenting vector $\boldsymbol{\theta}$ with θ_{j0}. To simplify the notation, only the model in Eq. (7) will be treated in the rest of the paper. If the matrix of the coefficients for the whole set of rules is written as $\mathbf{\Theta}^T = [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_m]$ and the vector of membership values as $\boldsymbol{\beta}^T(\mathbf{x}_p) = \left[\beta^1(\mathbf{x}_p), \ldots, \beta^m(\mathbf{x}_p)\right]$, then Eq. (6) can be rewritten in the matrix form

$$y = \boldsymbol{\beta}^T(\mathbf{x}_p)\mathbf{\Theta}\mathbf{x} \tag{8}$$

The fuzzy model in the form given in Eq. (8) is referred to as affine Takagi-Sugeno model and can be used to approximate any arbitrary function that maps the compact set $\mathbf{C} \subset \mathbb{R}^d$ to $\mathbb{R}$ with any desired degree of accuracy in [6], [17] and [18]. The generality can be proven by Stone-Weierstrass in [3] theorem which indicates that any continuous function can be approximated by fuzzy basis function expansion in [9].

To identify the process, the discrete static compensator was added to stabilize the process at high concentration values

$$\Delta u_{ff} = K_{ff} \left[T(k) - T(k-1) \right], \tag{9}$$

where K_{ff} was chosen to be 3. The process was identified in a form of discrete second order model with the premise defined as $\mathbf{x}_p^T = [C_A(k)]$ and the consequence vector as $\mathbf{x}^T = [C_A(k), C_A(k-1), q_c(k - T_{D_m}), 1]$. The functions $\phi_j(\cdot)$ can be arbitrary smooth functions in general, although linear or affine functions are usually used. Due to strong nonlinearity the structure with six rules and equidistantly shaped gaussian membership functions was chosen. The sampling time was chosen to be $T_s = 0.1min$. The nonlinearity depends mostly on concentration. The structure of fuzzy model is given in (10)

$$\mathbf{R}_j: \ \textbf{if } C_A(k) \textbf{ is } \mathbf{A}_j \textbf{ then } C_A(k+1) = a_{1j} C_A(k) + a_{2j} C_A(k-1) + b_{1j} q_c(k - T_{D_m}) + r_j \quad j = 1, \ldots, m \tag{10}$$

The parameters of the fuzzy form in (10) have been estimated using least square algorithm where the data have been preprocessed using QR factorization. The estimated parameters can be written as vectors $\mathbf{a}_1^T = [a_{11}, ..., a_{1m}]$, $\mathbf{a}_2^T = [a_{21}, ..., a_{2m}]$, $\mathbf{b}_1^T = [b_{11}, ..., b_{1m}]$ and $\mathbf{r}_1^T = [r_{11}, ..., r_{1m}]$. The estimated parameters in the case of CSTR are as follows:

$$\begin{aligned} \mathbf{a}_1^T &= [1.37, 1.45, 1.54, 1.69, 1.75, 1.85] \\ \mathbf{a}_2^T &= [-0.44, -0.51, -0.60, -0.74, -0.79, -0.88] \\ \mathbf{b}_1^T &= [1.42, 1.89, 2.30, 2.19, 2.77, 2.38] \cdot 10^{-4} \\ \mathbf{r}_1^T &= [-0.82, -1.37, -1.76, -0.185, -2.51, -2.23] \cdot 10^{-2} \end{aligned} \tag{11}$$

and $T_{D_m} = 5$.

In our case the TS fuzzy model was after estimation of the parameter , transformed into a state space form (14) because of the control purposes.

$$\mathbf{x}_m(k+1) = \sum_i \beta_i(\mathbf{x}_p(k)) \left(\mathbf{A}_{m_i} \mathbf{x}_m(k)\right) + \sum_i \beta_i(\mathbf{x}_p(k)) \left(\mathbf{B}_{m_i} u(k - T_{D_m}) + \mathbf{R}_{m_i}\right) \tag{12}$$

$$y_m(k) = \mathbf{C}_m \mathbf{x}_m(k) \tag{13}$$

$$\mathbf{A}_{m_i} = \begin{bmatrix} 0 & 1 \\ a_{2_i} & a_{1_i} \end{bmatrix} \quad \mathbf{B}_{m_i} = \begin{bmatrix} 0 \\ b_{1_i} \end{bmatrix} \quad \mathbf{C}_m = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad \mathbf{R}_{m_i} = \begin{bmatrix} 0 \\ r_i \end{bmatrix} \tag{14}$$

where the proces model output concentration C_A is denoted as y_m and the input flow q_c as u.

The frozen-time theory ([7], [8]) enables the relation between the nonlinear dynamical system and the associated linear time-varying system. The theory establish the following fuzzy model

$$\mathbf{x}_m(k+1) = \bar{\mathbf{A}}_m \mathbf{x}_m(k) + \bar{\mathbf{B}}_m u(k - T_{D_m}) + \bar{\mathbf{R}}_m \tag{15}$$

$$y_m(k) = \mathbf{C}_m \mathbf{x}_m(k) \tag{16}$$

where $\bar{\mathbf{A}}_m = \sum_i \beta_i(\mathbf{x}_p(k))\mathbf{A}_{m_i}$, $\bar{\mathbf{B}}_m = \sum_i \beta_i(\mathbf{x}_p(k))\mathbf{B}_{m_i}$ and $\bar{\mathbf{R}}_m = \sum_i \beta_i(\mathbf{x}_p(k))\mathbf{R}_{m_i}$.

3 Control Strategies

3.1 Fuzzy model based nonlinear predictive control

As first approach a predictive functional control ([13]) was applied to control the reactor. In this case the prediction of the plant output is given by its fuzzy model in the state-space domain ([14]).

The problem of delays in the plant is circumvented by constructing an auxiliary variable that serves as the output of the plant if there were no delay present. The so-called "undelayed" model of the plant will be introduced for that purpose. It is obtained by "removing" delays from the "delayed" model and converting it to the state space description:

$$\begin{aligned} \mathbf{x}_m(k+1) = \bar{\mathbf{A}}_m \mathbf{x}_m(k) + \bar{\mathbf{B}}_m \mathbf{u}(k) + \bar{\mathbf{R}}_m \\ y_m^0(k) = \mathbf{C}_m \mathbf{x}_m(k) \end{aligned} \quad (17)$$

where $y_m^0(k)$ models the "undelayed" output of the plant.

The behavior of the closed-loop system is defined by the reference trajectory which is given in the form of the reference model. The control goal is to determine the future control action so that the predicted output value coincide with the reference trajectory. The coincidence point is called a coincidence horizon and denoted by H. The prediction is calculated under assumption of constant future manipulated variables ($u(k) = u(k+1) = \ldots = u(k+H-1)$), the mean level control. The H-step ahead prediction of the "undelayed" plant output is then obtained from Eq. (17):

$$\begin{aligned} y_m^0(k+H) = \mathbf{C}_m \left(\bar{\mathbf{A}}_m^H \mathbf{x}_m(k) \right) + \\ \mathbf{C}_m \left(\left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \left(\bar{\mathbf{B}}_m \mathbf{u}(k) + \bar{\mathbf{R}}_m \right) \right) \end{aligned} \quad (18)$$

The reference model is given by the first order difference equation

$$\begin{aligned} \mathbf{x}_r(k+1) = \mathbf{A}_r \mathbf{x}_r(k) + \mathbf{B}_r w(k) \\ y_r(k) = \mathbf{C}_r \mathbf{x}_r(k) \end{aligned} \quad (19)$$

where w stands for the reference signal. The reference model parameters should be chosen to fulfil the following equation

$$\mathbf{C}_r \left(\mathbf{I} - \mathbf{A}_r \right)^{-1} \mathbf{B}_r = 1 \quad (20)$$

which results in the reference model unity gain. This enables the reference trajectory tracking.

The main goal of proposed algorithm is to find the control law which enables the reference trajectory tracking of the "undelayed" controlled signal ($y_r(k+i) = y_p^0(k+i), \quad i = 1, ..., H$). The idea of MPFC is introduced thought the equivalence of the objective increment vector Δ_p and the model output increment vector Δ_m:

$$\Delta_p = \Delta_m \quad (21)$$

The former is defined as the difference between the predicted reference signal vector $y_r(k+H)$ and the actual output vector of the "undelayed" plant $y_p^0(k)$

$$\Delta_p = y_r(k+H) - y_p^0(k) \quad (22)$$

The variable $y_p^0(k)$ cannot be measured directly. Rather, it will be estimated from the available signals:

$$y_p^0(k) = y_p(k) - y_m(k) + y_m^0(k) \quad (23)$$

It can be seen that the delay in the plant is compensated by the difference between the outputs of the "undelayed" and the "delayed" model. When the perfect model of the plant is available ($G_m = G_p$), the first two terms on the right side of Eq. (23) cancel and the result is actually the output of the "undelayed" plant. If this is not the case, only the approximation is obtained. The model output increment vector Δ_m is defined by the following formula:

$$\Delta_m = y_m^0(k+H) - y_m^0(k) \quad (24)$$

The following is obtained from (21) by using (22), and (24) and introducing (18):

$$\begin{aligned} u(k) = g_0^{-1} \left(\left(y_r(k+H) - y_p^0(k) + y_m^0(k) \right) \right) - \\ -g_0^{-1} \left(\mathbf{C}_m \bar{\mathbf{A}}_m^H \mathbf{x}_m(k) - \mathbf{C}_m \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \bar{\mathbf{R}}_m \right) \end{aligned} \quad (25)$$

where g_0 stands for:

$$g_0 = \mathbf{C}_m \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \bar{\mathbf{B}}_m \quad (26)$$

And the control law of MPFC in analytical form is finally obtained by introducing (23) into (25):

$$\begin{aligned} u(k) = g_0^{-1} \left(\left(y_r(k+H) - y_p(k) + y_m(k) \right) \right) - \\ -g_0^{-1} \left(\mathbf{C}_m \bar{\mathbf{A}}_m^H \mathbf{x}_m(k) - \mathbf{C}_m \left(\bar{\mathbf{A}}_m^H - \mathbf{I} \right) \left(\bar{\mathbf{A}}_m - \mathbf{I} \right)^{-1} \bar{\mathbf{R}}_m \right) \end{aligned} \quad (27)$$

Note that the control law (27) is realizable if the matrix g_0 is non-singular. This is true if the plant is stable, controllable and observable.

The stability analysis of the proposed predictive control can be performed using an approach of linear matrix inequalities (LMI) proposed in ([19]) and ([16]).

3.2 Wiener based model predictive control

In this case a Wiener Based MPC (WMPC) is implemented to control the reactor ([10], In press). To obtain a good representation of the process, the dynamic linear block of the Wiener model is obtained by linearizing of the nonlinear model (1-2) and the nonlinear stationary block is obtained by approximating the steady-state solutions of (1-2). Also in this case, a discrete static compensator was added (9) to stabilize the process at high concentration values.

In this case, the objective function to be minimized was

$$\begin{aligned} f_{obj} = 35 \sum_{i=1}^{P} (C_A(i * T_s) - C_A^{sp})^2 + \\ +0.25 \sum_{i=1}^{M} (q_c(i * T_s) - q_c((i-1) * T_s))^2 \end{aligned}$$

Note that the performance of this controller is poor. This is because along the operative region, the Wiener model is not a good description of the process. For example, consider that the eigenvalues of the dynamic model changes from $-1.3510 \pm j3.0347$ (at the nominal operative point, $q_c = 103.41 \ lmin^{-1}$) to $-0.0052 + j2.4221$ at the neighborhood of the point A (i.e., a single linear model can described booth situation).

3.3 Nonlinear model predictive control based on optimization

To compare the proposed control methodology a nonlinear MPC is used (NMPC). In this control, we use a discrete version of the model of Eqs. (1-2). The sample time is $T_s = 0.1\ min$, the prediction horizon $P = 20$ and the control horizon $M = 5$. The objective function to be minimized is defined as

$$f_{obj} = 10^5 \sum_{i=1}^{P} (C_A(i*T_s) - C_A^{sp})^2 +$$
$$+10^3 \sum_{i=1}^{M} (q_c(i*T_s) - q_c((i-1)*T_s))^2 +$$
$$10^{-2} \sum_{i=1}^{M} (q_c(i*T_s) - q_c(0))^2$$

where C_A^{sp} is the set point for the controlled variable. To ensure small steady state error a constraint is imposed to the control problem as $|C_A(PT_s) - C_A^{ss}| < \varepsilon$. It is necessary to note that in our case the error was chosen as $\varepsilon = 0.001$. For the implementation of this scheme we consider that the temperature is measured and that the disturbances are known at any time. This last assumption is quite strong, but it is included to stand in a more favorable situation in order to compare this controller with the proposed in this paper.

4 Simulation results and comparison of different nonlinear MBPC control algorithms

Wiener based model predictive control, nonlinear predictive control based on optimization and fuzzy model based predictive control have been simulated and compared with same conditions. We have studied the behavior of all those different control algorithms by the reference step response and the disturbance rejection. The simulation results of set-point tracking are presented in Fig. 2. Set-point in Fig. 2 is presented the with the dotted line, WMPC response is plotted with the dashed line, NMPC response is plotted with dash-dot line and FPFC with solid line. The upper diagram represents the output signals of all different control algorithms and the lower diagram represents control signals. The set-point was changed from 0.1 to 0.15, back to 0.1, then to 0.05 and again to 0.1 mol/l. The changes were made every 8 minutes.

The disturbance rejection performance can be seen in Fig. 3. The unmeasured feed concentration changes from 1 mol/l to 0.95 mol/l at 8 min and back to 1 mol/l at 24 min. The unmeasured coolant temperature decreases from $350°C$ to $340°C$ at 16 min and back to $350°C$ at 32 min. The obtained simulation results have shown the superiority of nonlinear optimization predictive control approach especially in disturbance rejection mode due

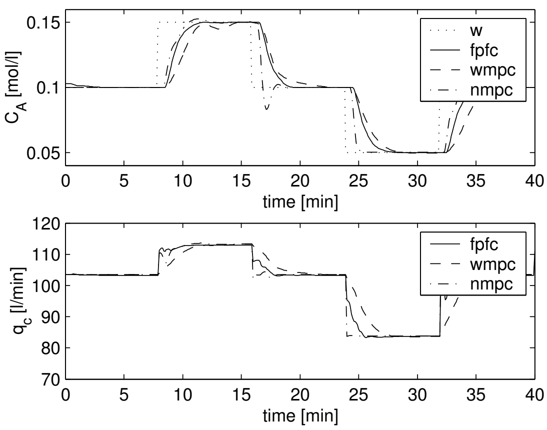

Figure 2: Control performance of different algorithms in the case of the reference trajectory tracking

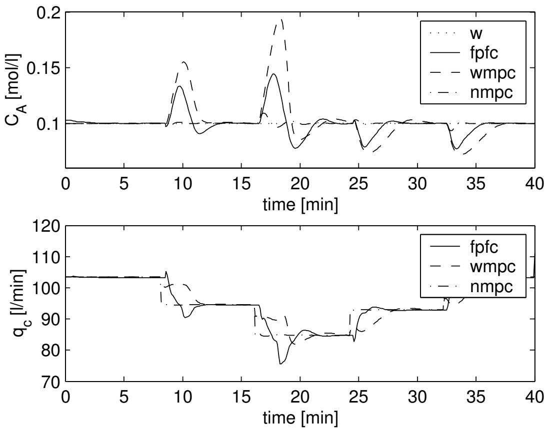

Figure 3: Control performance of different algorithms in the case of the disturbance rejections

to the measured disturbance. In reference trajectory mode the quality of performance is very similar to those obtained by FPFC. The control obtained by WMPC has given in all modes the poorest results.

5 Conclusion

In the paper a comparison of different nonlinear MBPC has been presented as a case study for a continuous stirred tank reactor. The focus is given to the fuzzy predictive control approach which is compared to Wiener based model predictive control and nonlinear model predictive control based on optimization. The first two approaches are based on models given in a form which approximate the proces nonlinear behavior and the last one is based on explicit mathematical model of the proces. Both, WMPC and NMPC are based on optimization which can be sometimes a bottleneck of the whole algorithm, because it could be very time consuming. The control law in the case of FPFC approach which is based on fuzzy model is given in analytical form.

This makes the approach very easy for implementation also in programmable logic controllers.

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References

[1] Bequette, B. W., "Nonlinear control of chemical processes: A review.", Ind. Eng. Chem. Res., vol. 30, pp. 1391-1413, 1991.

[2] Figueroa, J. L., "Piecewise linear models in model predictive control", Latin American Applied Research, vol. 31, no.4, pp.309-315, 2001.

[3] Goldberg, R. R., "Methods of Real Analysis", John Wiley and Sons, 1976.

[4] Henson, M.A., Seborg, D.E. (1990). "Input-output linerization of general processes", AIChE J., vol. 36, pp. 1753, 1990.

[5] Henson, M.A., "Nonlinear model predictive control: current status and future directions. Computer and Chemical Engineering, vol. 23, pp. 187-202, 1998.

[6] Kosko, B., "Fuzzy Systems as Universal Approximators", IEEE Transactions on Computers, vol. 43, no. 11, pp. 1329-1333, 1994.

[7] Leith, D. J., Leithead, W. E., "Gain-scheduled and nonlinear systems: dynamics analysis by velocity-based linearization families", International Journal of Control, vol. 70, no. 2, pp. 289-317, 1998.

[8] Leith, D. J., Leithead, W. E., "Analyitical framework for blended model systems using local linear models", International Journal of Control, vol. 72, no. 7-8, pp. 605-619, 1999.

[9] Lin, C-H, "Siso nonlinear system identification using a fuzzy-neural hybrid system", Int. Jour. of Neural Systems, vol. 8, no. 3, pp. 325-337, 1997.

[10] Lusson Cervantes, A., Agamennoni, O.E., Figueroa J.L. . A nonlinear model predictive control system based on Wiener piecewise linear models, Journal of Process Control, In Press, 2003.

[11] Morningred, J.D., Paden, B. E., Mellichamp, D.A.. An adaptive nonlinear predictive controller. Chemical Engineering Science, vol. 47, pp. 755-762, 1992.

[12] Padin, M. S., Figueroa, J.L., "Use of CPWL approximations in the design of a numerical nonlinear regulator", IEEE Transection on Automatic Control, vol. 45, no. 6, pp. 1175-1180, 2000.

[13] Škrjanc, I., Matko, D., Predictive functional control based on fuzzy model for heat-exchanger pilot plant. IEEE Transection on Fuzzy Systems, vol. 8, no. 6, pp. 705-712, 2000.

[14] Škrjanc, I., Matko, D., Fuzzy predictive functional control in the state space domain. J. Intell. robot. Syst., vol. 31, pp. 283-297, 2001.

[15] Takagi, T. and M.Sugeno, "Fuzzy identification of Systems and Its Applications to Modelling and Control", IEEE Trans. on Systems, Man, and Cybernetics, vol. 15, pp. 116-132, 1985.

[16] Tanaka, K., Ikeda, T., Wang, H. O., "Robust Stabilization of a Class of Uncertain Nonlinear Systems via Fuzzy Control: Quadratic Stabilizability, H^{∞} Control Theory, and Linear Matrix Inequalities", IEEE Trans. on Fuzzy Systems, vol. 4, no. 1, pp. 1-13, 1996.

[17] Ying, H. GC., "Necessary conditions for some typical fuzzy systems as universal approximators", Automatica, vol. 33, pp. 1333-1338, 1997.

[18] Wang, L.-X., Mendel, J. M., "Fuzzy basis functions, universal approximation, and orthogonal least-squares learning", IEEE Trans. Neural Networks, vol. 3, no. 5, pp. 807-814, 1992.

[19] Wang, H. O., Tanaka, K., Griffin, M. F., " An Approach to Fzzy Control of Nonlinear Systems: Stability and Design Issues", IEEE Trans. on Fuzzy Systems, vol. 4, no. 1, pp. 14-23, 1996.