Travelling Wave Influence in Lubricated Journal Bearings

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Abstract: - In this paper an investigation on the influence of travelling wave in cylindrical journal bearings is proposed. The travelling wave is assumed to be generated by means piezoelectric actuators placed on the external surface on the bearing. This actuators exciting modes of the bearing can influence the fluid film force in order to control the dynamic behavior of the journal.

Key-Words: - Journal bearing, travelling wave, piezoelectric actuator, oil whirl, instability

1 Introduction
Fluid bearing are useful and necessary in the basic rotating machinery used in industries. They allow:

- Optimization of the impedance to the rotor for stability;
- Passage of critical speed and normal function with vibrations and force transmitted to low-amplitude bearing;
- Adaptation to moderate misalignment;
- The possibility to remove calories by fluid, of which a small part is used for lubrication.

In this paper, a new method for the optimization of the impedance is proposed. This method is based on the control of the fluid film force by means piezoelectric actuators which induce a travelling wave on the internal surface of the bearing. The amplitude of this wave can become until hundred of microns and to modify the fluid film force in the gap in order to avoid oil whirl phenomena, for example. Furthermore this method can be used for supporting journal with angular velocity equal to 0.

2 Analysis
As well knows, the pressure distribution in journal bearings is obtained integrating the following Reynolds equation:

\[
\frac{\partial}{\partial x} \left( \frac{H^2}{\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial x} \left( \frac{H^2}{\mu} \frac{\partial p}{\partial x} \right) = 12(V_x - V_y) + 6(U_x - U_y) \frac{\partial H}{\partial x} + 6H \frac{\partial}{\partial x} (U_x + U_y)
\]

(1)

The variables used in (1) are indicated in Figg. 1 and 2 while the parameters \( \mu \) and \( \bar{p} \) are dynamic viscosity and pressure, respectively.

Only for “short” or “long” bearing it is possible to integrate the equation (1) even though many authors have proposed approximate fluid film force for finite journal bearings. A travelling wave, obtained exciting a particular mode of the bearing, induce a velocity of the points on the internal surface of the bearing such that \( V_1 \neq 0 \) (see Figg. 3 and 4). The solution to equation (1) may be obtained analytically for large \( L \) (= length of bearing) and for small \( L \).
The first case is \( \frac{\partial p}{\partial z} = 0 \) and in the latter is \( \frac{\partial p}{\partial x} = 0 \).

In this case by the equation (1) is obtained the following ordinary differential equation:

\[
\frac{C^3}{L} \frac{\partial}{\partial \xi} \left( \frac{h^3}{\mu} \frac{\partial p}{\partial \xi} \right) = 12C \omega (v_2 - v_1) + 6 \frac{C^2}{R} \omega (u_1 - u_2) \frac{\partial h}{\partial \theta} + \\
6 \frac{C^2}{R} \omega h \frac{\partial}{\partial \theta} (u_1 + u_2)
\]

having put

\[
H = Ch , V_1 = C \omega \varpi, V_2 = C \omega \varpi_2 , U_1 = C \omega \xi , \\
U_2 = C \omega \xi_2 , z = L \xi, x = R \theta
\]

where

- \( h \) = thickness gap
- \( u_i \) = dimensionless tangential velocity
- \( v_i \) = dimensionless radial velocity
- \( p \) = dimension pressure
- \( R \) = Bearing Radius
- \( C \) = radial clearance
- \( L \) = bearing length
- \( A \) = travelling wave amplitude
- \( n \) = lobe numbers
- \( \omega^n \) = nth mode frequency

\[ A \cos \omega^{(n)} t \]  

(3)

2.1 Travelling wave in cylindrical bearing

In Fig.5 are indicated the first twelve natural modes of vibration. Some modes excite exclusively radial displacement. If an external source is:
The radial motion is:

\[ s_{r(t)}(\theta, t) = A \cos n \vartheta \cos \omega^{(n)} t \]  

(4)

If one other external source of the type:

\[ A \cos(n \vartheta - \pi / 2) \]  

(5)

is summed to the equation (3) the following travelling wave is obtained:

\[ s_{tot(t)}(\theta, t) = A \cos(n \vartheta - \omega^{(n)} t) \]  

(6)

Assuming the bearing thickness less than bearing radius can be shown that the radial and tangential displacement of the points on the internal bearing surface are express by the relations:

\[ s_{r(t)}(\theta, t) = A \cos(n \vartheta - \omega^{(n)} t) \]
\[ s_{\tau(t)}(\theta, t) = \frac{nbA}{2R} \sin(n \vartheta - \omega^{(n)} t) \]  

(7)

Where b and R are thickness and radius of the cylindrical bearing. The velocity is obtained deriving the (7).

\[ v_{r(t)}(\vartheta, t) = \omega^{(n)} A \sin(n \vartheta - \omega^{(n)} t) \]
\[ v_{\tau(t)}(\vartheta, t) = -\omega^{(n)} \frac{nbA}{2R} \cos(n \vartheta - \omega^{(n)} t) \]  

(8)

While oil thickness is given by the relation:

\[ h = c(1 + \varepsilon \cos \vartheta) + A \cos(n \vartheta - \omega^{(n)} t) \]

In Fig.6 is shown the trajectory of the points belonging to internal surface of the bearing.

3 Results and conclusion

The equation (2) leads to the following relation for the pressure filed:

\[ p_{CC}(\vartheta, \tau) = \psi a^{-1} \frac{\partial (hs)}{\partial \vartheta} + 2\psi as \]
\[ -\frac{\tau}{12h^2} \left( \frac{\partial u}{\partial \vartheta} - u \frac{\partial h}{\partial \vartheta} \right) + 2\nu_2 \]  

(3)

Substituting in (3) the following relations:

\[ \nu_2 = \dot{\varepsilon} \cos \vartheta + \varepsilon (\dot{\varphi} - 1) \sin \vartheta \]
\[ u_2 = \frac{R}{C} + \dot{\varepsilon} \sin \vartheta - \varepsilon \dot{\varphi} \sin \vartheta \]

\[ h = c(1 + \varepsilon \cos \vartheta) + A \cos(n \vartheta - \omega^{(n)} t) \]
\[ v_1 = \psi as \]
\[ u_1 = -\psi a^{-1} s \]
\[ S_{\tau} = C S_{\tau} \]
\[ S_{\nu} = C S_{\nu} \]
\[ V_1 = \omega^{(n)} A \sin(n \vartheta - \omega^{(n)} t) \]
\[ U_1 = -\omega^{(n)} A \cos(n \vartheta - \omega^{(n)} t) \]
\[ s_{\tau} = \beta_{\tau} \cos(n(\vartheta + \varphi) - \psi \tau) \]
\[ s_{\nu} = \beta_{\nu} \sin(n(\vartheta + \varphi) - \psi \tau) \]
\[ A_{\tau} = C \beta_{\tau}, A_{\nu} = C \beta_{\nu} \]
\[ \gamma = \frac{C}{R}, \psi = \frac{\omega^{(n)}}{\omega} \]
\[ a = \frac{\beta_{\tau}}{\beta_{\nu}} \]
\[ p = \frac{C^2}{6L^2 \omega \mu} \bar{p} \]
\[ \tau = \omega \xi \]
and making any quantitative considerations an explicit formula that describe the pressure field in the short journal bearings is obtained:

\[
p(\vartheta,t) = \frac{2(\psi \alpha_s - v_2) + \frac{\partial h}{\partial \vartheta}}{12h^3}
\]

(4)

Or rendering explicit the numerator of the foregoing equation (4):

\[
p_{CC}(\vartheta, \tau) = \frac{(1-2\phi)e \sin \vartheta - 2e \cos \vartheta}{12(1 + e \cos \vartheta + \beta_n \cos(n(\vartheta + \phi) - \psi \tau))^3} + \frac{\beta_n (2\psi - n) \sin(n(\vartheta + \phi) - \psi \tau)}{12(1 + e \cos \vartheta + \beta_n \cos(n(\vartheta + \phi) - \psi \tau))^3}
\]

(5)

For \( \beta_n = 0\) the (5) becomes:

\[
p_{CC}(\vartheta, \tau) = \frac{(1-2\phi)e \sin \vartheta - 2e \cos \vartheta}{12(1 + e \cos \vartheta)^3}
\]

(6)

That is the well known expression of the pressure field in short journal bearing obtained by Ocvirk.

In Fig. 7 is shown the pressure field for a time course of one period. The lobe numbers is equal to five and \( \varepsilon = 0 \). It's possible to observe that the travelling wave can influence the pressure field in order to control the dynamic behaviour of the rotor.

References:

Fig.7_Rotating pressure field for n=5