Use of Non-Circular Gears in Pressing Machine Driving Systems

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Abstract: - The paper describes a new approach in non-circular gears design, based on the mathematical description of the meshing progress. The teeth profiles are generated by numerical integration of a differential equation, which describes the contact point displacement, along the line of action, as a function of the pitch lines shape and of the pressure angle.
In the typical arrangement, a pair of non-circular gears is used to drive a slider-crank mechanism, whose cursor follows a prescribed law of motion, thus working as a function generator. A procedure of inverse kinematics is illustrated, to determine the variable transmission ratio and then the pitch curves, once given the geometry of the slider-crank mechanism and the distance between the gears centres of rotation.
An application of the method is described, where the ram of a mechanical press is driven according to a cinematically optimized law of motion. First step is the pitch lines synthesis, starting from the prescribed output curve. The teeth profiles are then generated on the primitives previously determined.
A virtual analysis is performed to validate the method.

Key-Words: - Non-Circular Gears, Pitch Line Synthesis, Function Generator, Mechanical Presses, Virtual Prototyping

1 Introduction

Variable radius gears are essential in many industrial applications: automatic equipment in printing presses, textile industry, packaging machines, quick-return mechanisms, pumps, flow meters, flying shears and other applications where there is a need for a purely mechanical speed variation control during the working cycle [1-3]. Their diffusion, however, is very limited, since, with the exception of elliptical gears, only recently CAD-CAM technologies allow the implementation of a fully integrated design and manufacturing process.
Up to the present, several methods have been developed to design and manufacture non-circular gears. Generally they consist in defining the mathematical model of a cutter and then reproducing its pure rolling movement along the variable radius pitch curves [4-6].
The paper presents a new approach in non-circular gear design, based on the mathematical description of the contact point displacement along the line of action. The meshing evolution is described as a function of the pitch line radius and of the pressure angle, by means of a differential equation, whose integration generates the teeth profiles.
The method is applied to design the prototype of a pressing machine, whose ram is driven, according to an optimized law of motion, by a pair of non-circular gears, coupled to a slider-crank mechanism. At first, the pitch curves synthesis is performed by means of an inverse kinematical analysis of the system. The teeth profiles are then generated by assuming the pressure angle to be constant along each tooth, but variable from tooth to tooth. In that way, the conditions in which the differential equation is integrated are kept optimal along the primitive.
A CAD model of the gears is created and a virtual kinematical analysis of the whole mechanism performed, in order to validate the method.

2 Synthesis of the Pitch Lines

Starting point of the pitch lines synthesis is the prescribed law of motion of the piston *s(t)*. In the application described here, the purpose is to provide the ram of a pressing machine for deep metal forming with a cinematically optimized law of motion. For this reason, the requested output curve should have a reduced cycle time in comparison with the curve of a traditional pressing machine.
The typical pressing law consists of three phases: loading, forming and removing the part. In order to maximize the process productivity, the first and the third stage must be as short as possible.

Besides, the quality of the final product is strongly affected by the ram velocity during the forming phase. The best product characteristics are achieved if the process is slow enough to avoid too much work-hardening, but not so slow as to prevent a correct lubrication during the metal forming process. Starting from these considerations and on the basis of a FEM simulation, Doege [7, 8] suggests to provide the ram with a law of motion similar to the curve represented in figure 1. The curve refers to a prototype of pressing machine, whose ram has a stroke of 80 mm, one tenth of the stroke of the press built at the Institute for Metal Forming and Metal Forming Machine Tools of Hanover University. That is the curve we want to reproduce by designing a pair of non-circular gears as driving system, coupled to a slider-crank mechanism.

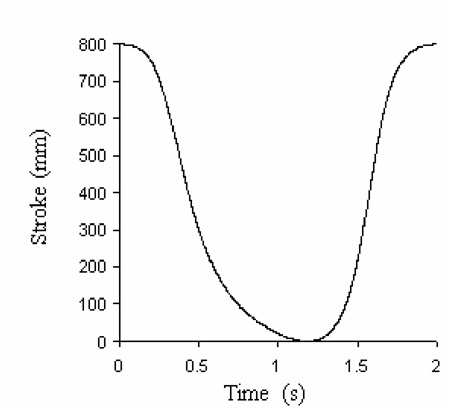

Fig.1 - Requested output curve

First step is the pitch lines synthesis. With reference to the scheme of figure 2, the relation between the displacement of the connecting-rod small end, identified with the point D, and the rotational angle δ on the driven gear is:

$$s_D(\delta)=r\left[1-\cos(\delta)-\frac{1}{\lambda}\left(1-\sqrt{1-\lambda^2\sin^2(\delta)}\right)\right] \qquad (1)$$

where λ is the crank-to-rod ratio, equal to 0.5 in the application described here, r=40 mm the crank length, ω the constant angular velocity of the driving gear, Ω the variable angular velocity of the driven one.

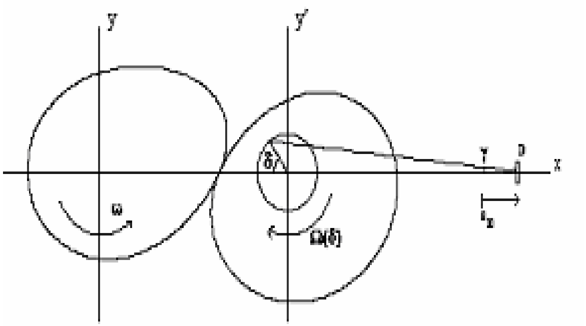

Fig.2 - Schematic representation of the system

By coupling the analytical law $s_D(\delta)$ and the numerical one $s_D(t)$, the time dependence of the rotational angle of the driven gear is established. Once known δ(t), it is possible to correlate this angle to the rotational angle θ of the driving gear, being the time dependence θ(t) the following:

$$\theta(t)=\omega\cdot t=\frac{2\pi}{T}\cdot t \qquad (2)$$

where T is the period of the motion, equal to 2 seconds in this application.

The derivation of the variable transmission ratio, as a function of the angle θ, is now immediate, being, by definition:

$$\tau(\theta)=\frac{\Omega(\theta)}{\omega}=\frac{d\delta(\theta)}{d\theta} \qquad (3)$$

In figure 3 the transmission ratio law is shown. It varies between 0.48 and 2.05, being equal to 1.00 its average value.

Fig.3 - Transmission ratio law

By imposing the condition of pure rolling of the primitives during the motion, a first relation between the pitch lines radii is found. In fact, the linear velocity of the actual contact point is the same on both primitives:

$$R_1(\theta)\cdot\omega = R_2(\delta(\theta))\cdot\Omega(\delta(\theta)) \qquad (4)$$

By combining the last two equations, the transmission ratio can be expressed as:

$$\tau(\theta)=\frac{\Omega(\theta)}{\omega}=\frac{R_1(\theta)}{R_2(\delta(\theta))} \qquad (5)$$

where $R_1(\theta)$ and $R_2(\delta(\theta))$ are the radii of the primitives at the instant t=θ/ω.

In order to determine the shape of the pitch lines a second relation is needed. For this reason, the distance Δr between the centres of rotation of the gears must be assigned. In this application a constant value of 100 millimetres has been chosen. The second relation is then given, being:

$$R_1(\theta)+R_2(\delta(\theta))=\Delta r \qquad (6)$$

The mathematical model of the pitch lines is now complete. The curves generated in this application, by starting from the requested output curve of figure 1, are represented in figure 4.

The process of variable-radius pitch lines synthesis requires that a series of constraints are respected [9, 10]. Note that these conditions are implicitly taken into account in this application, since the cycle time of the pressing machine is assigned as period of rotation to both the driven and the driving gear.

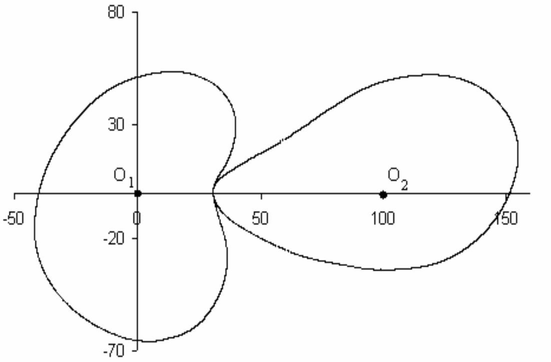

Fig.4 - Pitch lines generating the output curve of fig.1

In particular, since the average transmission ratio is equal to one, the pitch lines have the same length and each tooth of the driving gear always meshes with the corresponding tooth on the driven one.
Once determined the pitch lines shape, the teeth profiles can be generated.

3 Determination of Teeth Profiles

Several methods have been proposed by various authors in order to study and design non-circular gears.
In the method described here, the analytical determination of constant pressure-angle teeth profiles on variable-radius gears is based on the description of the meshing evolution. The mathematical model is based on the following differential equation:

$$\frac{dy}{dq} = \cos^2(a)\left[r(q) - \frac{dr}{dq}\tan(a)\right] \qquad (7)$$

where *dy* is the elemental displacement, projected on the y-axis, of the contact point along the line of action, in correspondence of a rotation dq of the primitive. Figure 5 represents the general scheme of meshing progress, on which Danieli [11] bases the derivation of equation 7.
This is the key equation of the method, since it allows the generation of meshing profiles on gears of any shapes, once the law of the pitch line radius r(θ) is defined and the constant pressure angle α assigned.
By integration of equation 7, starting from the intersection point of a tooth profile with the pitch line, the meshing point displacement on the contact segment is obtained, while the primitive of the driving gear rotates with a given value of angular velocity. From the actual contact point two different points on the conjugate profiles are derived, by means of a coordinate transformation based on opportune rotation matrices.
Once established the contact point between the first teeth pair on the pitch lines and given the diametral pitch, the starting integration points for any tooth can be determined.

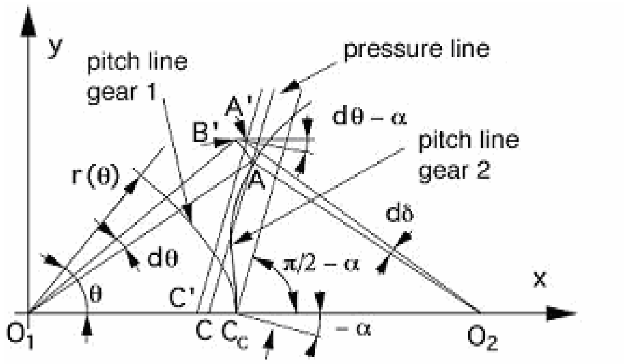

Fig.5 - Schematic representation of teeth meshing

In order to establish when the integration process must stop, two conditions are checked: the first concerns the maximum radial growth of each tooth, the second is used to avoid undercutting.
In particular, the integration procedure stops if the actual tooth height, in radial direction, beyond the pitch line, equals the addendum value, considered, by analogy to standard gears, as proportional to the pitch. The integration process also stops if the distance between the actual contact point and the gears centres of rotation is actually stationary. In fact, if the distance reaches a maximum or a minimum value, undercutting conditions occur.
By applying the method described above to the pitch lines of figure 4 and assuming a constant pressure angle of 20°, irregular teeth profiles are generated. The bad results can be observed in figure 6, where the driving gear teeth are represented.

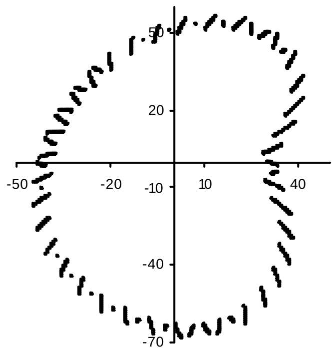

Fig.6 – Driving gear with constant pressure angle

A great part of the profiles is irregular, while many teeth fail at all. The bad performance of the method is due to the pitch lines radius variability. In fact, since the tangent line to the primitives in the contact point is variable, the pressure angle must be varied from tooth to tooth, so that the angle between the pressure line and the perpendicular to the pitch lines has always the same value.

For this reason a correction value must be added or subtracted from the basic pressure angle value.

A subroutine has been implemented to calculate the value of correction, computed as the angle between the line perpendicular to the primitive at the intersection point with the actual tooth profile and the local radial direction. As a consequence, the pressure line inclination is assumed constant within a certain tooth profile, but it generally varies from a profile to the other of a same tooth, and from tooth to tooth.

Obviously, two meshing teeth on different gears must have the same pressure angle, thus receiving the same correction.

Figures 7 and 8 show the teeth profiles generated by the proposed method on the driving and the driven pitch lines respectively. Both gears have 36 teeth and an average pressure angle of 20°.

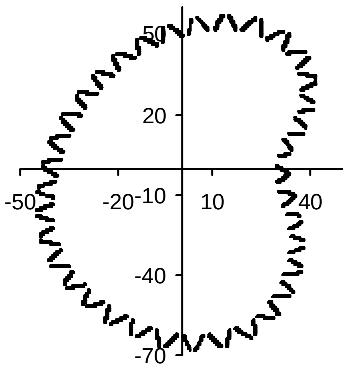

Fig.7 – Driving gear with variable pressure angle

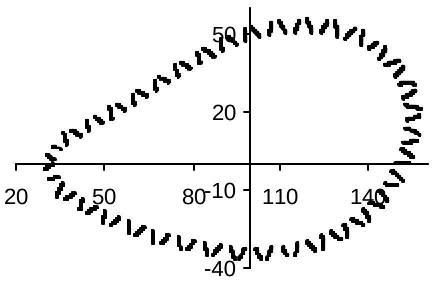

Fig.8 – Driven gear with variable pressure angle

Note that, thanks to the pressure angle variation, the teeth profiles are well-defined also where the pitch lines are particularly different from a circular shape.

In order to check if the continuity of power transmission is assured, an ad hoc formula has been introduced to evaluate how the contact ratio varies along the primitives:

$$c = 1 + \frac{\Delta q_{i-1,i}}{\Delta q_{i-1}} + \frac{\Delta q_{i,i+1}}{\Delta q_{i+1}} \qquad (8)$$

where:

- $\Delta\theta_{i-1,i}$ is the rotation of the gear 1 while the meshing evolves from the beginning of the contact on the i^{th} tooth to the end on the i-1^{th} one;
- $\Delta\theta_{i,i+1}$ is the rotation of the gear 1 while the meshing evolves from the beginning of contact on the i+1^{th} tooth to the end on the i^{th} one;
- $\Delta\theta_{i-1}$ is the i-1^{th} tooth angle-of-action;
- $\Delta\theta_{i+1}$ is the i+1^{th} tooth angle-of-action.

By applying this formula to the gears of figure 7 and 8, the meshing continuity shows to be assured, being the contact ratio grater then one along the whole pitch line, as shown in figure 9.

In this application the method generates spur gears working in a cinematically correct way.

However, the contact ratio has a value lightly higher than one in the region where the pitch lines radius variability is particularly severe.

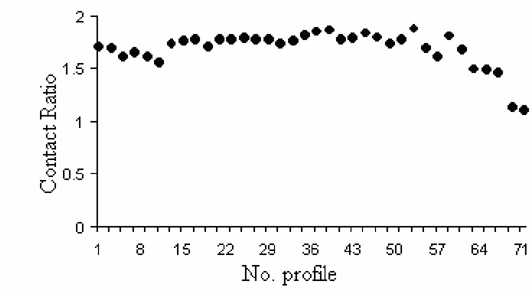

Fig.9 – Contact ratio

This situation can be improved by modifying the design parameters of the slider-crank mechanism, in order to obtain more regular primitive curves.

3 Virtual Prototyping

The output data have been supplied to a CAD software, thus generating the solid model of the gears, whose primitives are represented in figure 4. A procedure guides the profile extension in the fillet region, in order to avoid interference during the meshing.

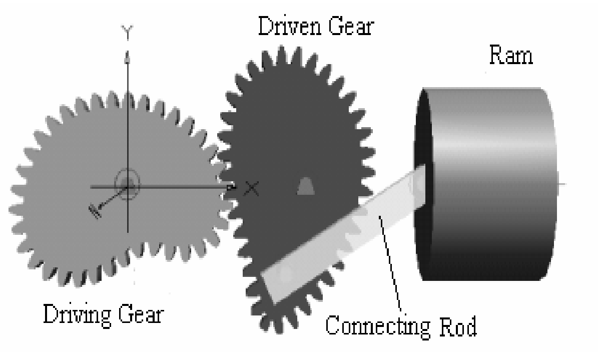

Fig.10 – Virtual prototype

The virtual mechanism, consisting of the slider-crank arrangement, driven by the pair of non-circular gears, has been built using a virtual prototyping software. In figure 10 the virtual prototype of the whole system is shown.

A simulation has been performed by providing the driving gear with a constant angular velocity of 30 r.p.m. and the ram law of motion monitored. The output curve, shown in figure 11, is similar to the curve of figure 1. Therefore, the virtual cinematic analysis seems to validate the method.

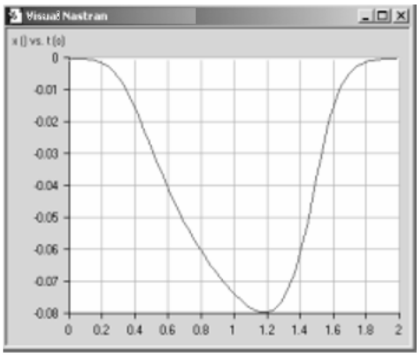

Fig.11 – Virtual output curve

4 Conclusion

The paper shows the actual results of the research developed at the University of Calabria about non-circular gears. The teeth profiles are generated by means of the mathematical description of the contact point displacement along the line of action.

The method has been used to design a purely mechanical driving system of a pressing machine.

At first, the variable radius pitch lines have been designed in order to obtain an optimized output curve. Then the teeth profiles have been design and the solid model of the gears built.

A virtual prototype of the mechanism has been obtained and a kinematical analysis simulated in order to validate the method.

Further studies will concern the design of helical gears, in order to improve the meshing conditions, and the implementation of a CIM procedure in order to build a real prototype.

Nomenclature:

α pressure angle
c contact ratio
δ rotational angle of the driven gear
D centre of the connecting rod small-end
λ crank-to-rod ratio
r radius of the pitch line
θ rotational angle on gear 1 (driving gear)
τ transmission ratio
ω speed of the driving gear
Ω speed of the driven gear

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