Recurent Rules-Based Fuzzy Decision-Making and Control

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Abstract – Many engineering problems involve decision-making and control processes. These are complex processes, entrangling rational, emotional, social and cultural factors. Including intelligence in mechanical systems and devices may require the inclusion of decision-making abilities. The development of decision-making and control capabilities similar to those of the humans by classical *If-Then* fuzzy logic rules is oversimplifying. We propose a new type of fuzzy logic rules that improve the modeling of the decision process by mimicking the human iterative process of decision-making. Based on such rules, a new class of fuzzy control systems is introduced. We investigate the analytical aspects of systems based on such rules. Applications are envisaged.

Key-Words: control theory, decision-making, fuzzy system, recurrent rules

1. Introduction

Humans in general and engineers frequently need to make decisions. Including intelligence in mechanical systems and devices may require the inclusion of decision-making abilities. Decision processes are complex and involve rational inferences, experience-derived patterns of behavior, various types of knowledge, emotional, social and cultural factors. Human decision-making may involve strategies based on various types of knowledge and on experience. The development of decision-making and control capabilities similar to those of the humans is a goal for intelligent systems, including mechatronic systems. The use of fuzzy logic has been frequently advocated as a tool to achieve this goal. However, using simple *If-Then* fuzzy logic rules to mimic human decision-making and control capabilities is, however, oversimplifying.

We propose a new type of fuzzy logic rules that improve the modeling of the decision process by mimicking the human iterative process of decision-making. Then, we investigate the analytical aspects of systems based on such rules.

We deal with fuzzy systems with confidence degrees attached to the rules, where the confidence (or belief) degrees are not fixed, but are dependent on the input, moreover may be dependent on the defuzzified output values. We consider that such systems are more appropriate to model human decision-makers, because humans have a tendency to adjust their reasoning based on "preliminary" results they obtain, in an iterative process. Also, we suggest that decision-making systems and control systems based on recurrent fuzzy rules may be implemented in mechatronic systems to increase their intelligence. Moreover, we consider that in some circumstances, replacing the classic fuzzy control by fuzzy controllers based on recurrent rules may increase the flexibility and quality of the control in mechatronic systems. A first attempt to establish fuzzy systems based on recurrent rules has been presented in [1]. More details have been provided in [2], but the topic remains up today only partly investigated.

The paper organization is as follows. In the next section, the rational of using recurrent rules is discussed. In Section 3, the basic model is introduced, while in Section 4, the analysis of the model is presented. An example is discussed in Section 5. In Section 6, an algorithm to determine the solutions of the recurrent rules is presented. In the final section, conclusions are derived and further work pointed to.

2. The rational of using recurrent rules

The rational to use recurrent rules is that the human decider adjusts the decision making by an iterative procedure, based on evaluation of results during the decision process. Thus, the human decision-making is a dynamic process, through which the confidence in the rule may evolve step by step. The change of the rule may be just momentarily, or may affect the process in the future, thus becoming "acquired knowledge".

There is an extensive literature on the confidence the decision-makers (DMs, in brief) have in their decisions. The confidence depends on a multitude of factors [3]. The decision-making process seems always to involve a confidence assignment to the decision made. Therefore, when such processes are modeled by rules, and specifically by fuzzy-valued rules, the rules should include confidence degrees. In [3], it is argued that the DMs have more confidence when having more information. This would imply that the confidence degree should be increasing for input values that have higher truth degrees, because this reflects more information.

On the other hand, there are proofs that the outcome of a decision influences the confidence in the decision. In an environment of uncertainty about the probabilities of several lotteries, decisions have been reported to take into account the values at stack [4] of probability information and prominence of outcomes). The same author, summarizing the literature, shows that *"Most theories that seek to describe decision making under uncertainty assume that the attractiveness of each option is closely related to its subjectively expected utility, that is, possible outcomes (or derivatives of these)…"*

The recurrent manner humans make decisions is intuitively well known. The recurrence is used not only to correct decisions, but also because of perceived change of information produced by the temporal result yielded by applying the rule, by the attractiveness of the result, as put by [4].

The way of establishing a reasonable confidence function is not our goal here. Such a goal is far beyond the possibility to deal with in a single paper. It is also an elusive goal to determine an "optimum" model for the confidence function. Such model may not exist. Indeed, it is well known that there are various types of approaching decisions, based on the strategy and the personality of the decider. For example, it is known that speed/accuracy decisions are influenced by personal abilities as well as by situational variables [5]. Therefore, we will make choices for the confidence degree function only for exemplifying purposes.

The organization of this paper is as follows. The basic model is presented in the subsequent section. The analysis of the model and the derivation of the characteristic function are presented in the third section. In the fourth section are presented several examples and selected results for specified cases. The last section is devoted to conclusions.

3. Basic model

Systems with recurrent rules, introduced in [1], have the rules in the form:

If x *is* $\tilde{A}_k$, *then* y *is* $b_{i(k)}$, with confidence $\gamma(x, y_0)$ (1)

where y_0 is the defuzzified output value, and the function $\gamma(\cdot): \mathbf{R}^2 \to [0,1]$ represents the element generating the recurrence. Here, we have considered a recurrent rule-based system, built on the frame of a 0-type Sugeno-kind system.

In case of multi-input Sugeno-kind systems, the rules have the form:

If x_1 *is* $\widetilde{A}_{k_1}$ *AND* x_2 *is* $\widetilde{A}_{k_2}$ *AND . . . AND* x_n *is* $\widetilde{A}_{k_n}$,

Then y *is* $b_{i(k_1, k_2, \ldots, k_n)}$, with confidence $\gamma(x_1, x_2, \ldots, x_n, y_0)$. (2)

These systems have not been studied in detail until now. In this section, we present several analysis aspects related to recurrent fuzzy systems.

Recurrent fuzzy systems based on Mamdani-type rules are introduced in a similar manner to the above-defined recurrent systems based on Sugeno-type rules. A recurrent Mamdani-type system is defined by rules as:

If x *is* $\widetilde{A}_k$, *Then* y *is* $\widetilde{B}_{i(k)}$, with the confidence $\gamma(x, y_0)$, (3)

where $y_0 \in \mathbf{R}$ is the defuzzified output value, and $\widetilde{B}_{i(k)}$ are the output fuzzy sets. The confidence degree functions must fulfill the conditions:

$$\forall x \in X, \forall y_0 \in Y: \quad \gamma(x, y_0) \in [0, 1] \qquad (4)$$

where X is the universe of discourse for x, and Y is the universe of discourse for y.

To be able to determine the value of the function $\gamma(x, y_0)$, the defuzzification operator has to be applied firsthand, to determine y_0. The center of gravity (*c.o.g.*) defuzzification method will be used throughout this paper. The general rule (1) becomes:

If x *is* A_k, *Then* y *is* B_k with confidence $\gamma_k(x, cog\, y)$.

Because the result of the rule recursively depends on itself, the output membership function is implicitly, not explicitly produced by the rule. Therefore, the rules have either to be analytically "solved" for the result, or they should be iteratively applied until the result is approached by approximation. This makes the systems with recurrent rules be much more difficult to solve than usual fuzzy rules. Solving the rules deserves a special attention in this case. The next sections are devoted to this problem.

4. Model analysis

We use the convention that the confidence degrees of the rule act by multiplication, $\mu(\cdot)\cdot\gamma(\cdot)$, not according to the minimum rule, $\min(\mu, \gamma)$. Therefore, the defuzzified output, denoted by z, is:

$$z = cog\, y = \frac{\sum_k \mu_k(x)\cdot\gamma_k(x, z)\beta_k}{\sum_k \mu_k(x)\cdot\gamma_k(x, z)} \Rightarrow$$
$$z\cdot\sum_k \mu_k(x)\cdot\gamma_k(x, z) = \sum_k \mu_k(x)\cdot\gamma_k(x, z)\cdot\beta_{i(k)}(x) \qquad (5)$$

Equation (5) is a non-linear equation in z. The equation becomes linear if the function $\gamma(\cdot)$ is constant in z, $\gamma_k(x, z) = \gamma_k^*(x) + ct$. However, this case is of low interest, because the system is not recurrent, but a mere modification of the usual Sugeno fuzzy system.

If the function $\gamma(\cdot)$ is linear in z, $\gamma_k(x, z) = \varphi_k(x)z + \psi_k(x)$, where the functions in x, $\varphi_k(x), \psi_k(x)$, are not restricted in form, then

$$z^2\cdot\sum_k \mu_k(x)\cdot\varphi_k(x) + z\cdot\sum_k \mu_k(x)\cdot\psi_k(x) =$$
$$= z\cdot\sum_k \mu_k(x)\cdot\varphi_k(x)\cdot\beta_{i(k)}\cdot\sum_k \mu_k(x)\cdot\psi_k(x)\cdot\beta_{i(k)} \qquad (6)$$

Equation (6) has two solutions, expressed by radicals. The radicals change the character of the usual response function of the system (characteristic function), which is a rational function [6]. Because of the existence of two solutions, the response of the system may be considered ambiguous. How to decide between the two solutions is to be dealt in the context of the problem: in a decision problem, specific methods to decide between the solutions may be accepted. Also, the existence of two solutions may be interpreted as a model of a "decision switching process" in decision-making problems.

Notice that the solution of the equation generating z may be unacceptable. Indeed the solutions may fall outside the universe of discourse of the output.

Also notice that the existence of several solutions implies a strategy to choose the "right" solution. Choosing a single solution out of several personalizes the interpretation of the rule outcome. This may be interpreted as the differences in decision as occur when several deciders are involved in solving a problem.

There is an issue that deserves special consideration. Because the output is the solution of a non-linear algebraic equation, the output may be either real or complex. This is a new feature, not encountered in classic fuzzy systems, and should be dealt with care. According to the application in hand, the existence of a complex solution may be considered unacceptable. This is the case in decision-making problems. In such situations, restrictive conditions for the coefficients to keep the solution real-valued must be imposed. In other applications, like control, complex solutions for real-valued inputs might be acceptable. In such applications, the discussed feature of the recurrent fuzzy system class is a significant bonus.

5. Numerical example

We assume a Sugeno-kind of recurrent fuzzy system, with beliefs in rules represented by functions $\gamma(\cdot)$ that are constant in z and linear in x, having the form

$$\gamma_k(x, z) = \varphi_k(x)z + \psi_k(x) = \alpha_k(x - x_k)z + \delta_k x + \lambda_k \quad (7)$$

where α_k, δ_k are coefficients. As an example, the rules describing a control system may read:

If the error x is High (A_1), then I will make a correction y that is Large (B_1) with the confidence

$\gamma_1(error\ x, cog(correction\ y)) = 3.(x-8)z + 1.2x + 0.5$.

If the error x is Average (A_2), then I will make a correction y that is Moderate (B_2) with the confidence:

$\gamma_2(error\ x, cog(correction\ y)) = 0.7(x-6)z + 0.5x + 0.3$.

If the error x is Small (A_3), then I will make a correction y that is Low (B_3) with the confidence

$\gamma_3(error\ x, cog(correction\ y)) = 0.3(x-2)z + 0.2x + 0.1$.

As a matter of example, assume that the input membership functions are isosceles triangular, and the outputs (B_i) are singletons, as in Figure 1.

The equations corresponding to the output, for several cases, are listed below. Notice that the rule aggregation may produce *no* real solution for the output. This situation must be avoided, because it means that the system is undefined for some values of the input, or we need to accept complex (imaginary) output values.

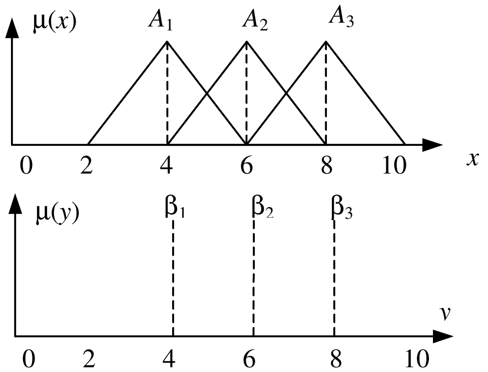

Fig. 1. Input membership functions and output singletons in the example

As discussed, in some cases, complex output values may be of interest, for instance in control problems. An example is the control of a mechanical or electrical impedance value of the controlled system. However, in some applications, the complex output values are unacceptable. Then, we must make sure that the parameters in the confidence degree allow for a real-valued solution. These issues are briefly discussed below.

Case $\gamma(cog\ y)$, that is, the function $\gamma(\cdot)$ does not depend on *x*.

A. The function $\gamma(\cdot)$ is rule-independent (i.e., does not depend on *k*) and linear. We denote $\gamma(z)=\alpha z+\delta$ and we obtain:

$$z^2\alpha\sum_k \mu_k(x)+z\delta\sum_k \mu_k(x)=$$

$$z\alpha\sum_k \mu_k(x)\beta_{i(k)}+\sum_k \mu_k(x)\beta_{i(k)} \quad \Rightarrow$$

$$z^2\left[\alpha\sum_k \mu_k(x)\right]+z\left[\sum_k \mu_k(x)\left(\delta-\beta_{i(k)}\right)\right]-$$

$$-\delta\sum_k \mu_k(x)\beta_{i(k)}=0$$

Because $\gamma(z)=\alpha z+\delta$ represents a confidence degree, the condition

$$\forall z\in Y: \quad \gamma(z)\in[0,1]\Leftrightarrow \alpha z+\delta\in[0,1]$$

must be fulfilled – see equ. (4). If the universe of discourse for the output is an interval of the real line, denoted by $[m,M]$, then the condition is equivalent to $\delta\in[-\alpha m,1-\alpha M]$.

For the system has a real-valued solution, the following condition must be satisfied for all input values:

$$\forall x\in \textit{universe of discourse}:$$

$$\left[\sum_k \mu_k(x)\cdot\left(\delta-\beta_{i(k)}\right)\right]^2+$$

$$+4\alpha\cdot\delta\cdot\sum_k \mu_k(x)\cdot\sum_k \mu_k(x)\beta_{i(k)}\geq 0$$

Because the values of the membership functions are positive, the condition is satisfied for all x provided that $\alpha\cdot\delta\geq 0$. In case $y\in[m,M]$, because $\delta\in[-\alpha m,1-\alpha M]$, the condition $\alpha\cdot\delta\geq 0$ can not be guaranteed in general.

B. For the case of rule-dependent belief functions, $\gamma_k(z)=\alpha_k z+\delta_k$, where α_k,δ_k are coefficients changing from rule to rule (i.e., the function γ is rule dependent), the equation is

$$z^2\sum_k \alpha_k\mu_k(x)+z\sum_k \delta_k\mu_k(x)=$$

$$z\sum_k \alpha_k\mu_k(x)\beta_{i(k)}+\sum_k \mu_k(x)\beta_{i(k)} \quad \Rightarrow$$

$$z^2\left[\sum_k \alpha_k\mu_k(x)\right]+z\left[\sum_k \mu_k(x)\left(\delta-\beta_{i(k)}\right)\right]-$$

$$-\sum_k \delta_k\mu_k(x)\beta_{i(k)}=0$$

C. The case $\gamma_k(z)=\gamma_k\left((z-\beta_k)^2\right)$. One may interpret this confidence degree function as 'the value of the function decreases whenever the final result is far from the respective singleton". Computations are similar to the computation in previous cases.

6. Algorithm for recurrent rules solving

In case the rule can not be analytically resolved, we need an algorithm to iteratively determine the rule outcome. In this section, such an algorithm is proposed. We assume the rules are in the form

If x *is* $\tilde{A}_{k,y}$ y *is* $B_{i(k)}$ with confidence degree $\gamma_k(x,y)$.

The algorithm is:

1. Determine all rules that are activated (fired), that is all the rules in the system above that correspond to the input condition $x\in\tilde{A}_k$.

2. Determine, for all fired rules, the least values x such that $\mu_{\tilde{A}_k}(x) > 0$; let these values be denoted by $x_{k\,\min}$.
3. Determine, for all fired rules, the largest values x, such that $\mu_{\tilde{A}_k}(x) > 0$; let these values be denoted by $x_{k\,\max}$.
4. Determine:
   $a_1 = \min_k (x_{k\,\min}), \quad b_1 = \max_k (x_{k\,\max})$
5. Randomly choose $y_0 \in [a, b]$.
6. For the chosen y_0, determine $\gamma(x, y_0)$.
7. Using the computed $\gamma(x, y_0)$, compute y_1, y_2, the solutions of the characteristic function. If $y_1, y_2 \in [a, b]$, go back to step #6. If no value belongs to the given interval, go back to step #5.

7. Conclusions and further work

In this paper, we have introduced and analyzed a new type of fuzzy logic systems, continuing our previous work [1,2]. Applications to mechatronics of the new class of systems have been envisaged in two ways: by increasing the control capabilities, and by adding decision-making capabilities that are similar to those of human deciders. Robotic systems with a high flexibility in making decisions under difficult conditions, using decision making and control strategies similar to those used by the human operators, may best benefit of the new class of fuzzy systems.

Further work to extend the analysis of the new class of fuzzy systems, of the control and of the decision-making based on these systems is needed. Also, applications based on such systems have to be developed. In this respect, the analysis of the dynamics of loops including such systems should be carefully performed, because chaotic behavior may appear [7,8]. Remarkably, this type of fuzzy systems generalizes classic fuzzy systems in several ways: not only recurrent rules include as a particular case the non-recurrent rules, but these systems may yield complex (imaginary) results starting from real-valued inputs.

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