# Measure of Quality in State Space

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*Abstrac:* Phase trajectory, as the presentation of dynamic systems is a common and well known description of second order homogenous systems, and it is often used for analysing of nonlinear systems. Sensitivity to system parameters changes can result in distinction among some nominal phase trajectory. This article deals with question how to define boundaries in phase plane that include permitted variations. Evaluation is done by statistical analysis and is illustrated by case example of human locomotion.

*Key words*: System Theory, Phase Plane, Clustering Algorithm, Human Locomotion

## 1. Introduction

The terms phase plane and norm are basic facts and starting points from which we developed the presentation of error in phase plane, using norms and cluster classification. The system that is observed in this article is not modelled. Instead, the phase trajectory which presents output from an observed system is obtained as a result of measurements and is presented by a periodically repeated curve. The procedure for the determination and classification of errors that depend on system parameter variations is illustrated in human locomotion example. Varying parameters in human gait are different anthropologic characteristic of observed persons. A representative group of subjects is selected and gait kinematics data are measured. After processing measured data, the hip, the knee and the ankle angles, angle velocities and accelerations, as functions of time, are obtained. By considering human gait as periodic motion, obtained signals are also periodic. Therefore, they are observed within a time interval, which is denoted as one gait cycle. Synthesising the angle and the angle velocity curves on the same graph, we obtained phase trajectories that are carrying the angle and the angle velocity information at the same time.

The values of these trajectories in particular moments, considered as discrete events, are processed. Obtained data are classified into clusters associated with a certain quality factor.

The quality factor presents measure of variation of observed data (points in phase plane) from a statistically defined referent point. Following the selection of data between clusters, each cluster will contain data of the same quality. The number of clusters depends on wanted preciseness of the evaluation as a quality norm. Normatives of quality i.e. clusters defined in that way, are presented in a form of concentric ellipses. From these ellipses, the area of existing and allowed variations from the referent point, which is located on a nominal phase trajectory, is easily observed.

On the other hand, if the known model is used, it is possible to define referent phase trajectory in advance, based on this ideal model. That gives us possibility of quality classification that is especially valuable in the construction of different objects, vehicles, robots etc.

## 2. Error in correlation with norm

Let's consider the output of some model as nonstationary time function f(t). If n subsequent measurements are performed on the model, obtained functions are: $f_1(t)$, $f_2(t)$,.., $f_n(t)$. In the classical approach, measured data are determined by mean value $f_{mn}(t)=[f_1(t)+f_2(t)+...+ f_n(t)]/n$ and standard deviation $\sigma_f(t)$, as shown in Fig.1.

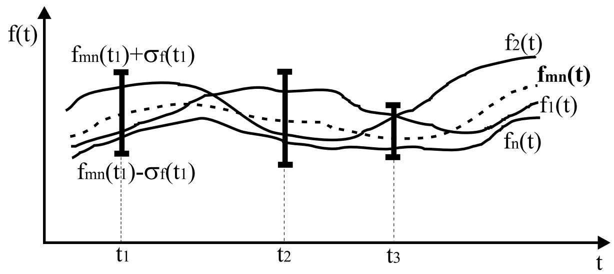


Fig. 1: Classical interpretation of nonstationary ensemble

In this work we will visualize the data in phase plane [1] in a manner that synthesises the graphs of measured function f(t) and it's derivation df(t)/dt, onto a unique graph.
As a result of synthesis, the trajectories will be obtained in phase plane, with x axis presenting f(t) and y axis presenting df(t)/dt, as shown in Fig. 2.

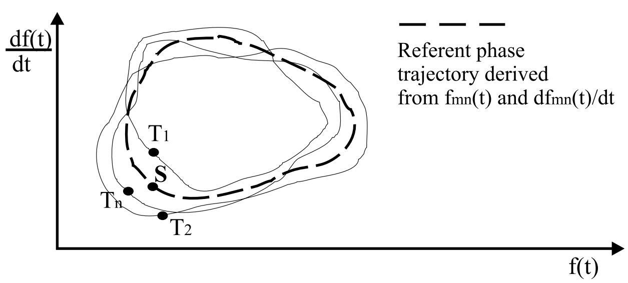


Fig.2. Phase trajectories in phase plane

Phase trajectories are associated to system with varying parameters. Change of any system parameter results with change of phase trajectory. Trajectories in a phase plane will be observed for specific time discrete events, whose selection depends on the physical system we explore. For example, if we choose time instant $t=t_x$, we will get the points from phase plane for that time (fig.2.):

- referent point $S\ [\, f_{mn}(t_x),\ df_{mn}(t_x)/dt \,]$
- point $T_1\ [\, f_1(t_x),\ df_1(t_x)/dt \,]$
- point $T_2\ [\, f_2(t_x),\ df_2(t_x)/dt \,]$
- ...
- point $T_n\ [\, f_n(t_x),\ df_n(t_x)/dt \,]$

Obtained points in phase plane represent data that will be classified using a cluster statistical method. Error, which is denoted as distinction of an observed point from a referent one, is used as classification criteria.

Before we start with classification process, let us describe the term of norm in state space.
Generally, norm is a single number, which gives an overall measure of the size of a vector, a matrix, a signal or a system. The most commonly used norm is the Euclidean vector norm [1]. The Euclidean norm of a particular vector **x** in n–dimensional space is the inner product:

$$\textit{norm}\ \mathbf{x} = \|\mathbf{x}\| = (\mathbf{x},\mathbf{x})^{1/2} \tag{1}$$

Generally, an orthonormal basis is understood, so that Eq. (1) is usually interpreted as

$$\|\mathbf{x}\| = (\mathbf{x},\mathbf{x})^{1/2} = \left(\sum_{i=1}^{n} x_i^2\right)^{1/2} \tag{2}$$

Vector **x** is n-dimensional vector in n-dimensional Euclidean space, which can also be presented using notation R(n) or $R^n$ . Further, we shall consider the $R^2$ Euclidean inner-product space. In an inner-product space, the distance between two vectors is defined as

$$\rho(\mathbf{x},\mathbf{s}) = \|\mathbf{x}-\mathbf{s}\| = (\mathbf{x}-\mathbf{s},\mathbf{x}-\mathbf{s})^{1/2} \tag{3}$$

Suppose we consider the distance between vectors in $R^2$. Let's have coordinates $s_1$ and $s_2$ in some orthonormal basis, and let r be the distance between x and s. In other words,

$$\|\mathbf{x}-\mathbf{s}\|^2 = (x_1-s_1)^2 + (x_2-s_2)^2 = r^2 \tag{4}$$

Clearly, Eq. (4) defines a circle of radius r centered at $(s_1,s_2)$. Fig. 3 gives a graphical interpretation of Eq. (4).

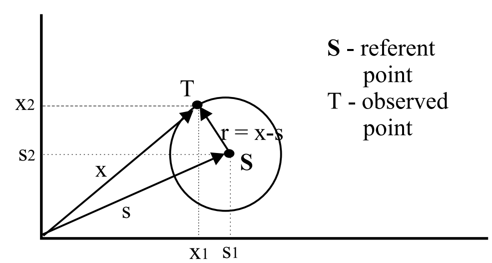


Fig.3. Graphical interpretation of the Euclide norm

The graphical interpretation of difference between two vectors **x** and **s**, will be denoted as norm r. Therefore, the distinction of observed point T from referent point S in phase plane is determined by the norm r, as shown in Fig.3. Each point $T_i$ in phase plane is associated with norm $r_i$, see Fig.4.

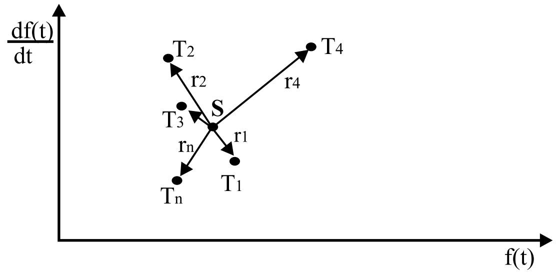


Fig. 4: Norms joined to points in phase plane

After defining how many clusters we will use, and the performing of cluster algorithm regarding each point's norm, we will get a points distribution between clusters. Cluster 1. will contain points with the least error of distinction from the referent point. The clusters with higher numbers will contain points with higher error i.e. the variation from referent point, see Fig. 5.

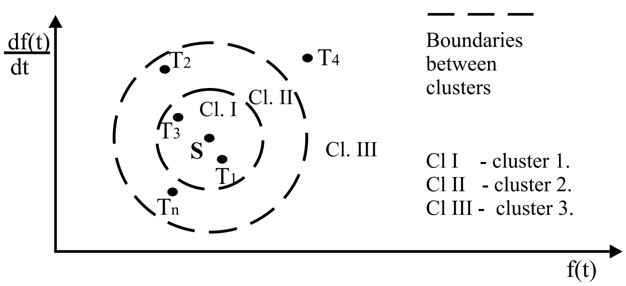


Fig.5: Points distribution within clusters

## 3. K-mean clustering algorithm

Generally, cluster is a set of similar objects or data. All clustering algorithms (including K-mean method) are statistical methods and iterative procedures for searching through a set of all possible clustering to find one that fits the data reasonably well [2]. The clustering algorithm is expected to group similar data together (in the same cluster) such that it reduces the within-cluster variation.

The following is a description of K-means algorithm:

A data matrix $\{A(I,J),\ 1 \le I \le M,\ 1 \le J \le N\}$ is given with M cases and N variables. The partition P(M,K) is composed of the clusters 1,2,...,K. Each of M cases lies in just one of the K clusters. The mean of Jth variable over the cases in the Lth cluster is denoted by B(L,J). The number of cases in L is N(L). The distance between the Ith case and Lth cluster is

$$D(I,L)=\left[\sum_{J=1}^{N}\left(A(I,J)-B(L,J)\right)^2\right]^{1/2} \tag{5}$$

The error of the partition is:

$$e[P(M,K)]=\sum_{I=1}^{M} D[I,L(I)]^2 \tag{6}$$

where L(I) is cluster containing the Ith case and D[I,L(I)] denotes the Euclidean distance between I and cluster mean of the cluster containing I. The general procedure is to search for a partition with small e by moving cases from one cluster to another. The search ends when no such movement reduces e.

STEP 1. Assume initial clusters 1,2,...,K. Compute the cluster means B(L,J); ( $1 \le L \le K$, $1 \le J \le N$ ) and the initial error e.

STEP 2. For the first case, compute for every cluster $L \neq L(1)$:

$$\frac{N(L)D(1,L)^2}{N(L)+1}-\frac{N[L(1)]D[1,L(1)]^2}{N[L(1)]-1}, \tag{7}$$

which presents the increase in error in transferring the first case from cluster L(1), to which it belongs at present, to cluster L. If the minimum of this quantity over all $L \neq L(1)$ is negative, transfer the first case from cluster L(1) to this minimal L, adjust the cluster of L(1) and the minimal L, and add the increase in error (which is negative) to e[P(M,K)].

STEP 3. Repeat Step 2 for the Ith case ($2 \le I \le M$).

STEP 4. If no movement of a case from one cluster to another occurs for any case, stop. Otherwise, return to Step 2.

## 4. Case example – human locomotion

Human movement measurements [3] were recorded by a TV Camera-PC system [4],[5] for 18 persons. The experiment is presented in Fig.6.

The marker positions were measured and processed [6], [7]. Using this data, the hip, knee and ankle angles, angle velocities and accelerations are obtained and presented in phase plane. The statistical classification of the quality of locomotion was performed.

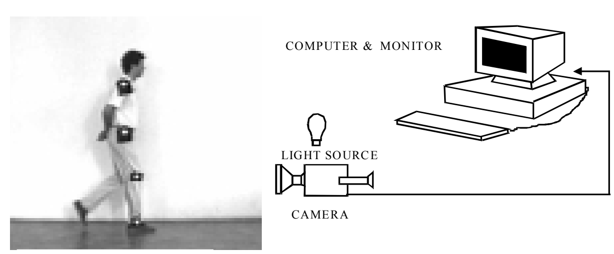


Fig. 6: Camera – PC system

The previously described clustering method will be done according to hip angle ($\Theta$) and hip angle velocity ($\dot{\Theta}$),shown in Fig. 7.a. and 7.b.

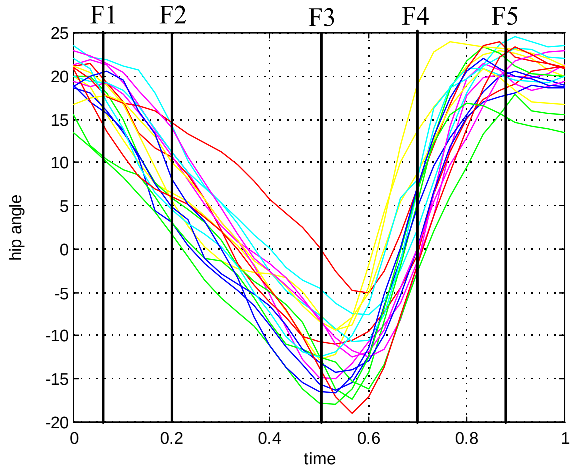


Fig 7.a. Hip angle as a function of time (for all 18 subjects)

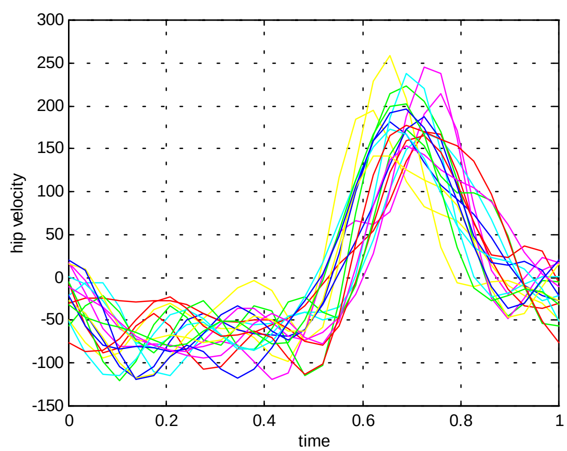


Fig 7.b. Hip angle velocity as a function of time (for all 18 subjects)

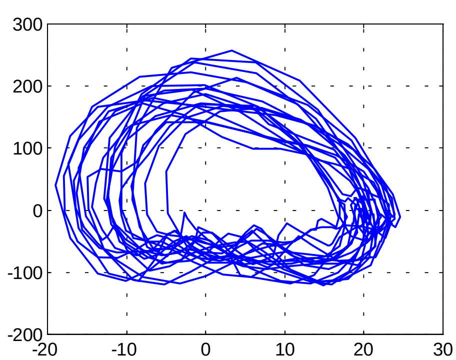

Fig 8. Phase-plane for hip angle - hip angle velocity (all 18 subjects)

The time interval of one second represents one gait cycle. Hip angle and hip angle velocity presented in phase plane are shown in Fig. 8.

Five discrete events that present dominant gait phases are chosen for observation [8] and marked by lines in Fig. 7.a.: initial contact (F1), loading response (F2), terminal stance (F3), mid swing (F4), terminal swing (F5). In this case example we will be focused on discrete event F2, the loading response.

### 4.1 Data standardisation and presentation in phase plane using norms

The angle and angle velocity values for each person were standardised by well known statistical law [2], [9]:

$$\Theta_{st\_p} = (\Theta_p - \Theta_{mn}) / \sigma_{an}$$

$$\dot{\Theta}_{st\_p} = (\dot{\Theta}_p - \dot{\Theta}_{mn}) / \sigma_{ve}$$

where:

- $\Theta_p$ hip angle for person p
- $\dot{\Theta}_p$ hip angle velocity for person p ($\dot{\Theta}_p \equiv d/dt(\Theta_p)$)
- $\Theta_{st\_p}$ standardised value of $\Theta_p$
- $\dot{\Theta}_{st\_p}$ standardised value of $\dot{\Theta}_p$
- $\Theta_{mn}$ arithmetic mean of $\Theta$
- $\dot{\Theta}_{mn}$ arithmetic mean of $\dot{\Theta}$
- $\sigma_{an}$ standard deviation of hip angle $\Theta$
- $\sigma_{ve}$ standard deviation hip angle velocity $\dot{\Theta}$

(person p=1 to 18)

As the result of standardisation, the referent point will become (0,0) point in phase plane.

Standardised data for each discrete event are presented in angle-angle velocity phase plane. Fig.9. presents phase plane for observed discrete event F2.

Vector $\vec{r}_p$ is joined to each particular person. The absolute value of these vectors is norm:

$$r_p = \left\| \vec{r}_p \right\| = \sqrt{\Theta^2_{st\_p} + \dot{\Theta}^2_{st\_p}} \qquad (8)$$

The norm $r_p$ will be fundamental value for clustering the quality of human locomotion. Table 1. contains angle and velocity values and associated norms for all 18 subjects.

| F2-LOADING RESPONCE | | | | | |
|---|---|---|---|---|---|
| pers. | $\Theta_p$ | $\Theta_{st\_p}$ | $\dot{\Theta}_p$ | $\dot{\Theta}_{st\_p}$ | $r_p$ |
| 1 | 7.5 | -0.70 | -37.0 | 1.34 | 1,52 |
| 2 | 13.8 | 0.76 | -76.8 | -1.10 | 0,76 |
| 3 | 18.0 | 1.73 | -110.3 | -1.32 | 2,18 |
| 4 | 11.7 | 0.27 | -42.36 | 1.15 | 1,18 |
| 5 | 7.0 | -0.65 | -55.48 | 0.67 | 0,94 |
| 6 | 4.4 | -1.42 | -104.0 | -1.09 | 1,79 |
| 7 | 9.4 | -0.26 | -113.4 | -1.43 | 1,46 |
| 8 | 13.4 | 0.66 | -78.7 | -0.17 | 0,68 |
| 9 | 13.9 | 0.78 | -82.0 | -0.29 | 0,83 |
| 10 | 15.9 | 1.24 | -27.6 | 1.69 | 2,10 |
| 11 | 6.0 | -1.05 | -87.7 | -0.50 | 1,13 |
| 12 | 7.5 | -0.70 | -82.1 | -0.29 | 0,76 |
| 13 | 13.0 | 0.57 | -71.5 | 0.08 | 0,58 |
| 14 | 16.2 | 1.31 | -79.0 | -0.18 | 1,33 |
| 15 | 6.5 | -0.93 | -65.6 | 0.3 | 0,98 |
| 16 | 7.0 | -0.81 | -29.2 | 1.63 | 1,82 |
| 17 | 4.3 | -1.44 | -73.6 | 0.01 | 1,44 |
| 18 | 13.3 | 0.64 | -114.9 | -1.49 | 1,62 |

Table 1. Angle and velocity values and associated norms ( all 18 subjects)

### 4.2 Clustering procedure of kinematic data

As already mentioned, 18 persons are classified into clusters, using the value of norms $r_p$. The K-mean clustering method, which is contained in the Statistica 5.0. software package, is performed and 3 clusters are pre-defined:

CLUSTER I: **H**igh **Q**uality **G**ait cluster (**HQG**), contains the closest points to the referent point which is the center of phase plane

CLUSTER II: **N**ormal **G**ait cluster (**NG**)

CLUSTER III: **L**ow **Q**uality **G**ait cluster (**LQG**), contains the most distant points to the center of phase plane

After cluster algorithm is performed, cluster's centres and members are obtained.

The boundary between two adjacent clusters is taken as the arithmetic mean of that cluster's centers. Defining clusters' boundaries, normative areas are established. For example, points inside the second cluster boundaries represent normal gait values. Obtained results for second gait phase F2 (loading response) are presented in Table 2. and Fig. 9.

We should also mention that this method is inadequate for some specific cases. One extremely atypical subject, with an enormous high value of norm r, completely changes cluster boundaries and effects distribution of normative boundaries.

| F2 | CLUSTER I: **HQG** | CLUSTER II: **NG** | CLUSTER III:**LQG** |
|---|---|---|---|
| CLUSTER'S CENTER | 0.793 | 1.391 | 1.976 |
| STANDARD DEVIATION σ | 0.139 | 0.172 | 0.194 |
| CLUSTER'S BOUNDARIES | (0,1.092) | (1.092, 1.683) | (1.683,+∞) |
| CLUSTER'S MEMBERSHIP | 7 PERS. (38.8%) | 7 PERS. (38.8%) | 4 PERS. (22.4%) |

Tab 2. Cluster's results for F2 (loading response)

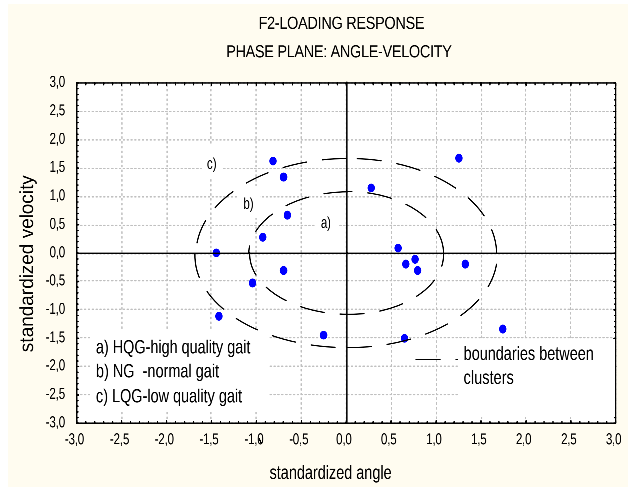


Fig 9.Clustering of variables $r_p$ for F2 (loading response)

To prevent this happening, it is necessary to increase the number of observed subjects and exclude those extremely atypical subjects by doing preselection. In the end, they will be presented as a point in a phase plane outside of normative curves, i.e. included into the cluster with the highest number.

## 5. Conclusion

Development of this method has resulted from the need to evaluate a dynamic system's accuracy i.e. it's time response, and it's derivations: not only the first, but also second and third as well.

Applied to human gait analysis, or other moving objects, this method enables us to evaluate quality of movement, movement derivation (speed), speed derivation (acceleration) and acceleration derivation (shock). Shock is especially important in evaluating gait quality, and this method provides the ability to make such an evaluation.

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