A Canonical Representation for the Solution of Fuzzy Linear System and Fuzzy Linear Programming Problem

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Abstract: In this paper first, we find a canonical symmetrical trapezoidal(triangular) for the solution of the fuzzy linear system $A\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$, where the elements in A and $\tilde{\mathbf{b}}$ are crisp and arbitrary fuzzy numbers, respectively. Then, a model for fuzzy linear programming problem with fuzzy variables (FLPFV), in which, the right hand side of constraints are arbitrary numbers, and coefficients of the objective function and constraint matrix are regarded as crisp numbers, is discussed. A numerical procedure for calculating a canonical symmetrical trapezoidal representation for the solution of fuzzy linear system and the optimal solution of FLPFV, (if there exist) is proposed. Several examples illustrate these ideas.

Key-Words: Fuzzy number, fuzzy linear system, fuzzy linear programming, value, ambiguity, fuzziness, canonical representation.

1 Introduction

An approach to solve a fuzzy linear system was given by Buckley et al. [1] and Friedman et al. [4]. Also, several kinds of fuzzy linear programming problems and corresponding approaches to solving them have appeared in the literature and valuable results are obtained. Fuzzy linear programming are usually divided into two categories: fuzzy linear programming with fuzzy constraints and fuzzy linear programming with fuzzy coefficients, where the variables in both models are regarded as crisp. In this paper first, we propose a new method for solving an $n \times n$ fuzzy linear system of algebraic equations, and then propose a method for solving a FLPFV based on a canonical representation of fuzzy numbers and ranking method which is introduced by Delgado et al. [2, 3]. They introduced three real indices called value, ambiguity and fuzziness to obtain "simple", fuzzy numbers, that could be used to represent more arbitrary fuzzy numbers. In section 2 we review basic definitions and the notions of value, ambiguity and fuzziness defined in [2, 3] and then give a canonical representation of fuzzy numbers by means of symmetrical trapezoidal ones [2, 3]. In section 3 we present a fuzzy linear system and propose a method for solving them. Some numerical examples are given in section 3. In section 4, first we introduce fuzzy linear programming with fuzzy variables and then a method for solving this problem is given. Also some numerical examples are presented.

2 Preliminaries

In this section we review some preliminaries which are needed in the next section. For more details see [2, 3].

2.1 Basic definitions and notations

A fuzzy set $\tilde{a}$ on $I\!R$ is a *fuzzy number* provided:
1)Its membership function is upper semi continuous.
2)There exist three interval [a,b], [b,c], [c,d] such that $\tilde{a}$ is increasing on [a,b], equal to 1 on [b,c], decreasing on [c,d] and equal to 0 elsewhere.

We denote the *support* of $\tilde{a}$ by $\tilde{a}_0 = \{x \in I\!R \mid \tilde{a}(x) > 0\}$ and the *mode* of $\tilde{a}$ by $\tilde{a}_1 = \{x \in I\!R \mid \tilde{a}(x) = 1\}$. Suppose $\tilde{a}$ is a fuzzy number and $r \in (0, 1)$, then the *r-cut* of $\tilde{a}$ is defined by $\tilde{a}_r = \{x \in I\!R \mid \tilde{a}(x) \geq r\}$. The r-cut representation of $\tilde{a}$ is the pair of functions $(L_{\tilde{a}}, R_{\tilde{a}})$ both from [0,1] to $I\!R$ defined by

$$L_{\tilde{a}}(r) = \inf\{x|x \in \tilde{a}_r\}, R_{\tilde{a}}(r) = \sup\{x|x \in \tilde{a}_r\}$$

for $r \in [0, 1]$. We will let $\tilde{T} = (a, b, \alpha, \beta)$ denote the trapezoidal fuzzy number, where $[a-\alpha, b+\beta]$ is the support of $\tilde{T}$ and $[a, b]$ is the mode of $\tilde{T}$. If $a = b$ then we obtain a triangular fuzzy number. A fuzzy number with $a = m - c$, $b = m + c$, $\alpha = \beta = d$ is said to be a symmetrical trapezoidal fuzzy number which we denote it by $\tilde{T} = (m, c, d)$. If $c = 0$, we have a symmetrical triangular fuzzy number.

2.2 Canonical trapezoidal representation of a fuzzy number

Delgado et al. [2, 3] introduced three parameters value, ambiguity and fuzziness for a fuzzy number in order to capture the relevant information about the fuzzy number's ill-defined magnitude. These parameter where defined as follows. A function $s : [0, 1] \to [0, 1]$ which is said to be a reducing function, if s is increasing, $s(0) = 0$ and $s(1) = 1$. The simplest and most natural reducing function is the uniform $s(r) = r$ [2, 3]. Let $\tilde{a}$ be a fuzzy number with r-cut representation $(L_{\tilde{a}}, R_{\tilde{a}})$. With respect to the uniform reducing function value of $\tilde{a}, V(\tilde{a})$, ambiguity of $\tilde{a}, A(\tilde{a})$, and fuzziness of $\tilde{a}, F(\tilde{a})$,defined:

$$V(\tilde{a}) = \int_0^1 r[L_{\tilde{a}}(r) + R_{\tilde{a}}(r)]dr,$$

$$A(\tilde{a}) = \int_0^1 r[L_{\tilde{a}}(r) - R_{\tilde{a}}(r)]dr,$$

$$F(\tilde{a}) = \int_0^{1/2} [R_{\tilde{a}}(r) - L_{\tilde{a}}(r)]dr + \int_{1/2}^1 [L_{\tilde{a}}(r) - R_{\tilde{a}}(r)]dr.$$

$V(\tilde{a})$ may be seen as a central value which represent the value of a fuzzy number, $A(\tilde{a})$ the "global spread" of the membership function of $\tilde{a}$, the measure of the vagueness of $\tilde{a}$ and $F(\tilde{a})$ the global difference between $\tilde{a}$ and $\tilde{a}_c$ the complement of $\tilde{a}$. For a trapezoidal fuzzy number $\tilde{T} = (a, b, \alpha, \beta)$, we have

$$V(\tilde{T}) = 1/2(a + b) + 1/6(\beta - \alpha),$$

$$A(\tilde{T}) = 1/2(b - a) + 1/6(\beta + \alpha),$$

$$F(\tilde{T}) = 1/4(\alpha + \beta).$$

Also for a symmetrical trapezoidal fuzzy number $\tilde{T} = (m, c, d)$ we have $V(\tilde{T}) = m, A(\tilde{T}) = d/3 + c$ and $F(\tilde{T}) = d/2$. Suppose now we

Fig. 1.

are given a fuzzy number $\tilde{a}$ with parameters $V(\tilde{a}), A(\tilde{a})$ and $F(\tilde{a})$. Then from the above results we can construct a symmetrical trapezoidal fuzzy number $\tilde{T}$ such that $V(\tilde{T}) = V(\tilde{a}), A(\tilde{T}) = A(\tilde{a})$ and $F(\tilde{T}) = F(\tilde{a})$ provided $c = A(\tilde{a}) - 2/3F(\tilde{a}) \geq 0$. This symmetrical trapezoidal fuzzy number is said to be canonical trapezoidal representation of $\tilde{a}$. If for a fuzzy number $\tilde{a}$ we have $c = A(\tilde{a}) - 2/3F(\tilde{a}) < 0$, then this fuzzy number will not have a canonical trapezoidal representation. In this case we have a typical quasi-trapezoidal representation for $\tilde{a}$. In Fig. 1, we sketch a typical quasi-trapezoidal [2] fuzzy number $\tilde{a}$ with r-cut representation $(L_{\tilde{a}}(r), R_{\tilde{a}}(r))$, where

$$L_{\tilde{a}(r)} = \begin{cases} m - s - t & if \;\; r = 0, \\ rt/h - t - s & if \;\; 0 < r < h, \\ m - s & if \;\; h \leq r \leq 1, \end{cases}$$

$$R_{\tilde{a}}(r) = \begin{cases} m + s + t & if \;\; r = 0, \\ -rt/h + t + s & if \;\; 0 < r < h, \\ m + s & if \;\; h \leq r \leq 1. \end{cases}$$

If $A(\tilde{a}) \leq F(\tilde{a})/12$, then we can consider $s = 0,\ h = [3A(\tilde{a})/F(\tilde{a})]^{1/2}$, $t = (F(\tilde{a}))^{3/2}/(3A(\tilde{a}))^{1/2}$. If $F(\tilde{a})/12 \leq A(\tilde{a}) \leq 2F(\tilde{a})/3$, then in this case we have $th^3/3 + s = A(\tilde{a}),\ t(-1/(2h) + 2 - h) = F(\tilde{a})$.
If $c = 0$, then the corresponding canonical representation is a triangular number.

Note that for any two trapezoidal fuzzy number $\tilde{a}_1$ and $\tilde{a}_2$ and any real number k we have

$$V(k\tilde{a}_1 + \tilde{a}_2) = kV(\tilde{a}_1) + V(\tilde{a}_2), \quad (1)$$

$$A(k\tilde{a}_1 + \tilde{a}_2) = |k|A(\tilde{a}_1) + A(\tilde{a}_2), \quad (2)$$

$$F(k\tilde{a}_1 + \tilde{a}_2) = |k|F(\tilde{a}_1) + F(\tilde{a}_2). \quad (3)$$

Algorithm 2.2.1 As a ranking method, we compare two fuzzy number $\tilde{a}$ and $\tilde{b}$ into the following steps:

1)Compare $V(\tilde{a})$ and $V(\tilde{b})$. If they are equal, then go to the next step. Otherwise rank $\tilde{a}$ and $\tilde{b}$ according to the relative position of $V(\tilde{a})$ and $V(\tilde{b})$.

2)Compare $A(\tilde{a})$ and $A(\tilde{b})$. If they are equal, then go to the next step. Otherwise rank $\tilde{a}$ and $\tilde{b}$ according to the relative position of $A(\tilde{a})$ and $A(\tilde{b})$.

3)Compare $F(\tilde{a})$ and $F(\tilde{b})$. If they are equal, then conclude that $\tilde{a}$ and $\tilde{b}$ are equal. Otherwise rank $\tilde{a}$ and $\tilde{b}$ according to the relative position of $F(\tilde{a})$ and $F(\tilde{b})$.

Note that, we say that $\tilde{a}_1 = \tilde{a}_2$ if and only if

$$V(\tilde{a}_1) = V(\tilde{a}_2),\ A(\tilde{a}_1) = A(\tilde{a}_2),\ F(\tilde{a}_1) = F(\tilde{a}_2). \quad (4)$$

3 A fuzzy linear system

In this section we will investigate a fuzzy linear system. We define a fuzzy vector solution and propose a procedure for calculating it.

Definition 3.1 The linear system

$$\sum_{j=1}^{n} a_{ij}\tilde{y}_j = \tilde{b}_i \quad \text{for } i = 1, \ldots, n, \quad (5)$$

where the coefficient matrix $A = (a_{ij}), 1 \leq i,j \leq n$ is a crisp $n \times n$ matrix and $\tilde{b} = (\tilde{b}_i), 1 \leq i \leq n$ is a fuzzy $n \times 1$ vector of fuzzy numbers, is called *a fuzzy linear system.*

A vector of fuzzy number $\tilde{\mathbf{y}} = (\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_n)^t$ is called *a fuzzy solution* of (5) if it satisfies (5). In this method we will find a canonical symmetrical trapezoidal representation for this solution. Now let $\tilde{\mathbf{y}}$be a solution for (5), then by (4) for $i = 1, \ldots, n$ we have

$$V(\sum_{j=1}^{n} a_{ij}\tilde{y}_j) = V(\tilde{\mathbf{b}}_i),$$

$$A(\sum_{j=1}^{n} a_{ij}\tilde{y}_j) = A(\tilde{\mathbf{b}}_i),$$

$$\text{and} \quad F(\sum_{j=1}^{n} a_{ij}\tilde{y}_j) = F(\tilde{\mathbf{b}}_i).$$

(1),(2) and(3) implies

$$\sum_{j=1}^{n} a_{ij}V(\tilde{y}_j) = v(\tilde{\mathbf{b}}_i),$$

$$\sum_{j=1}^{n} |a_{ij}|A(\tilde{y}_j) = A(\tilde{\mathbf{b}}_i),$$

$$\text{and} \quad \sum_{j=1}^{n} |a_{ij}|F(\tilde{y}_j) = F(\tilde{\mathbf{b}}_i).$$

Now let $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n)$ be the canonical symmetrical trapezoidal representation for this solution. Hence for finding this representation we solve the following crisp linear system

$$\begin{aligned}
&\sum_{j=1}^{n} a_{ij}V(\tilde{x}_j) = V(\tilde{\mathbf{b}}_i),\\
&\sum_{j=1}^{n} |a_{ij}|A(\tilde{x}_j) = A(\tilde{\mathbf{b}}_i), \qquad (6)\\
&\sum_{j=1}^{n} |a_{ij}|F(\tilde{x}_j) = F(\tilde{\mathbf{b}}_i),\\
&V(\tilde{x}_j), A(\tilde{x}_j), F(\tilde{x}_j) \geq 0 \quad 1 \leq j \leq n.
\end{aligned}$$

After obtaining $V(\tilde{x}_j), A(\tilde{x}_j)$ and $F(\tilde{x}_j)$, if $c_j = A(\tilde{x}_j) - 2/3F(\tilde{x}_j) \geq 0$ for all j, then we can find a symmetrical trapezoidal fuzzy number $\tilde{x}_j = (m_j, c_j, d_j)$. But if $c_j = A(\tilde{x}_j) - 2/3F(\tilde{x}_j) < 0$ for some j, then we can only find a symmetrical quasi-trapezoidal fuzzy number $\tilde{x}_j = (m_j, c_j, d_j)$ as defined in section 2.2.

Some examples are presented here to show the applicability of canonical representation of fuzzy numbers.

Example 3.2 Consider the following 2×2 fuzzy linear system

$$\tilde{x}_1 - \tilde{x}_2 = \tilde{b}_1, \quad \tilde{x}_1 + 3\tilde{x}_2 = \tilde{b}_2,$$

where $\tilde{b}_1 = (1, 1, 1)$, $\tilde{b}_2 = (5, 1, 2)$ are triangular fuzzy numbers. Consider the classical linear system (6) for this fuzzy linear system. we find the following values:

$$V(\tilde{x}_1) = 2.0417, A(\tilde{x}_1) = 0.25, F(\tilde{x}_1) = 0.375,$$

$$V(\tilde{x}_2) = 1.0417, A(\tilde{x}_2) = 0.0834, F(\tilde{x}_2) = 0.125.$$

By simple calculations, we find $m_1 = 2.0417, c_1 = 0, d_1 = 0.75, m_2 = 1.0417, c_2 = 0, d_2 = 0.125$. Hence $\tilde{x}_1 = (2.0417, 0.75, 0.75), \tilde{x}_2 = (1.0417, 0.125, 0.125)$ are the canonical triangular representation solution. Note that, these solution are close to the solution $\tilde{x}_1 = (2, 0.625, 0.875)$ and $\tilde{x}_2 = (1, 0.125, 0.375)$ of Example 4 of [4].

Example 3.3 Consider the following 2×2 fuzzy linear system

$$\tilde{x}_1 - 2\tilde{x}_2 = \tilde{b}_1, \quad \tilde{x}_1 + 5\tilde{x}_2 = \tilde{b}_2,$$

where $\tilde{b}_2 = (4, 6, 1, 3)$ is a trapezoidal fuzzy number and

$$\tilde{b}_1(x) = \begin{cases} 1 - x^2 & if \;\; -1 \leq x \leq 0,\\ (-1/4)x + 1 & if \;\; 0 \leq x \leq 4,\\ 0 & otherwise. \end{cases}$$

for which $V(\tilde{b}_1) = 0.4$, $A(\tilde{b}_1) = 0.9333$, $F(\tilde{b}_1) = 0.6953$ as in Example 3 of [2]. The solution of the classical system (6) of this example is as follow:

$$V(\tilde{x}_1) = 1.8095, A(\tilde{x}_1) = 0.4444, F(\tilde{x}_1) = 0.4922,$$

$$V(\tilde{x}_2) = 0.7048, A(\tilde{x}_2) = 0.2445, F(\tilde{x}_2) = 0.1016.$$

Simple calculation implies $m_1 = 1.8095, c_1 = 0.1163, d_1 = 0.9844, m_2 = 0.7048, c_2 = 0.1768, d_2 = 0.2032$. Hence $\tilde{x}_1 = (1.7488, 1.8702, 0.9844, 0.9844)$, $\tilde{x}_2 = (0.5003, 0.9093, 0.2032, 0.2032)$.

Example 3.4 Consider the following 2×2 fuzzy linear system

$$\tilde{x}_1 - \tilde{x}_2 = \tilde{b}_1, \quad \tilde{x}_1 + 3\tilde{x}_2 = \tilde{b}_2,$$

where $\tilde{b}_1 = (-15.7667, -7.2333, 6.4, 6.4)$ and $\tilde{b}_2 = (6.8667, 15.1333, 10.4, 10.4)$ are symmetrical trapezoidal fuzzy numbers. Consider the classical linear system (6) for this fuzzy linear system. we find the following values:

$$V(\tilde{x}_1) = 0.5, A(\tilde{x}_1) = 1.2, F(\tilde{x}_1) = 2,$$
$$V(\tilde{x}_2) = 12, A(\tilde{x}_2) = 5.2, F(\tilde{x}_2) = 1.2.$$

By simple calculations, we find $m_1 = 0.5, c_1 = -0.1333, d_1 = 4, m_2 = 12, c_2 = 4.4, d_2 = 2.4$, and hence $\tilde{x}_1$ does not have a canonical trapezoidal representation and the canonical representation for $\tilde{x}_2$ is $\tilde{x}_2 = (7.6, 16.4, 2.4, 2.4)$. There are a lot of quasi-trapezoidal representation for $\tilde{x}_1$ such that

$$th^3/3 + s = 1.2, t(-1/(2h) + 2 - h) = 2.$$

One of these which we use as a typical quasi-trapezoidal representation for $\tilde{x}_1$ is obtained in [2] by:

$$h = 0.875, t = 3.6127, s = 0.3932.$$

4 A fuzzy linear programming problem with fuzzy variables

In this section, we describe a method for solving linear programming problem with fuzzy variables.

Definition 4.1 The problem

$$min : \tilde{z} = \mathbf{c}\tilde{\mathbf{x}}, \quad s.t. \quad \mathbf{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}}, \quad \tilde{\mathbf{x}} \geq \tilde{0}, \quad (7)$$

where $\mathbf{c}^t \in I\!R^n, \mathbf{A} \in I\!R^{m\times n}, \tilde{\mathbf{b}} \in (F(I\!R))^m$, and $\tilde{\mathbf{x}} \in (F(I\!R))^n$ is called an *linear programming problem with fuzzy variables.* We say that $\tilde{\mathbf{x}}$ is a *fuzzy feasible solution* to (4) if and only if $\tilde{\mathbf{x}}$ satisfies the constraints (4). A fuzzy feasible solution $\tilde{\mathbf{x}}^*$is called to be a *fuzzy optimal solution* (4), if $\mathbf{c}\tilde{\mathbf{x}}^* \leq \mathbf{c}\tilde{\mathbf{x}}$, for all $\tilde{\mathbf{x}}$ belonging to the set of all fuzzy feasible solutions of (4).

By ranking method described in algorithm 2.2.1, we solve (4) as:

$$\begin{aligned} min : \quad & z = V(\mathbf{c}\tilde{\mathbf{x}}) + A(\mathbf{c}\tilde{\mathbf{x}}) + F(\mathbf{c}\tilde{\mathbf{x}}), \\ s.t. \quad & V(\mathbf{A}\tilde{\mathbf{x}}) = V(\tilde{\mathbf{b}}), \\ & A(\mathbf{A}\tilde{\mathbf{x}}) = A(\tilde{\mathbf{b}}), \\ & F(\mathbf{A}\tilde{\mathbf{x}}) = F(\tilde{\mathbf{b}}), \\ & V(\tilde{\mathbf{x}}) - A(\tilde{\mathbf{x}}) - 4/3F(\tilde{\mathbf{x}}) \geq 0, \\ & V(\tilde{\mathbf{x}}), A(\tilde{\mathbf{x}}), F(\tilde{\mathbf{x}}) \geq 0. \end{aligned}$$

From (1), (2) and (3), this model is equivalent to the following classical linear programming problem:

$$\begin{aligned} & min : \\ & z = \sum_{j=1}^{n} c_j V(\tilde{x}_j) + \sum_{j=1}^{n} |c_j| A(\tilde{x}_j) + \sum_{j=1}^{n} |c_j| F(\tilde{x}_j), \\ & s.t. \quad \sum_{j=1}^{n} a_{ij} V(\tilde{x}_j) = V(\tilde{b}_i), \\ & \sum_{j=1}^{n} |a_{ij}| A(\tilde{x}_j) = A(\tilde{b}_i), \\ & \sum_{j=1}^{n} |a_{ij}| F(\tilde{x}_j) = F(\tilde{b}_i), \\ & -V(\tilde{x}_j) + A(\tilde{x}_j) + 4/3F(\tilde{x}_j) + S_j = 0, \\ & V(\tilde{x}_j), A(\tilde{x}_j), F(\tilde{x}_j) \text{ and } S_j \geq 0, \; j = 1, \ldots, n. \end{aligned}$$

This is a crisp linear programming problem with $3m + n$ constraints and $4n$ variables, but many of elements of the constraints matrix are zero. After obtaining the optimal solution of (6), if for some $\tilde{x}_j$ we have $c_j = A(\tilde{x}_j) - 2/3F(\tilde{x}_j) \geq 0$, then we can find a canonical symmetric trapezoidal representation for $\tilde{x}_j$. But if for some $\tilde{x}_j$ we have $c_j = A(\tilde{x}_j) - 2/3F(\tilde{x}_j) < 0$, then we can find a canonical symmetric quasi-trapezoidal representation for $\tilde{x}_j$ as defined in section 2.2.

Some examples are presented here to show the applicability of the canonical representation of fuzzy numbers.

Example 4.2 Consider the following FLPFV:

$$\begin{aligned} min : \quad & \tilde{Z} = 2.5\tilde{x}_1 + 3\tilde{x}_2, \\ s.t. \quad & 7\tilde{x}_1 - 3\tilde{x}_2 - \tilde{x}_3 = \tilde{23}, \\ & -3\tilde{x}_1 + 8\tilde{x}_2 - \tilde{x}_4 = \tilde{34}, \\ & \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq \tilde{0}, \end{aligned} \quad (9)$$

where $\tilde{23} = (22, 24, 3, 3)$, $\tilde{34} = (33, 35, 3, 3)$. The classical linear programming problem(6), corresponding to (7) have the following optimal solution:

$V(\tilde{x}_1) = 6.0851,\ A(\tilde{x}_1) = 0.2128,\ F(\tilde{x}_1) = 0.1596,\ V(\tilde{x}_2) = 6.5319,\ A(\tilde{x}_2) = 0.1702,\ F(\tilde{x}_2) = 0.1277,\ S_1 = 5.6596,\ S_2 = 6.1915,$ and other variables are zero.

Simple calculations give the following canonical symmetrical trapezoidal representation for (7):

$$\tilde{x}_1 = (5.9787, 6.1915, 0.3192, 0.3192),$$

$$\tilde{x}_2 = (6.4468, 6.617, 0.0851, 0.0851).$$

Example 4.3 Consider the following FLPFV:

$$\begin{array}{ll} min: & \tilde{Z} = 7\tilde{x}_1 + 2\tilde{x}_2, \\ s.t. & \tilde{x}_1 - 2\tilde{x}_2 - \tilde{x}_3 = \tilde{b}_1,\ \tilde{x}_1 + 5\tilde{x}_2 - \tilde{x}_4 = \tilde{b}_2, \\ & \tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq \tilde{0}, \end{array}$$

where and $\tilde{b}_2 = (4, 6, 1, 3)$ is a trapezoidal fuzzy number and

$$\tilde{b}_1(x) = \begin{cases} 1 - x^2 & if\ -1 \leq x \leq 0, \\ (-1/4)x + 1 & if\ \ 0 \leq x \leq 4, \\ 0 & otherwise, \end{cases}$$

for which $V(\tilde{b}_1) = 0.4,\ A(\tilde{b}_1) = 0.9333,\ F(\tilde{b}_1) = 0.6953$ as in Example 3 of [3]. $\tilde{b}_2 = (4, 6, 1, 3)$ is a trapezoidal fuzzy number. The classical linear programming corresponding to (8) have the following optimal solution:

$V(\tilde{x}_1) = 2.2812,\ A(\tilde{x}_1) = 0,\ F(\tilde{x}_1) = 0,\ V(\tilde{x}_2) = 0.6104,\ A(\tilde{x}_2) = 0.3333,\ F(\tilde{x}_2) = 0.2,\ V(\tilde{x}_3) = 0.6603,\ A(\tilde{x}_3) = 0.2666,\ F(\tilde{x}_3) = 0.2953,\ S_1 = 2.2812,\ S_2 = .0104,\ V(\tilde{x}_4) = A(\tilde{x}_4) = F(\tilde{x}_4) = S_3 = S_4 = 0.$

Simple calculations give the following canonical symmetrical trapezoidal representation for the optimal solution (8):

$$\tilde{x}_1 = (2.2812, 2.2812, 0, 0),$$

$$\tilde{x}_2 = (0.4104, 0.8104, 0.4, 0.4),$$

$$\tilde{x}_3 = (0.5906, 0.73, 0.5906, 0.5906).$$

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