On the Design of State Observers for Flexible Link Mechanisms

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Abstract: - This paper presents a versatile approach to the synthesis of accurate state observers for flexible link mechanisms and manipulators. The design of a state observer should be always based on a model providing an adequate description of the system dynamics. However, when flexible link mechanisms are considered, the synthesis of a state observer becomes very challenging, since only nonlinear models may be adopted to reproduce the system dynamic response with adequate accuracy. In this work the possibility of employing a Kalman estimator together with a suitable piecewise-linear model is investigated. Numerical results prove the effectiveness of the proposed approach when it is applied to the synthesis of a state observer for a four-bar linkage with all the links flexible.

Key-Words: - modeling, flexible, link, mechanism, linearization, state, observer

1 Introduction

Considerable research effort has been recently devoted to the development of adequate models and control schemes for manipulators, and more in general mechanisms, with light and flexible structures. These mechanical systems might be profitably employed in such fields as industrial manufacturing, biomechanics and aerospace engineering.

The chief problems to be faced when flexible structure mechanisms are employed are related to the not negligible elastic deformations which generally occur in the links. Such deformations, and the consequential undesired vibrational phenomena, make it difficult to control the mechanism motion accurately, and to achieve satisfactory precision and repeatability levels in task execution [1]. On the other hand, it is well known that contemporary industrial manipulators prevent the aforementioned problems through low payload-to-weight ratios. Ratios close to 1:15 are normally employed [2]. When the payload-to-weight ratio increases, and so when the lightness of the links increases, it is no longer possible to assume that the links are rigid and to neglect, in the control scheme synthesis, the elastic effects. The development of models ensuring an accurate description of the dynamic behavior of flexible link mechanisms has therefore assumed primary scientific relevance. In particular, such models take a crucial role in the synthesis of effective position and vibration control schemes.

The prevalent approaches to flexible mechanism modeling make use of discrete representations obtained through the finite element method [3, 4] or modal expansion techniques applied to either continuous [5, 6] or discrete [7, 8] models. In these models it is often assumed that the overall motion of a flexible link mechanism can be split into the rigid-body motion of a moving reference and the superimposed small elastic motion caused by the link elasticity [9].

Numerical and experimental investigations [10, 11] have proven that accurate dynamic models for flexible link mechanisms cannot neglect the mutual influence between rigid-body and elastic motion nor the system chief inertial and geometric nonlinearities. Such “fully coupled” nonlinear models [3, 4] provide a very accurate description of the system dynamic response, but only have minor utility in the synthesis of control schemes. In fact, though fully coupled nonlinear models can be effectively employed in the implementation of simulation environments where new control schemes may be tuned and tested [12], they do not allow a direct use of the classical control design and stability analysis techniques developed for linear systems.

Notwithstanding, linear control schemes have been widely employed in this field, with varying degrees of success. In particular, linear quadratic optimal (LQ) control strategies have been effectively adopted by some researchers both to control single-link [13, 14] and multi-body systems (e.g. two-link arms [14], and four-bar linkages [15]). In these works approximate state-space linearizations of the dynamic models are used, and the control actions are based on the values assumed by the state variables of the systems. A typical assumption made when LQ regulators are employed is that the vector of the full state of the system is available for feedback. Yet, the direct measurement of all the state variables of a flexible link mechanism is almost ever impossible, hence, in order to practically employ such control strategies it is necessary to design a state observer which reconstructs the state vector from the value of the sensed output (i.e. the measured variables). For example, the use of an observer, has allowed attaining satisfactory experimental results in [13], where a single-link manipulator is considered. Clearly, practical difficulties arise when designing observers for flexible link mechanisms of a higher complexity: the dynamic models which should be employed are highly nonlinear and the real-time computation of the state variables may become very expensive.

On the basis of the considerations above, the use of fully coupled nonlinear models appears to be inconvenient in the synthesis of regulators for flexible link manipulators. This fact, in addition to the increased availability of control system design [16, 17] and stability analysis methods [18, 19] for switched linear systems (i.e. dynamic systems capable of reproducing the response of physical systems by means of a finite number of linear and time invariant models), makes the use of piecewise-linear models attractive also when dealing with flexible link mechanisms.

The objective of this work is to present and to prove the effectiveness of a versatile approach to the synthesis of accurate state observers for flexible link mechanisms and manipulators treated as switched linear systems. The approach is based on the use of a Kalman estimator together with a piecewise-linear model obtained by linearizing a fully coupled nonlinear model about a suitable set of operating points (equilibrium configurations). As underlined above, the design of accurate state observers represents a critical requirement for the synthesis and real-time implementation of regulators for the simultaneous control of rigid-body motion and vibration.

In some previous works, a very accurate finite element and fully coupled nonlinear model has been developed [3] and validated experimentally with reference to four-link [3] and five-link [20] planar mechanisms with all the links flexible, except for the ground link. In [21] a linear model in state-space form has been obtained by linearizing the nonlinear model proposed in [3] about a generic operating point. A numerical and experimental validation of the linearized model has been provided in [22] with reference to a planar four-bar linkage with all the links flexible. Both small and large displacements of the mechanism with respect to an equilibrium configuration have been considered, proving that when large displacements are to be reproduced, a piecewise-linear model can be successfully employed.

In this paper a piecewise-linear model, based on the linearized model proposed in [21], is employed in the synthesis of an observer of the state of the flexible link mechanism also considered in [22]. The observer performances are assessed in simulation by comparing the actual values of the state variables with those computed by the observer on the basis of the sensed output. The state variables considered are the elastic displacements and velocities at the nodes, and the value of the generalized coordinate (and of its first time derivative) of the moving reference configuration (‘ERLS’ [9]) from which the elastic displacements are measured. The sensed output, instead, only comprises a limited number of variables which can be easily measured: the crank position and the link curvatures.

A piecewise-linear model has been chosen to ensure accurate estimates of the observer even when large displacements form an initial equilibrium configuration are considered. The switching among different linear models is regulated by the value taken by the ERLS generalized coordinate. Finally, in order to make it possible to use the observer together with a controller operating in real-time at a reasonable sample time (0.001 s), the linear models employed have been appropriately truncated not to include high frequency modes of vibration, whose dynamics could not be reproduced. Such a truncation is proven to cause minor decrease of the estimate accuracy

Section 2 briefly outlines the chief aspects of the nonlinear model and of the one linearized. Only the most relevant equations are reported: the interested reader should refer to [3] and [21] for further details. The approach followed in the synthesis of the state observer is described in Section 3, as well as the mechanism studied. In Section 4 the implementation of the observer is discussed and the simulation results are presented. Finally concluding remarks and future directions are given in Section 5.

2 The Dynamic Model

2.1 Nonlinear Model

This section provides a synthetic description of the discrete nonlinear model from which the linear model is derived. Both the models are valid for any chain of flexible bodies, which increases the versatility of the approach followed.

So as to get a model with a finite number of degrees of freedom, the links of the mechanism are subdivided into finite elements. Moreover the total motion of each flexible link is separated into the large rigid-body motion of an equivalent rigid-link system (ERLS) and the small elastic deflection of the link with respect to the ERLS itself. In particular, the following definitions are employed:

- $\mathbf{r}_i$ and $\mathbf{u}_i$ are respectively the vectors of the positions of the nodes in the i-th element of the ERLS and of their elastic displacements; the sum of these vectors provides the global motion of the nodes of the i-th element.
- $\mathbf{p}_i$ is the position vector of a point of the i-th element;
- $\mathbf{q}$ is the vector of the ERLS generalized coordinates.

All these vectors are defined in a common fixed reference frame, a local reference frame following the motion of the ERLS is also defined for each element so as to simplify the use of the finite element method.

The equations of motion are obtained through the direct application of the principle of virtual work. The total virtual work is split into the elemental contributions and damping is initially neglected. It holds:

$$\begin{aligned}&\sum_i \int_{v_i} \delta\mathbf{p}_i^T \ddot{\mathbf{p}}_i \rho_i \, dv + \sum_i \int_{v_i} \delta\boldsymbol{\varepsilon}_i^T \mathbf{D}_i \boldsymbol{\varepsilon}_i \, dv = \\ &= \sum_i \int_{v_i} \delta\mathbf{p}_i^T \mathbf{g} \rho_i \, dv + (\delta\mathbf{u}^T + \delta\mathbf{r}^T)\mathbf{f}\end{aligned} \tag{1}$$

where, $\boldsymbol{\varepsilon}_i$, $\mathbf{D}_i$ and ρ_i are respectively the strain vector, the stress-strain matrix, and the mass density for the i-th element, $\mathbf{g}$ is the gravity acceleration vector, $\mathbf{f}$ is the vector of the concentrated external forces and torques, and $\delta\mathbf{u}$ and $\delta\mathbf{r}$ refer to the virtual displacements of all the nodes of the model. The following interpolations are employed for the virtual displacement and real acceleration of a generic point:

$$\delta \mathbf{p}_i = \mathbf{R}_i \mathbf{N}_i \mathbf{T}_i \delta \mathbf{r}_i + \mathbf{R}_i \mathbf{N}_i \mathbf{T}_i \delta \mathbf{u}_i \quad (2)$$

$$\ddot{\mathbf{p}}_i = \mathbf{R}_i \mathbf{N}_i \mathbf{T}_i \ddot{\mathbf{r}}_i + \mathbf{R}_i \mathbf{N}_i \mathbf{T}_i \ddot{\mathbf{u}}_i + 2(\dot{\mathbf{R}}_i \mathbf{N}_i \mathbf{T}_i + \mathbf{R}_i \mathbf{N}_i \dot{\mathbf{T}}_i)\dot{\mathbf{u}}_i \quad (3)$$

in the equations above, $\mathbf{T}_i$ is the transformation matrix from the global to the local reference frame of the i-th element and $\mathbf{R}_i$ is the local-to-global rotation matrix; $\mathbf{N}_i$ is the shape function matrix, which is defined in the local frame.

As for the real and virtual strains, by introducing the strain-displacement matrix ($\mathbf{B}_i$) the following hold locally:

$$\boldsymbol{\varepsilon}_i = \mathbf{B}_i \mathbf{T}_i \mathbf{u}_i \qquad \delta \boldsymbol{\varepsilon}_i = \mathbf{B}_i \delta \mathbf{T}_i \mathbf{u}_i + \mathbf{B}_i \mathbf{T}_i \delta \mathbf{u}_i \quad (4)\text{-}(5)$$

Because the nodal elastic virtual displacements ($\boldsymbol{\delta}\mathbf{u}$) and the nodal virtual displacements of the ERLS ($\boldsymbol{\delta}\mathbf{r}$) are completely independent of each other, from the reported basic relations, it is possible to get this final expression of the system equations of motion:

$$\left[\begin{array}{c|c} \mathbf{M}_{in} & (\mathbf{MS})_{in} \\ \hline (\mathbf{S}^T\mathbf{M})_{in} & \mathbf{S}^T\mathbf{MS} \end{array}\right] \begin{bmatrix} \ddot{\mathbf{u}}_{in} \\ \ddot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{t}_{in} \\ \mathbf{S}^T\mathbf{t} \end{bmatrix} \quad (6)$$

In the equation above, vector $\mathbf{t}$ accounts for all the forces excluding those related to the second derivatives of the generalized coordinates (see Eq.(7)), $\mathbf{M}$ is the matrix obtained assembling the consistent mass matrices of the elements, and $\mathbf{S}$ is the ERLS sensitivity coefficient matrix for all the nodes. The subscript 'in' has been introduced to underline that Eq.(6) includes only the nodal elastic displacements (and the corresponding matrix elements) which are not forced to zero to define the ERLS position.

Admittedly, the model reproduces the system dynamics very accurately because inertia coupling between rigid-body motion and vibrations is accounted for (through the off-diagonal submatrices $(\mathbf{MS})_{in}$ and $(\mathbf{S}^T\mathbf{M})_{in}$) as well as the chief geometric and inertial nonlinearities of the system.

2.2 Linearized Model

A linearization procedure has been applied to the nonlinear model described above, so as to get a state-space linear model capable of reproducing the dynamic behavior of a flexible link mechanism about an equilibrium configuration. The state vector has been defined: $\mathbf{x} = [\dot{\mathbf{u}} \quad \dot{\mathbf{q}} \quad \mathbf{u} \quad \mathbf{q}]^T$. By reorganizing Eq.(6) the following state space representation of the system dynamic model may be adopted:

$$\begin{bmatrix} \mathbf{M} & \mathbf{MS} & \mathbf{0} & \mathbf{0} \\ \mathbf{S}^T\mathbf{M} & \mathbf{S}^T\mathbf{MS} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{q}} \\ \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{I} \\ \mathbf{S}^T\mathbf{M} & \mathbf{S}^T \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \mathbf{f} \end{bmatrix} + $$
$$+ \begin{bmatrix} -2\mathbf{M}_G - \alpha\mathbf{M} - \beta\mathbf{K} & -\mathbf{M}\dot{\mathbf{S}} & -\mathbf{K} & \mathbf{0} \\ \mathbf{S}^T(-2\mathbf{M}_G - \alpha\mathbf{M}) & \mathbf{S}^T\mathbf{M}\dot{\mathbf{S}} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{q}} \\ \mathbf{u} \\ \mathbf{q} \end{bmatrix} \quad (7)$$

where all the components of vector $\mathbf{t}$ have been made explicit: in particular the matrix $\mathbf{M_G}$ allows keeping into account the Coriolis contributions, $\mathbf{K}$ is the matrix obtained assembling the stiffness matrices of all the elements, and α and β are the Rayleigh damping coefficients. A more compact form for Eq.(7) is: $\mathbf{A}(\mathbf{x})\dot{\mathbf{x}} = \mathbf{B}(\mathbf{x})\mathbf{x} + \mathbf{C}(\mathbf{x})\mathbf{v}$. In general, the state vector $\mathbf{x}$ and the system input $\mathbf{v}$ depend on time, while the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ only depend on $\mathbf{x}$.

The system of nonlinear differential equations shown in Eq.(7) has been linearized considering the linear terms of a Taylor series expansion about an equilibrium configuration where $\mathbf{x} = \mathbf{x_e}$, $\mathbf{v} = \mathbf{v_e}$, and $\dot{\mathbf{x}}_\mathbf{e} = 0$ In the neighborhood of the operating point, the state vector and the input vector can therefore be expressed in the form: $\mathbf{x}(t) = \mathbf{x_e} + \Delta\mathbf{x}(t)$, $\mathbf{v}(t) = \mathbf{v_e} + \Delta\mathbf{v}(t)$. By introducing these relations into Eq.(7) and by adopting the acceptable approximation $\mathbf{A}(\mathbf{x_e} + \Delta\mathbf{x})\Delta\dot{\mathbf{x}} \cong \mathbf{A}(\mathbf{x_e})\Delta\dot{\mathbf{x}}$, the following can be obtained:

$$\mathbf{A}(\mathbf{x_e})\Delta\dot{\mathbf{x}} = \mathbf{B}(\mathbf{x_e} + \Delta\mathbf{x})(\mathbf{x_e} + \Delta\mathbf{x}) + \mathbf{C}(\mathbf{x_e} + \Delta\mathbf{x})(\mathbf{v_e} + \Delta\mathbf{v}) \quad (8)$$

Some algebraic computations, which are reported in [21], allow yielding the linear expression shown below, which holds in the neighborhood of any equilibrium configuration:

$$\mathbf{A}(\mathbf{x_e})\Delta\dot{\mathbf{x}} = \left[\mathbf{B}(\mathbf{x_e}) + \left(\left.\frac{\partial \mathbf{B}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x_e}} \otimes \mathbf{x_e}\right) + \left(\left.\frac{\partial \mathbf{C}}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x_e}} \otimes \mathbf{v_e}\right)\right]\Delta\mathbf{x} + \quad (9)$$
$$+ \mathbf{C}(\mathbf{x_e})\Delta\mathbf{v} = \widetilde{\mathbf{B}}(\mathbf{x_e}, \mathbf{v_e})\Delta\mathbf{x} + \mathbf{C}(\mathbf{x_e})\Delta\mathbf{v}$$

where the symbol "$\otimes$" indicates the inner product of $\left\lfloor \partial B_{i,1}/\partial x_j \cdots \partial B_{i,n}/\partial x_j \right\rfloor_{x=x_e}$ and $\left\lfloor \partial C_{i,1}/\partial x_j \cdots \partial C_{i,n}/\partial x_j \right\rfloor_{x=x_e}$ with respectively $\mathbf{x_e}$ and $\mathbf{v_e}$, for all the subscripts 'i' e 'j'.

It should be pointed out that since a symbolic form of the linearized model has been obtained, once an equilibrium configuration is set, the matrices $\mathbf{A}$, $\widetilde{\mathbf{B}}$ and $\mathbf{C}$ can be immediately computed. Finally, by setting $\mathbf{F} = \mathbf{A}^{-1}\,\widetilde{\mathbf{B}}$ and $\mathbf{G} = \mathbf{A}^{-1}\,\mathbf{C}$ Eq.(9) can be rewritten in the standard form:

$$\Delta\dot{\mathbf{x}} = \mathbf{F}\,\Delta\mathbf{x} + \mathbf{G}\,\Delta\mathbf{v} \quad (10)$$

3 Design of a State Observer for a Flexible Link Mechanism

The dynamic models described above may be applied to any mechanism. In this work they have been employed to develop a state observer for the test case considered in [22]. The mechanism is a four-bar planar linkage with all the links flexible. Fig.1 illustrates the finite element representation of the mechanism: the links, the joints, the beam elements and the nodes are shown. Two elements of the same length are used to model link 2 and 3 while a single beam element is employed for link 1, which is the shortest link. Lumped masses and inertias are used to account for the joints and the motor driving the mechanism at joint A. Table 1 reports the chief characteristics of the mechanism. The elastic degree of freedom forced to zero to define the position of the ERLS with respect to the deformed mechanism is the horizontal displacement at node 5. Finally, the ERLS generalized coordinate q is the (rigid-body) rotation of the crank (link 1). The finite element representation employed for the mechanism, leads to a dynamic model with 32 state variables.

The mechanism is supposed to move on a vertical pane, and so the effect of the force of gravity cannot be neglected.

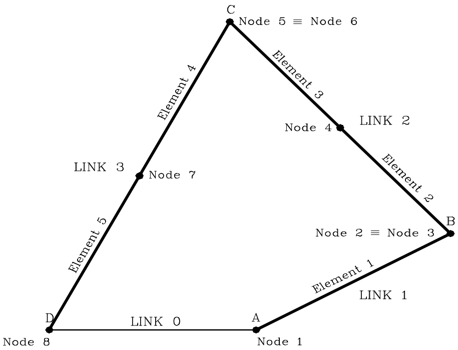

Fig.1 Finite element representation of the four-bar linkage

Table 1 Mechanical parameters of the flexible link system

Link	**0**	**1**	**2**	**3**
Length [m]	0.35965	0.37285	0.52500	0.63200
Flexural Stiffness: 21.6 Nm^2		*Cross-Sectional Area:* 36E-6 m^2		
Joint	**A**	**B**	**C**	**D**
Mass [kg]	-	81E-3	80E-3	-
Inertia [kgm²]	4.9E-4	-	-	12E-6

As a consequence, the components of the state vector associated to the nodal elastic displacements generally take values different from zero, even when the mechanism is in static equilibrium. The values taken by such displacements in static equilibrium only depend on the position of the ERLS, and consequently on the equilibrium value of the generalized coordinate q_e. Moreover, it can be proven that the equilibrium values of all the matrices in Eq.(9) (and Eq. (10)) only depend on q_e. There follows, that the sole knowledge of q_e in an equilibrium point, allows the off-line computation of the matrices of the linearized dynamic model and of the state vectors in the corresponding static equilibrium configuration. This property is successfully employed in the design of the state observer.
The observer synthesized is based on a Kalman estimator. So as to implement such an estimator, two requirements have to be met: a linear time invariant dynamic model of the system must be available, and a linear relation between the state variables and the sensed output must exist. Equation (10) meets the first requirement about any equilibrium configuration. As for the output variables, the mechanism is supposed to be instrumented with two encoders measuring the absolute rotation of link 1 and 3, at respectively joint A (α_A) and D (α_D). Moreover, calibrated strain gages are supposed to be employed to measure the curvatures of the same links at approximately the bar midpoints (C_1 and C_3).
The vector of the output variables therefore takes the form: $\mathbf{y} = [\alpha_A, \alpha_D, C_1, C_3]^T$. Only α_A is a linear combination of state variables, while nonlinear expressions relate the other measured variables to the states. Linearized expressions have been obtained for α_D and the curvature C_i of the generic i-th link: these expressions clearly hold only about an equilibrium configuration. The following expressions have been used to approximate finite changes of the measured variables with respect to their equilibrium values:

$$\Delta\alpha_A = \Delta q + \Delta u_{\phi 1} \quad \Delta\alpha_D = S_{\theta_8,q}\Big|_{q=q_e} \Delta q + \Delta u_{\phi 8} \qquad (11)\text{-}(12)$$

$$\Delta C_i = \mathbf{B}_i \left[(\partial \mathbf{T}_i / \partial q) \mathbf{u}_i \Big|_{q=q_e} \Delta q + \mathbf{T}_i \Big|_{q=q_e} \Delta \mathbf{u}_i \right] \qquad (13)$$

where, $\Delta u_{\phi i}$ is the elastic rotation at node i and $S_{\theta_8,q}$ is the sensitivity coefficient between the generalized coordinate and the rigid-body rotation at node 8. The equations above can be aggregated in the usual matrix form $\Delta\mathbf{y} = \mathbf{H}\,\Delta\mathbf{x}$. The PBH test (involving the matrices **F** and **H**) has been used to assess the system observability, i.e. to verify that the observation of the output variables at all times is sufficient to determine the initial values of all the state variables.
Now, let **e** and **L** be, respectively, the vector of the errors of the state variable estimates ($\hat{\mathbf{x}}$) and the time invariant gain matrix of the asymptotic Kalman estimator [23]. Additionally, let **W** be the time invariant gain vector of a linear regulator, whose control action is proportional to the difference between the actual and the equilibrium values of the state variables. In this work, an appropriate value for **W** has been found by the solution of an optimal linear quadratic problem (LQ) in which a performance index is minimized. The design and implementation of the LQ regulator is not discussed here because it not pertinent to the observer design and performance assessment.
The dynamics of the overall system, including the linear regulator and the estimator, is described by the following system of equations:

$$\begin{cases} \Delta\dot{\hat{\mathbf{x}}} = \mathbf{F}\ \Delta\hat{\mathbf{x}} + \mathbf{G}\ \ \Delta\mathbf{u} + \mathbf{L}\ (\Delta\mathbf{y} - \Delta\hat{\mathbf{y}}) \\ \Delta\hat{\mathbf{y}} = \mathbf{H}\ \ \Delta\hat{\mathbf{x}} \\ \Delta\mathbf{u} = -\mathbf{W}\ \ \Delta\hat{\mathbf{x}} \\ \mathbf{e} = \Delta\hat{\mathbf{x}} - \Delta\mathbf{x} \end{cases} \qquad (14)$$

The matrix **L** is chosen so as to minimize the mean square error between the estimated and the actual values of the state variables. It can be proven that the solution of this problem is: $\mathbf{L} = \mathbf{P}\ \mathbf{H}^T\ \mathbf{R}^{-1}$, where **P** is the symmetric and positive semidefinite solution of the Riccati equation $\dot{\mathbf{P}} = \mathbf{F}\,\mathbf{P} + \mathbf{P}\mathbf{F}^T - \mathbf{P}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{P} + \mathbf{Q}$ and **R** and **Q** are the measurement and process noise covariance matrices [23].
Admittedly, these equations only hold in the neighborhood of an equilibrium configuration. However, when the state of the system shifts significantly from the equilibrium configuration, the elements of the coefficient matrices of the system may be re-computed to keep a reliable description of the physical system dynamics. Such re-computation corresponds to a switch (i.e. a discrete transition) between linear models. The switch can be made on the basis of the value assumed just by q_e, since the small amplitude of the elastic displacements with respect to the ERLS does not impose using further linearized models. This fact simplifies the design and implementation of a state observer based on a piecewise-linear model.

4 Implementation of the State Observer and Performance Analysis

The theory described in the forgoing sections has allowed implementing both a nonlinear dynamic simulator of the mechanism studied and a piecewise-linear state observer. The performances of the observer have been assessed by comparing the values of the state variables computed by the simulator with the observer estimates on a test case. All the simulations have been carried out using Matlab/Simulink.

The practical need of implementing the proposed observer (and controller) on a real-time hardware, for future experimental investigations, imposes a bound to the frequency content of the simulated system dynamics (in accordance with Shannon's sampling theorem) and suggests reducing the amount of computational power necessary to get the state variable estimates. Both these requirements may be met if a reduced-order model is employed. It should be noticed that the adopted finite element model, with 32 state variables, is itself a reduced-order representation of the infinite-dimensional physical system. However, given the marginal influence on the system response of the high frequency modes of the system, discarding such modes represents an efficient process of further model reduction. For this reason, only the first seven modes of the system are retained in the linear model adopted for the implemented observer. The figures in this section confirm that ignoring the high frequency modes enables computing the state estimates with adequate rapidity and with minor errors.

The test carried out to assess the observer performances consists in a fast rotation of the drive shaft from an initial to a final static equilibrium configuration. This motion is obtained by introducing a step change in the reference path of the controlled variable α_A (providing a detailed description of the control system goes beyond the scope of this paper). While moving from the initial to the final configuration, different linear models should be used to reproduce the system dynamics locally. Hence a single linear model cannot be employed in the observer synthesis.

In this work, in order to simplify the observer implementation, two linearized models of the mechanism have been used. They have been obtained by evaluating the matrices **F** and **G** about the initial and the final equilibrium configurations of the test. The switching among the models takes place when the intermediate position is crossed.

A first evidence of the observer effectiveness is reported in Fig.2. In fact, the availability of an accurate estimator also affects the performances of the closed-loop control system: the better the estimates, the more effective the control action. The advantages offered by the proposed observer are apparent in Fig.2, where a comparison is made between the step responses of the closed-loop system recorded using the proposed observer (blue line) and an observer employing one linear model holding about the final position (red line).

The figures from 3 to 6 refer to the test carried out to assess the observer performances: a step change of α_A of 45°.

A comparison between the actual (simulated) and the estimated values of the ERLS generalized coordinate q is shown in Fig.3. The time-history of the error between such values (Fig.4) proves that the error quickly converges to zero, as theoretically predicted. Fig.4 also shows that the maximum error values are achieved when the switching between the linear models occurs. This is an obvious consequence of the discontinuity introduced be the switching, and of the resulting free response of the observer.

The observer capability of providing accurate estimates of also the elastic states is proved by Figs. 5 and 6. These figures refer to the elastic displacement in the horizontal direction at node 4 (u_{x4}) on link 2. Good agreement is confirmed to exist between the actual and the estimated displacements, both in terms of amplitude and frequency content of the time-histories, even though no sensors are assumed to be placed on link 2. Similar results are obtained by comparing the actual and estimated values of the other state vector components, including rigid body and elastic velocities. These evidences are not reported here for brevity.

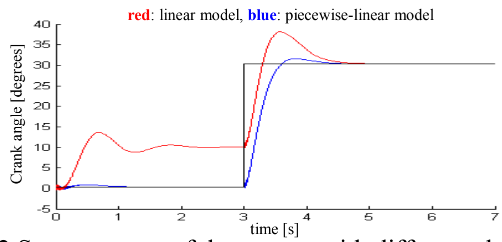

Fig.2 Step response of the system with different observers

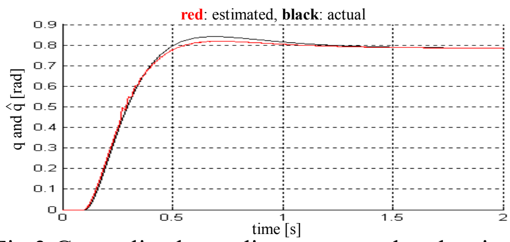

Fig.3 Generalized coordinate q: actual and estimated

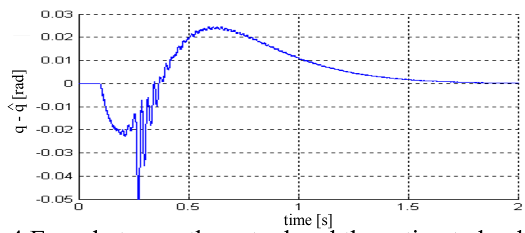

Fig.4 Error between the actual and the estimated values of q

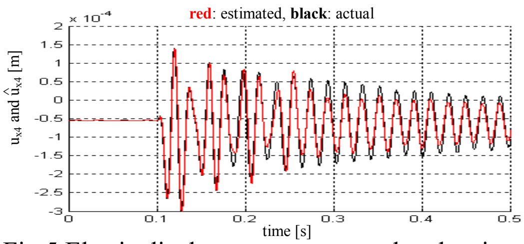

Fig.5 Elastic displacement u_{x4}: actual and estimated

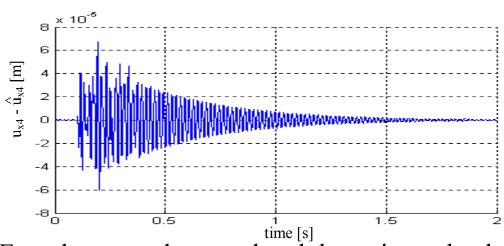

Fig.6 Error between the actual and the estimated values of u_{x4}

5 Conclusion

In this work a general approach for designing effective state observers for flexible-link mechanisms has been presented. The proposed method makes use of an accurate piecewise-linear dynamic model and leads to the synthesis of an observer with a structure combining the features of an asymptotic Kalman estimator and a switched linear system. This structure allows overcoming the difficulties arising from the geometric and inertial nonlinearities of flexible link mechanism dynamic models. Moreover, in order to make the implementation of the observer possible on a real-time hardware, a reduced order dynamic model is employed, which only accounts for the most significant modes of vibration of the system. The results achieved on a numerical test prove that the estimates are accurate and that the estimate error dynamics promptly converges to zero.

The availability of an effective and computationally efficient state observer represents an essential starting point for the synthesis and experimental validation of position and vibration control schemes based on the state vector feedback, such as LQ control strategies.

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