Diagnosis Utilizing the P Integration Method for Rotating Machinery

Yuan Kang, Chun-Chieh Wang, Yeon-Pun Chang
Department of Mechanical Engineering
Chung Yuan Christian University
Chung Li, 320, Taiwan, R.O.C.

Abstract：This study proposes a P integration method to diagnose rotor systems using vibration responses. An integral is defined by integrating the distance of trajectories and origin in phase plane. The responses of a nonlinear system for rotating machinery are used to identify the relationship between the P integration value and the integrated interval. In this study, the P integration method is utilized to diagnose responses in three case situations including an unbalanced rotor, a loose base and defects in ball bearings. Furthermore, in these case analyses, the P integration method is compared with spectrum analysis, the Poincaré section method, correlation dimension and Kolmogorov entropy. These results are regarded as fault diagnosis methods for rotating machinery.

Key-Words：P integration method, rotating machinery, Poincaré section method, correlation dimension, Kolmogorov entropy, fault diagnosis.

1 Introduction

A vibration signal can indicate abnormal operating and fault conditions in rotating machinery. However, in practical applications, rotating machinery exhibits strongly nonlinear behaviors. The vibration spectrum is not equal to rotating speed. Additionally, rotating machinery not only has rotating speed, but also has multiple speed and other frequencies.

The nonlinear effect almost occurs with the clearance of ball bearings, the reaction of hydrodynamic bearing, rubbing force, loose screws (loosening), etc. In case situations [1-3] symptoms of nonlinear vibration reaction are investigated by experimental record and simple analysis in former research reports. Goldman and Muszynska [4] explained that having the vibration of rotary and sub-harmonic frequency in loose screws in machinery for one degree of freedom of impact model in 1994. In numerical analysis, the system has chaos motion in certain parameters, in addition to the sub-harmonic vibration. Berry [5] used spectrum symptoms to establish a fault diagnosis rule for rotary machine. Grassberger and Procaccia [6-7] carried out a process of reconstructed phase space for a variable of time sequence in 1983. They used a calculation of correlation dimension integration function to diagnose the fault of the motor. K_2, Kolmogorov entropy is Kolmogorov [8] expanded by the concept of Shannon’s entropy in 1958. This method can be used to identify or diagnose regular motion, chaos motion or random motion.However, the Kolmogorov entropy method cannot identify regular motion. In this study, one can analyze responses of systems correctly for finite measuring data by using the P integration method. The P integration method is compared with spectrum analysis, the Poincaré section method, correlation dimension and Kolmogorov entropy. This analysis covers three cases including an unbalanced rotor, a loose base and defects in ball bearings. The results obtained from this analysis will be utilized for fault diagnosis. Comparison of each method utilized in motor systems will be made.

2 P Integration Observers

An integral defined by integrating the distance of trajectories and origin in phase plane has been presented as

$$P_T(t_c) = \int_{t_c}^{t_c+T} \left([\dot{x}(\tau)]^2 + [x(\tau)]^2 \right)^{1/2} d\tau \qquad (1)$$

where $x(\tau)$ and $\dot{x}(\tau)$ are time histories of displacement and velocity, respectively, τ is time and, t_c is an arbitrary time for nondimensional parameters, taken at the time of steady state. The integration interval T is the nondimensional period of a distinct harmonic excitation or the least common multiple of periods of multi-harmonic excitations. Constant P_T can be obtained from time history, when response is P-1 motion. Consequently, the integration interval nT is set for determining the period of P-n motion and the n-th main or differential sub-harmonic responses.

This integration observer can also be expressed by

$$P_{f(x,\dot{x})} = \int_{t_c}^{t_c+T} f(x,\dot{x})d\tau \tag{2}$$

where the integrand function can be presented by $f(x,\dot{x}) = x(\tau), \dot{x}(\tau), x(\tau)+\dot{x}(\tau)$, $x^2(\tau)$ or $\dot{x}^2(\tau)$. These functions will exhibit similar results, where the integrand represents the distance between trajectories and origin or trajectories and one of the coordinate axes in phase plane.

Equation (2) can be modified by replacing the integration interval T by $\hat{T}$. Thus the modified P integration method is defined by

$$P_{\hat{T}}(t_c) = \int_{t_c}^{t_c+\hat{T}} f(x,\dot{x})d\tau \tag{3}$$

where the integration interval $\hat{T}$ is set to be the predicted, confirmed, oscillating period. For example, if the system undergoes period-doubling, $\hat{T}$ might be multiples of T, i.e. $\hat{T} = nT$ where T is the excitation period and n an integer. The integer n is set to identify the response period of P-n motion. Note that "P-n" denotes (for convenience) that the system response undergoes motion of period nT through period-doubling. Furthermore, in Eq. (3), t_c will be varied to assist in the determination of the response period. Therefore, for a specific period $\hat{T}$, the value of $P_{\hat{T}}$ is a function of t_c.

Period $\hat{T}$ in Eq. (3) is set to be the excitation period i.e. $\hat{T} = T$ to calculate P_T for constructing the bifurcation diagram in order to observe nonlinear behavior. This is the same as the sectioning period for the Poincaré method.

When the steady state response of a nonlinear system is periodic with $\hat{T}$, there is

$$f(x,\dot{x}) = f(\tau) = f(\tau+\hat{T}) = f(x(\tau+\hat{T}), \dot{x}(\tau+\hat{T})) \tag{4}$$

and one obtains

$$\frac{dP_{\hat{T}}}{dt_c} = f(\tau+\hat{T}) - f(\tau) = 0 \tag{5}$$

or $P_{\hat{T}}(t_c) =$ constant (6)

As proof of the above for periodic motion, see Eq. (6). With period $\hat{T}$, the integral $P_{\hat{T}}$ is constant against the starting time of integration. Thus, the period $\hat{T}$ of a steady state response can be identified due to the existence of constant $P_{\hat{T}}$.

It should be noted that the above procedure is structured on the basis that the integral value $P_{\hat{T}}$ would be constant as opposed to varied starting times t_c's, if the chosen integration interval $\hat{T}$ is equal to the response period. Another point addressed is that the period is determined on the basis that the $P_{\hat{T}}$ value remains constant over t_c. This avoids possible error, by measurement or computation tolerance, occurring in the process of distinguishing the geometric points of Poincaré sectioning in phase space. Furthermore, by utilizing the method of $P_{\hat{T}}$ integration mentioned above, the set of simulated or experimental data needed to determine response period all reside in the time range $[t_{c,0}, t_{c,0} + \hat{T} + \Delta t_c]$, the duration of which is $(\hat{T} + \Delta t_c)$. With the freedom to set Δt_c small, the duration $(\hat{T} + \Delta t_c)$ is less than $\hat{T}$'s, which is usually the time span utilized by Poincaré sectioning to collect data for identifying the response period due to measurement or numerical tolerance.

3 Experiments and analysis

3.1 Unbalanced rotor

An unbalanced rotor is a common fault type in a rotary machine. This is due to the fact that the mass distributed along the rotor is not uniform. When the rotor is running and creates centrifugal force, the major vibration frequency is rotary frequency. Fig. 1 demonstrates an unbalanced rotor and loose base system using the experiment rotor kit. Fig. 2 shows the displacement spectrum of an unbalanced rotor system and has rotary and harmonic frequency. This study analyzes the fault response by using the P integration method, Poincaré section method, correlation dimension and Kolmogorov entropy.

(a) P integration method

The integrand represents the distance between the rotor's motion trajectories and origin point by using the P integration method. According to the P integration method calculation procedure, Fig. 3 demonstrates integration curves at different integration intervals. The integration interval $\hat{T}$ is set to be the rotary period. Fig. 3(a)-(b) shows both values P_T and P_{2T} remain constants, when integration intervals T and 2T are indicated. Because both errors are 0.42% and 0.32%, respectively, the system response might be P-1 regular motion.

(b) Poincaré section method

Fig. 4 shows time histories of experimental measuring signals and the solid points are Poincaré section points. When rotary frequency is 32 Hz, the vibrations of the system are regular and periodic. The Poincaré section points remain constant and change only slightly. The system response might be P-1 regular motion.

(c) Correlation dimension and Kolmogorov entropy

Moreover, the system responses from the results of experiments are used to carry out the process of reconstructed phase space. Its correlation dimension and Kolmogorov entropy are calculated so as to distinguish the types of system responses. In the analysis, the corresponding time equal to 24 is seen as the reconstructed delaying time when the auto-correlation dimension of system response reaches 1/exp value. Correlation function numbers of embedding dimension, $m=1$, 2,…., 30 are also calculated. Correlation dimension, D_2 and Kolmogorov entropy, K_2 are calculated in the linear intervals according to the changes of embedding dimension. This relationship is shown in Figs. 5(a)-(d). The figures indicate that D_2 and K_2 of reconstructed time sequence of system response tend to become stable when the embedding dimension increases to a certain value. The correlation dimension tends to be near 1.027 and 1.025 and Kolmogorov entropy, K_2 values become closer to 0.29 and 0.28. According to the definition, the correlation dimension regarding periodic motion is equal to 1 and Kolmogorov entropy is equal to zero. The results demonstrate that the system response might be regular motion.

3.2 Loose base

Screws, not securely fastening the ball bearings (loosening) on rotor systems is a common fault type in a rotary machine. These loose screws in rotor operation will create an abnormal vibration phenomenon. These loosening faults in a rotary machine sometime follow the nonlinear phenomenon. The vibration frequency also has sub-harmonic and harmonic frequency, in addition to the rotary frequency. Fig. 6 demonstrates the displacement spectrum of screws loosened in a rotor system. Analysis of the fault response is accomplished utilizing the P integration method, Poincaré section method, correlation dimension and Kolmogorov entropy.

(a) P integration method

The integrand represents the distance between the rotor's motion trajectories and origin point utilizing the P integration method. According to the P integration method calculation procedure, Fig. 7 demonstrates integration curves that are calculated with the P integration method at different integration intervals. The integration interval $\hat{T}$ is set to be the rotary period. When integration intervals increase from T to 4T. Fig. 7 demonstrates that only the P_{2T} and P_{4T} value remain constant and errors are kept to 3%, P_T and P_{3T} values are oscillating irregularly. The system response might be P-2 regular motion.

(b) Poincaré section method

Fig. 8 shows time histories of experimental measuring signal and the solid points are Poincaré section points. When rotary frequency is 32 Hz, the vibrations of the system are regular and periodic. The Poincaré section points remain two constants and change only slightly. The system response might be P-2 regular motion.

(c) Correlation dimension and Kolmogorov entropy

The system responses from the results of experiments are used to carry out the process of reconstructed phase space. Its correlation dimension and Kolmogorov entropy are calculated to distinguish the types of system responses. In the analysis, the corresponding time equal to 34 is seen as the reconstructed delaying time when the auto-correlation dimension of system response reaches 1/exp value. Correlation function numbers of embedding dimension, $m=1$, 2,…., 30 are also calculated. Correlation dimension, D_2 and Kolmogorov entropy, K_2 are calculated in the linear intervals according to the changes of embedding dimension. This relationship is shown in Figs. 9(a)-(d). The figures indicate that D_2 and K_2 of reconstructed time sequence of system response tend to become stable when the embedding dimension increases to a certain value. The correlation dimension tends to be near 1.07 and 1.15 and Kolmogorov entropy, K_2 values become closer to 0.32 and 0.47. According to the definition, the correlation dimension regarding periodic motion is equal to 1 and Kolmogorov entropy is equal to zero. The results demonstrate that the system response might be regular motion.

3.3 Defects in ball bearings

Ball bearings are an essential part in the motor equipment. Fig.10, it was a three-phase, two-pole, 350Hp, induction motor the shaft's height being 355mm with defects in the ball bearings. When the ball bearings break there is an occurrence of defective frequency, defective harmonic frequency and sideband frequency. The signal measurement method is used to measure defective ball bearings.

When some fault exhibit itself during normal rotor operation, abnormal vibration phenomenon will occur Fig. 11 shows the displacement spectrum of defective ball bearings of a rotor system. The vibration frequency has defective frequency, defective harmonic frequency and sideband frequency, in addition to the rotary frequency. The fault response is analyzed using the P integration method, Poincaré section method, correlation dimension and Kolmogorov entropy.

(a) P integration method

The integrand represents the distance between the rotor's motion trajectories and origin point using the P integration method. According to the P integration method calculation procedure, Fig. 12 demonstrates integration curves at different integration intervals. The integration interval $\hat{T}$ is set to be the rotary period. When integration intervals from T increase to 16T, all of the $P_{\hat{T}}$ values are oscillating irregularly. The system response might be chaos motion.

(b) Poincaré section method

Fig. 13 shows time histories of experimental measuring signals. The solid points are Poincaré section points. When rotary frequency is 60 Hz, the vibrations of the system are irregular. The Poincaré section points are distributed randomly. The system response might be chaos motion.

(c) Correlation dimension and Kolmogorov entropy

The system responses from the results of experiments are used to carry out the process of reconstructed phase space. Its correlation dimension and Kolmogorov entropy are calculated to distinguish between the types of system responses. In the analysis, the corresponding time equal to 1 is seen as the reconstructed delaying time when the auto-correlation dimension of system response reaches 1/exp value. Correlation function numbers of embedding dimension, $m=1$, 2,…., 30 are also calculated. Correlation dimension, D_2 and Kolmogorov entropy, K_2 are calculated in the linear intervals according to the changes of embedding dimension. This relationship is shown in Figs. 14(a)-(d). The figures indicate that D_2 and K_2 of reconstructed time sequence of system response tend to become stable when the embedding dimension increases to a certain value. The correlation dimension tends to be near certain fractions and Kolmogorov entropy, K_2 values become closer to 2500 and 4000. According to the definition, the correlation dimension regarding chaos motion is equal to a certain fraction and Kolmogorov entropy is equal to a positive number. The results demonstrate that the system response might be chaos motion.

4 Conclusions

In practical experiments, it is difficult to identify or diagnose the responses of regular motion in rotating machinery by using the Poincaré section method, correlation dimension or Kolmogorov entropy for finite measuring data. However, one can get a constant value using the P integration method with the integration interval nT, so that P-n regular motion can be identified. Table1 shows all methods that diagnose nonlinear motion responses in the rotating machinery.

References

[1] Tondl, A., " Notes on the Identification of Subharmonic Resonances of Rotors, " Journal of Sound and Vibration, Vol. 31(1), pp. 119-127, 1973.

[2] Botman, M., and Samaha, A. M., " Experiments on the Dynamic Behavior of a Supercritical Rotor, " ASME J. Mech. Des., Vol. 104, pp. 364-369, April, 1982.

[3] Holmes, R., and Dade, M. M., " Non-linear Phenomena in Aero-engine Rotor Vibration, " Proc. Instn. Mech. Engrs, Vol. 203, pp. 25-34, 1989.

[4] Goldman, P., and Muszynska, A., "Dynamic Effects in Mechanical Structures with Gaps and Impacting: Order and Chaos," ASME Journal of Vibration and Acoustics, Vol. 116(4), pp. 541-547, 1994.

[5] Berry, E, J., " Proven Method for Specifying Spectral Band Alarm Levels and Frequencies Using Today's Predictive Maintenance Software System, " Journal of Sound and Vibration, pp. 1-24, September, 1990.

[6] Grassberger, P., and Procaccia, J., "Measuring the Strangeness of Strange Attractors," Physical D, Vol. 9, pp. 189-208, 1983.

[7] Grassberger, P., and Procaccia, J., "Characterization of Strange Attractors," Physical Review Letters, Vol. 50(5), pp. 346-349, 1983.

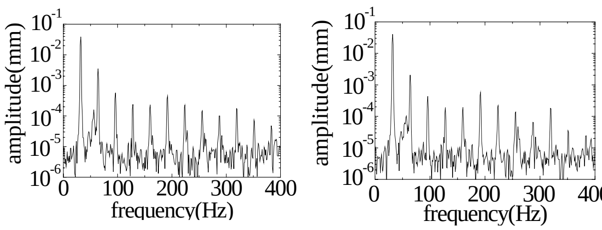
[8] Grassberger, P., and Procaccia, J., "Estimation of the Kolmogorov entropy from a chaotic signal," Physical Review A, Vol. 28(4), pp. 2591-2593, 1983.

Table1 All methods that diagnose nonlinear motion responses in the rotating machinery.

Method \ Motion type	Periodic	Quasi-period	Chaotic	Random
P integration	Constant (yes)	Constant (yes)	Not positive constant (no)	∞ (no)
Correlation dimension	1 (no)	2 or 3 (no)	Fraction (yes)	∞ (yes)
Kolmogorov entropy	Zero (no)	Zero (no)	Constant (yes)	∞ (yes)

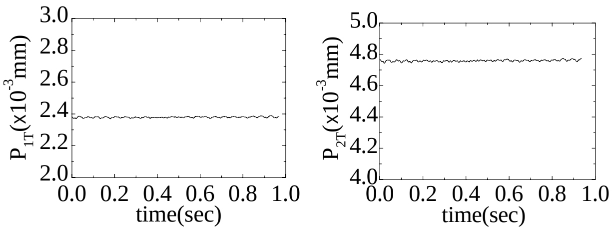
(yes): has identification ability. (no): has no identification ability.

Fig.1 Unbalanced rotor and loose base system of the experiment rotor kit.



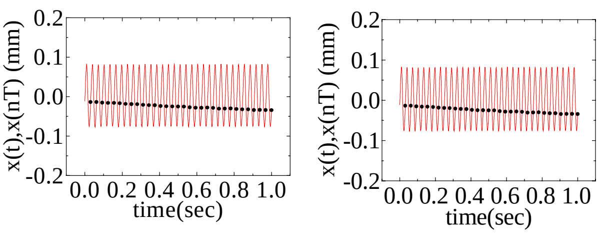
(a) vertical response (b) horizontal response

Fig.2 The displacement spectrum for unbalanced of the experiment rotor kit.



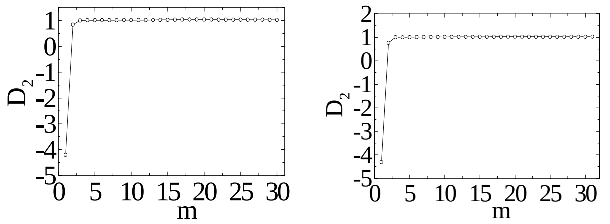
(a) integral interval $\hat{T} = T$ (b) integral interval $\hat{T} = 2T$

Fig.3 P_T integration value for unbalanced of the experiment rotor kit.



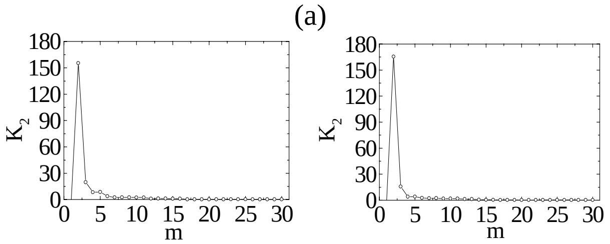
(a) vertical response (b) horizontal response

Fig.4 Poincaré section points for unbalanced of the experiment rotor kit.



vertical response horizontal response

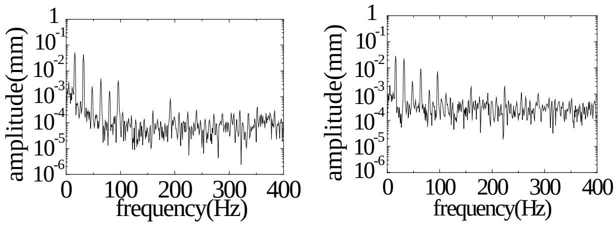
(a)



vertical response horizontal response

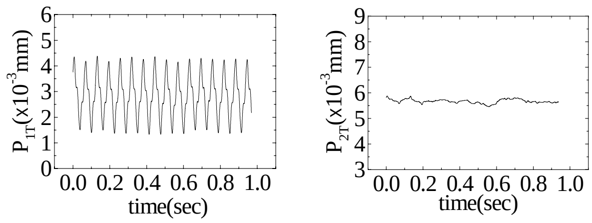
(b)

Fig.5 (a) Correlation dimension D_2 and (b) Kolmogorov entropy K_2 versus embedding dimension for unbalanced of the experiment rotor kit.

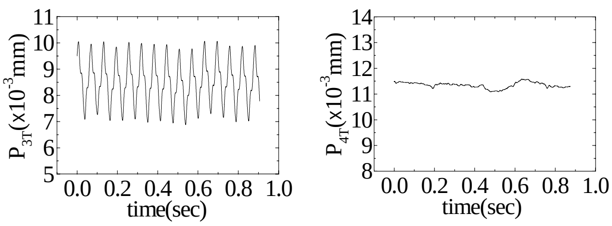


(a) vertical response (b) horizontal response

Fig.6 The displacement spectrum for loose base of the experiment rotor kit.

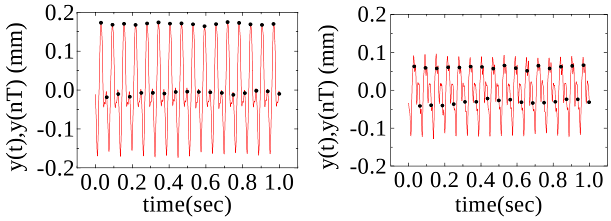


(a) integral interval $\hat{T} = T$ (b) integral interval $\hat{T} = 2T$



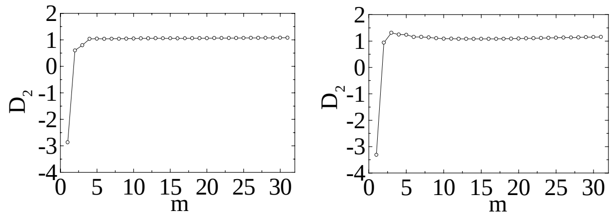
(c) integral interval $\hat{T} = 3T$ (d) integral interval $\hat{T} = 4T$

Fig.7 P_T integration value for loose base of the experiment rotor kit.



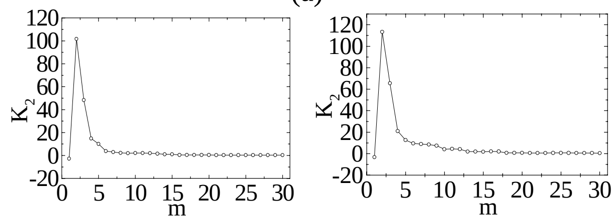
(a) vertical response (b) horizontal response

Fig.8 Poincaré section points for loose base of the experiment rotor kit.



vertical response horizontal response

(a)

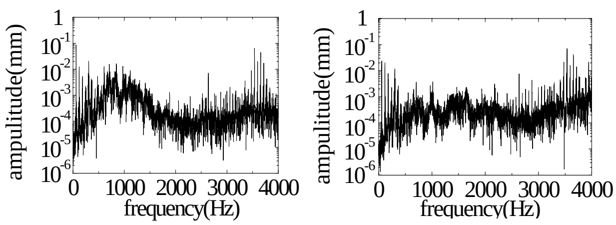


vertical response horizontal response

(b)

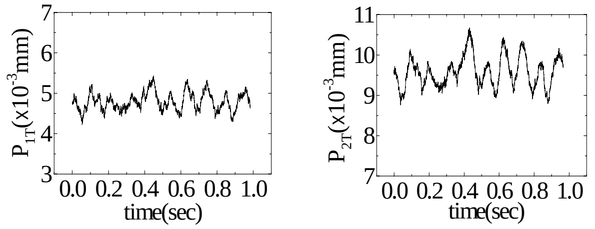
Fig.9 (a) Correlation dimension D_2 and (b) Kolmogorov entropy K_2 versus embedding dimension for loose base of the experiment rotor kit.

Fig. 10 Defecting in ball bearings of the motor.

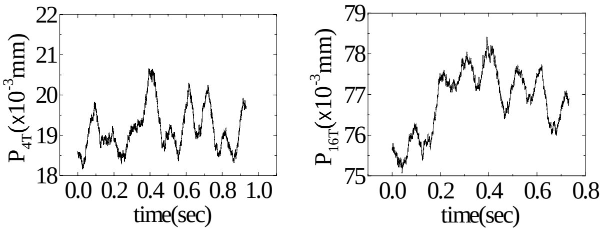


(a) vertical response (b) horizontal response

Fig.11 The displacement spectrum for defects in ball bearings of the motor.

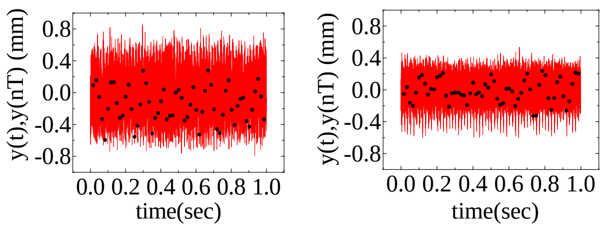


(a) integral interval $\hat{T} = T$ (b) integral interval $\hat{T} = 2T$



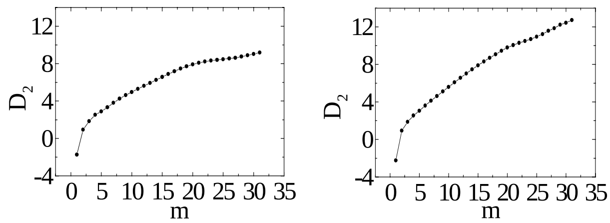
(c) integral interval $\hat{T} = 4T$ (d) integral interval $\hat{T} = 16T$

Fig.12 P_T integration value for defects in ball bearings of the motor.



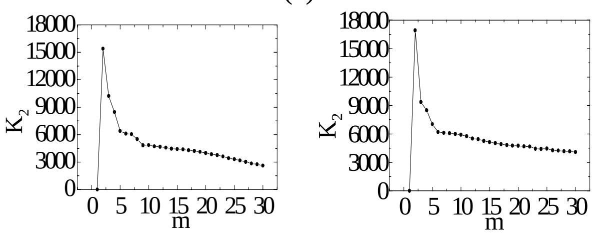
(a) vertical response (b) horizontal response

Fig.13 Poincaré section points for defects in ball bearings of the motor.



vertical response horizontal response

(a)



vertical response horizontal response

(b)

Fig.14 (a) Correlation dimension D_2 and (b) Kolmogorov entropy K_2 versus embedding dimension for defects in ball bearings of the motor.