A new approach in Mechanical Design: a preliminary proposal. Theoretical derivations and potential applications

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Abstract: - In this paper, we propose a new approach in mechanical engineering. This approach is based on conceptual mechanics and we propose a method to modelize and solve problems involving machine analysis and design. This approach would try to be used as a conceptual analysis system or as help for the designer during his decision-making.

Key-Words: - conceptual mechanics, machine design, machine theory, kinematics

1 Introduction

The problems of applied mechanical engineering often involve knowledge and approaches that are difficult to describe in the framework of the usual modeling instruments of classical mechanics [12]. A difficult task is to define an easy method to study complex problems, like selection of the components during the design stage, feasibility of a machine project, etc. We can consider, as an example, the realization process of a machine. During the designing stage, the designer should understand the needs of his "client" and should project a new machine. The first problem is deciding the most suitable technology to gain the desired functionalities, with the desired performances. Just during the designing stage, the designer tries to compare many machine models in his mind and his concept of the "right machine" is always transforming during the designing activity. The first aim of our research is to answer the question: how describing this transformation process with mathematical formulations to construct a new instrument to help the designer? This new instrument is a theory involving mechanics, concepts and transformations of concepts [14]. The second, and perhaps more reachable goal, refers to the differences between the designed machine and the manufactured machine. As a matter of fact, during the manufacturing stage some uncertainties intervene: tools precision, unconsidered physical phenomena, the human factor, etc. So, the realized machine and the designed machine exhibit different behaviors [7]. The reader should also observe that the designed machine exists only in a virtual space: "the space of the designs" while the realized machine exists only in "the real space". To compare these two machines, we should construct a transformation from a space (designs) to another space (real) able to transform a designed machine into a real machine. This transformation should be able to modelize the uncertainty of the manufacturing activity, so should be an instruments for the designer to forecast the behavior of the realized machine or to make the correct decision during his activity. Notice that the behavior of the real machine, when this forecasting instrument is not used, can be known only through measurements. Different measuring systems can give different results, so there differences between the "realized machine" and the "measured machine". These differences can be forecasted by another transformation from the "space of measurements" to the "real space". This new instrument should be a help for the designer to consider the measurement imprecision during his evaluation of the machine performances.

All these aspects of the design, manufacturing and measuring of a machine cannot be easily and rigorously considered only with the help of classical physics, so we are proposing a new approach to help a mechanical engineer during his work, adding new instruments to his classical physical and empirical knowledge. Based on the preliminary works [1-4, 10, 13, 15], and after a theoretic study [10], we propose a more detailed view. Precisely, we define mathematical structures, according to the developed theory [10] of fuzzy kinematics of particles. We propose some potential applications for this approach as well.

2 Fundamental Structures

We introduce a few theoretical concepts to create a general framework to deal with the problem of correspondence between designed, measured and actual systems.

We will work on a space as given in (Eq. 1).

$$E_n(R) = [A \quad V_n(R) \quad \circ \quad \underline{a}] \tag{1}$$

$E_n(R)$ is composed by a structure $A_n(R)$, as defined in Eq. 2, and a scalar product $\circ$ can operate.

$$A_n(R) = \begin{bmatrix} A & V_n(R) & \underline{a} \end{bmatrix} \tag{2}$$

where A is an non-empty set and its element are called points, $V_n(R)$ is an n-dimensional vectorial space on the real field R, while $\underline{a}$ is a function, defined in (Eq. 3) and satisfying the axioms in (Eq. 4).

$$\underline{a} : A \times V_n(R) \rightarrow A \tag{3}$$

$$\begin{array}{ll} 1) \quad \forall P, Q \in A & \exists! v \in V_n \wedge v \equiv \{\overrightarrow{PQ}\} \equiv Q - P \neg \\ & \neg \quad \underline{a}(< P, v >) = Q \\ 2) \quad \forall P, Q, R \in A & v \equiv \{\overrightarrow{PQ}\}, w \equiv \{\overrightarrow{QR}\}, u \equiv \{\overrightarrow{PR}\} \neg \\ & \neg \quad u = v + w \end{array} \tag{4}$$

The operator $\circ$ is a scalar product and it satisfies the (Eq. 5).

$$\circ : V_n(R) \times V_n(R) \rightarrow R \tag{5}$$

The fundamental concept of particle is introduced. X is an Euclidean three-dimensional space and T is an Euclidean one-dimensional space, P is an elementary particle: then an element p of the defined space $X \times T$ can be associated to this particle (Eq. 6).

$$\begin{array}{l} X = E_3(R) = \begin{bmatrix} A^X & V_3^X(R) & \circ & \underline{a}^X \end{bmatrix} \\ T = E_1(R) = \begin{bmatrix} A^T & V_1^T(R) & \circ & \underline{a}^T \end{bmatrix} \\ p \in X \times T \end{array} \tag{6}$$

After having defined a suitable reference system for the space $X \times T$, p can be identified with a set of coordinates like in (Eq. 7).

$$(x, t) \in V_3^X(R) \times V_1^T(R) \tag{7}$$

Then we can write: *P occupies (x,t)* or, concisely, $P(x,t)$.

3 Correspondence function

Two spaces $X^1 \times T^1$ and $X^2 \times T^2$ are defined and they have the same properties of the space $X \times T$. An elementary particle P is considered. Two elements, p_1 and p_2, are associated to it and they belong, respectively, to the spaces $X^1 \times T^1$ and $X^2 \times T^2$. Having defined suitable reference systems, we can write *P occupies* (x_1,t_1) *in the space* $X^1 \times T^1$ and *P occupies* (x_2,t_2) *in the space* $X^2 \times T^2$ or, concisely,

$$\begin{array}{l} P(x_1, t_1) \in V_3^{X^1}(R) \times V_1^{T^1}(R) \quad \wedge \\ P(x_2, t_2) \in V_3^{X^2}(R) \times V_1^{T^2}(R) \end{array} \tag{8}$$

One can ask which is the numerical correspondence between the couple of coordinates (x_1,t_1) and the couple of coordinates (x_2,t_2). To answer this question, the correspondence function $f_{1,2}$ is defined in Eq. 9.

$$\begin{array}{l} f_{1,2} : \left(V_3^{X^1}(R) \times V_1^{T^1}(R)\right) \times \left(R^3 \times R\right) \rightarrow \\ \qquad \rightarrow \left(V_3^{X^2}(R) \times V_1^{T^2}(R)\right) \\ f_{1,2}\left((x_1, t_1) \quad (\mu_{1,2}^x, \mu_{1,2}^t)\right) = (x_2, t_2) \end{array} \tag{9}$$

More generally, a succession of spaces $X^i \times T^i$ is defined and they have the same properties of the just defined space $X \times T$, the elementary particle P is considered again: a succession of elements p_i is associated to it and they belong to the spaces $X^i \times T^i$. Having defined suitable reference systems, we can write *P occupies* (x_i,t_i) *in the space* $X^i \times T^i$ or, concisely,

$$P(x_i, t_i) \in V_3^{X^i}(R) \times V_1^{T^i}(R) \quad , \quad i \in N \tag{10}$$

The respective correspondence functions are, then, defined by the Eq. 11.

$$\begin{array}{l} f_{i,i+1} : \left(V_3^{X^i}(R) \times V_1^{T^i}(R)\right) \times \left(R^3 \times R\right) \\ \qquad \rightarrow \left(V_3^{X^{i+1}}(R) \times V_1^{T^{i+1}}(R)\right) \qquad , \quad i \in N \\ f_{i,i+1}\left((x_i, t_i) \quad (\mu_{i,i+1}^x, \mu_{i,i+1}^t)\right) = (x_{i+1}, t_{i+1}) \end{array} \tag{11}$$

The element p_i can, now, be transformed in the element p_j with the correspondence function $f_{i,j}$. The function $f_{i,j}$ can be generated with a composition of the functions $f_{i,i+1}$ (Eq. 11) or can be directly defined like in the Eq. 12.

$$\begin{array}{l} f_{i,j} : \left(V_3^{X^i}(R) \times V_1^{T^i}(R)\right) \times \left(R^3 \times R\right) \rightarrow \\ \qquad \rightarrow \left(V_3^{X^j}(R) \times V_1^{T^j}(R)\right) \qquad , \quad i, j \in N \\ f_{i,j}\left((x_i, t_i) \quad (\mu_{i,j}^x, \mu_{i,j}^t)\right) = (x_j, t_j) \end{array} \tag{12}$$

To simplify the expressions, a reference element p_k, or a reference space $X^k \times T^k$, can be considered. Then all the elements p_i , and all the spaces $X^i \times T^i$, are referred to it. The element p_1 in the space $X^1 \times T^1$ is chosen as reference. Then, we can write *P occupies* (x_1,t_1) *in the space* $X^1 \times T^1$ *to which corresponds the "position" of* (x_i,t_i) *in the space* $X^i \times T^i$ or, concisely,

$$P(x_1, t_1)_{\mu_{1,i}^x, \mu_{1,i}^t} \quad , \quad i \in N \tag{13}$$

4 Discrete Movement

A succession of elements p_{ih} is considered and it is ordered with growing times t_{ij} (Eq. 14): it represents a discrete movement of P in the space $X^i \times T^i$.

$$
\begin{array}{l}
(x_{i1},t_{i1}),(x_{i2},t_{i2})\ldots(x_{ih},t_{ih}),(x_{ih+1},t_{ih+1})\ldots(x_{im},t_{im}) \\
t_{i1} < t_{i2} < \ldots t_{ih} < t_{ih+1} < \ldots < t_{im} \\
with \quad m \in N
\end{array}
\tag{14}
$$

Then, we can write *P moves along (x_{ij},t_{ij}) in the space $X^i \times T^i$* or concisely,

$$
P\{(x_{ih},t_{ih})\}_{h=1,2\ldots m} \tag{15}
$$

The correspondence function f_{ij} between the movement $\{(x_{ih},t_{ih})\}_{h=1,2\ldots m}$ and the movement $\{(x_{jh},t_{jh})\}_{h=1,2\ldots m}$ can be defined (Eq. 16).

$$
\begin{array}{l}
f_{i,j}:\{(V_3^{X^i}(R)\times V_1^{T^i}(R))\}_{h=1,2\ldots m}\times\{R^3\times R\}_{h=1,2\ldots m}\to \\
\quad\to\{(V_3^{X^j}(R)\times V_1^{T^j}(R))\}_{h=1,2\ldots m} \\
f_{i,j}\left(\{(x_{ih},t_{ih})\}_{h=1,2\ldots m} \quad \{(\mu^x_{i,j,h},\mu^t_{i,j,h})\}_{h=1,2\ldots m}\right)= \\
\quad=\{(x_{jh},t_{jh})\}_{h=1,2\ldots m}
\end{array}
\tag{16}
$$

If the i-movement and the j-movement are defined with a different number of elements p_{kh}, the correspondence function f_{ij} takes the form of the Eq. 17.

$$
\begin{array}{l}
f_{i,j}:\{(V_3^{X^i}(R)\times V_1^{T^i}(R))\}_{h=1,2\ldots mi}\times\{R^3\times R\}_{h=1,2\ldots mc}\to \\
\quad\to\{(V_3^{X^j}(R)\times V_1^{T^j}(R))\}_{h=1,2\ldots mj} \\
f_{i,j}\left(\{(x_{ih},t_{ih})\}_{h=1,2\ldots mi} \quad \{(\mu^x_{i,j,h},\mu^t_{i,j,h})\}_{h=1,2\ldots mc}\right)= \\
\quad=\{(x_{jh},t_{jh})\}_{h=1,2\ldots mj} \\
mc=\max(mi,mj)
\end{array}
\tag{17}
$$

One can observe that, if the i-movement and j-movement are defined in the same space and are considered two representations of the same movement, because they are defined by a different number of elements, the correspondence function f_{ij} can be seen as an interpolator from the representation with a low number of points to a representation with a higher number of points. In a more general sense, the correspondence function can be used to construct new representations, starting from a base representation.
The second problem regards the temporal order of the sequence. This property can be removed to simplify our theory, but its practical role is useful in the computation of other non fundamental properties of the movement like the speed or the acceleration. The example in Fig. 1 can be considered.

$$
\begin{tikzcd}
(x_{11},0) & (x_{12},1) & (x_{13},2) \arrow[d, "g"] & \arrow[dd, bend left, "f"] \\
(x_{21},0) \quad (x_{22},1.5) & & (x_{23},1) \arrow[d, "r"] & (x_{24},2) \\
(x_{21},0) & (x_{23},1) & (x_{22},1.5) & (x_{24},2)
\end{tikzcd}
$$

Fig. 1 Example of generation of an ordered movement

In Fig. 1 three successions of elements p_{kh} are considered, the first represents a movement in the space $X^1 \times T^1$, the second is a disordered succession of elements p_{kh}, while the third represents a movement in the space $X^2 \times T^2$. The correspondence function f_{12} is decomposed in an intermediate correspondence function g and an ordering function r. This approach is useful when the correspondence between the two movement takes a temporal fuzzy, or statistical, form. In this situation, the presence of an intermediate correspondence function g can be observed. These hinted concepts are exactly exposed in the Eq. 18.

$$
\begin{array}{l}
g_{i,j}:\{(V_3^{X^i}(R)\times V_1^{T^i}(R))\}_{h=1,2\ldots mi}\times\{R^3\times R\}_{h=1,2\ldots mc}\to \\
\quad\to\{(V_3^{X^j}(R)\times V_1^{T^j}(R))\}_{h=1,2\ldots mj} \\
g_{i,k}\left(\{(x_{ih},t_{ih})\}_{h=1,2\ldots mi} \quad \{(\gamma^x_{i,k,h},\gamma^t_{i,k,h})\}_{h=1,2\ldots mc}\right)= \\
\quad=\{(x'_{kh},t'_{kh})\}_{h=1,2\ldots mj} \\
r_{k,j}:\{(V_3^{X^i}(R)\times V_1^{T^i}(R))\}_{h=1,2\ldots mc}\to \\
\quad\to\{(V_3^{X^j}(R)\times V_1^{T^j}(R))\}_{h=1,2\ldots mc} \\
r_{k,j}\left(\{(x'_{kh},t'_{kh})\}_{h=1,2\ldots mc}\right)=\{(x_{jh},t_{jh})\}_{h=1,2\ldots mc} \\
f_{ij}=g_{ik}\circ r_{kj}
\end{array}
\tag{18}
$$

5 Continuous Movement

The discrete movement defined in the Eq. 15 is considered. A continuous movement is defined from a discrete movement with a suitable correspondence function (Eq. 19). It represents nothing else than an interpolator/extrapolator, which generates a continuous function, from a sequence of elements. This operation can be performed, i. e. with a fuzzy function, where the independent variable is the time of the element p_j of the codomain of c_{ij}.

$$
\begin{array}{l}
c_{i,j}:\{(V_3^{X^i}(R)\times V_1^{T^i}(R))\}_{h=1,2\ldots m}\times\{R^3\times R\}_{h=1,2\ldots m}\times V_1^{T^j}(R) \\
\quad\to(V_3^{X^j}(R)\times V_1^{T^j}(R)) \\
c_{i,j}\left(\{(x_{ih},t_{ih})\}_{h=1,2\ldots m} \quad \{(\mu^x_{i,j,h},\mu^t_{i,j,h})\}_{h=1,2\ldots m} \quad t_j\right)=(x_j,t_j)
\end{array}
\tag{19}
$$

Applying continuously the correspondence function of Eq. 19, a continuous movement is obtained (Eq. 20).

$$
\begin{gathered}
c_{i,j}:\left\{\left(V_3^{X^i}(R)\times V_1^{T^i}(R)\right)\right\}_{h=1,2\ldots m}\times\left\{R^3\times R\right\}_{h=1,2\ldots m}\times \\
\times I\left(V_1^{T^j}(R)\right)\rightarrow\left(I\left(V_3^{X^j}(R)\right)\times I\left(V_1^{T^j}(R)\right)\right) \\
c_{i,j}\left(\left\{\left(x_{ih},t_{ih}\right)\right\}_{h=1,2\ldots m}\quad\left\{\left(\mu^x_{i,j,h},\mu^t_{i,j,h}\right)\right\}_{h=1,2\ldots m}\quad I\left(t_j\right)\right)= \\
=\left(I\left(x_j\right),I\left(t_j\right)\right)
\end{gathered}
\tag{20}
$$

We can write P *moves continuously along* $(I(x_j), I(t_j))$ *in the space* $X^j \times T^j$ or concisely $P(t_j)=(x_j, t_j)$. A correspondence function between the movement $P(t_i)=(x_i, t_i)$ and the movement $P(t_j)=(x_j, t_j)$ can be defined (Eq. 21).

$$
\begin{gathered}
f_{i,j}:\left(I\left(V_3^{X^i}(R)\right)\times I\left(V_1^{T^i}(R)\right)\right)\times\left(I\left(R^3\right)\times I(R)\right)\rightarrow \\
\rightarrow\left(I\left(V_3^{X^j}(R)\right)\times I\left(V_1^{T^j}(R)\right)\right) \\
f_{i,j}\left(\left(I\left(x_i\right),I\left(t_i\right)\right)\quad\left(\mu^x_{i,j},\mu^t_{i,j}\right)\right)=\left(I\left(x_j\right),I\left(t_j\right)\right)
\end{gathered}
\tag{21}
$$

At the present stage of this paper we have gained an interesting result: the possibility of transforming a continuous movement in another continuous movement with suitable correspondence functions. Undoubtedly, more generality can be achieved, but this aim will be referred to future extended works. The exposed theory brings useful results for a set of applications, which will be hinted in the next section.

6 Applications

Consider, as first example, the process to realize a cam system [4-5, 9]. The designer selects a cam profile and projects the cam with the desired profile [8]. To project the proper profile, the designer needs to predict the behavior of the cam. The designer typically uses a model with ideal connections between the cam and the follower, but without considering backlash, compliances (stiffness and friction) in the connections [6]. Some more expert designers are able to estimate backlash, stiffness and friction in the connections, so they can use more precise models. However, this estimation is always affected by errors, because the manufacturing activity is affected by uncertainty in tools, ability of the workers, errors during assembly stage, etc. To know the value of the imprecision, we can measure the performances of the realized cam system, but different measuring systems give different results. Therefore, another problem is what is the relation between measures and the real machine performances. More general, we face the described problems for every sort of machine, so the first proposed application consists in rigorously codifying the relation between the model of a mechanical system during the design phase, the model after the physical realization of the system, the measured behavior and the real behavior. The difference between the exposed models is intuitive, but a rigorous coding is not easy. This coding allows a representation in a single software and allows to clarify the relationship between the models with explicit correspondence functions. A machine, from a mechanical viewpoint, is a system able to produce mechanical work, that is a force and a movement. Because our theory has, for the present, only a kinematical nature, we will point out only the movement production. From a kinematical viewpoint, the machine behavior can be described only with its generated movement. Usually, this movement can be described by the movement of a rigid body, but, for a wide category of machines, a movement of an elementary particle is sufficient. Because our theory treats, for the present, only elementary particles, we will consider only this class of machines.

During the design phase, compliances and control errors are not considered, or are approximately considered: this machine is described by the movement $P(t_1)=(x_1, t_1)$. After the machine realization, compliances and control errors can be evaluated with some precision: this machine is described by the movement $P(t_2)=(x_2, t_2)$. The behavior of the considered model, even if it is accurate, diverges from the measurements, which are described by the movement $P(t_3)=(x_3, t_3)$. Even the best measurements differ from the real behavior of the machine, which is described by the unknown movement $P(t_4)=(x_4, t_4)$. Now the correspondences between the models can be exposed with the Fig. 2.

$$
\begin{tikzcd}
P(t_1)=(x_1,t_1) \arrow[d, bend left, "f_{12}"] \\
P(t_2)=(x_2,t_2) \arrow[d, bend left, "f_{23}"] \\
P(t_3)=(x_3,t_3) \arrow[d, bend left, "f_{34}"] \\
P(t_4)=(x_4,t_4)
\end{tikzcd}
$$

Fig. 2 Correspondence between different models of a mechanical machine

Some observations are underlined. The correspondence function f_{23} can be fuzzy [11, 16] (or statistical) depending on the nature of the measurements. Because the model developed after the realization of the machine is extremely accurate uses a fuzzy (statistical) modelizing approach, the correspondence function f_{12} also exhibits a fuzzy (or statistical) nature. Furthermore, the best knowledge of the machine behavior is, usually, based on measurements, so the correspondence function f_{34} remains always unknown.

Often, commercial bonds impose some realizing choices of a machine; so, in spite of a extremely precise design, the real machine exhibits an unexpected behavior. Therefore, it can be interesting, as well, to carry out a rather imprecise design, so as to reduce unnecessary efforts. But how much should the design be imprecise? To answer this question, we propose a second application of our theory. The second application regards on the evaluation of the quality of a machine starting from the quality of its components.

In this application we point out the central component of a machine: the transmission. This component transforms the motion produced by the motor in the desired motion. The better the transmission is, closer the produced motion is to the desired motion. The motion produced by the motor is described by the movement $P(t_1)=(x_1, t_1)$. A transmission composed by a series of two components is considered. The output motion of the first component is described by $Q(t_1)=(x_1, t_1)$, while the output motion of the second component is described by $R(t_1)=(x_1, t_1)$. So the transmission can be described by the Fig. 3.

$$\begin{tikzcd}
P(t_1)=(x_1,t_1) \arrow[d, bend left, "t_{PQ}"] \\
Q(t_1)=(x_1,t_1) \arrow[d, bend left, "t_{QR}"] \\
R(t_1)=(x_1,t_1)
\end{tikzcd}$$

Fig. 3 Desired behavior of a machine

The real machine exhibits a different behavior, and the measured behavior is described in the space $X^2 \times T^2$ (Fig. 4).

$$\begin{tikzcd}
P(t_1)=(x_1,t_1) \arrow[d, bend left, "t_{PQ}"] \arrow[r, "f^1_{12}"] & P(t_2)=(x_2,t_2) \arrow[d, bend left, "t'_{PQ}"] \\
Q(t_1)=(x_1,t_1) \arrow[d, bend left, "t_{QR}"] \arrow[r, "f^2_{12}"] & Q(t_2)=(x_2,t_2) \arrow[d, bend left, "t'_{QR}"] \\
R(t_1)=(x_1,t_1) \arrow[r, "f^3_{12}"] & R(t_2)=(x_2,t_2)
\end{tikzcd}$$

Fig. 4 Differences between measured and desired behavior for a machine

The correspondence functions f^1_{12}, f^2_{12} and f^3_{12} represents the correspondence between desired and measured behavior, respectively, of the motor, of the first transmission component and of the second transmission component. We suppose that the designer should plan the output motion of the motor: therefore he can decide the level of precision for his activity, only evaluating the quality of the transmission components.

7 Conclusions

A theory to describe movement of elementary particles was exposed. A hint on two very felt applications in mechanical engineering was showed. We reckon that an overall evaluation of the obtained results should pass through a close examination of the proposed applications. Therefore we will evaluate the opportunity to extend our theory, considering dynamical aspects or rigid body motions.

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