Analysis of the Time-Optimal System Design Algorithm Structure

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Abstract: - The formulation of the process of analog system design has been done on the basis of the control theory application as the problem of a special functional minimization. This approach generalizes the design process and produces the different design trajectories inside the same optimization procedure. Numerical examples show that the potential computer time gain of the optimal design strategy with respect to the traditional design strategy increases when the size and the complexity of the system increase. Some properties of an additional acceleration effect of the design process were analyzed. The special selection of the optimization process start points provides the acceleration effect with a great probability. The positions of the optimal switch points of the control vector were found on the basis of the analysis of the special Lyapunov function of the design process by means of the time derivative minimization. The combination of the acceleration effect with the start point preliminary selection and the optimal switch points of the control vector serve as the principal ideas to the time-optimal design algorithm construction.

Key-Words: - General design algorithm, control theory application, acceleration effect, start point selection, Lyapunov function.

1 Introduction

The problem of the computer time reduction of a large system design is one of the essential problems of the total quality design improvement. This problem has a special significance for the VLSI electronic circuit design. The reduction of the necessary time for the circuit analysis and improvement of the optimization algorithms are two main sources for the reduction of the total design time. There are some powerful methods that reduce the necessary time for the circuit analysis. Because a matrix of the large-scale circuit is very sparse, the special sparse matrix techniques are used successfully for this purpose [1]-[3]. Other approach to reduce the amount of computational required for the linear and nonlinear equations is based on the decomposition techniques. The well-known ideas for partitioning of a circuit matrix into bordered-block diagonal form were described in original works using the branches tearing [4] or nodes tearing [5] and jointly with direct solution algorithms give the solution of the problem. The extension of the direct solution methods can be obtained by hierarchical decomposition and macro model representation [6]. The optimization technique is developed both for the unconstrained and for the constrained optimization and can be improved in future. However there is another way to reduce the total computer time for the analog system design. The reformulation of the optimization process on heuristic level was proposed decades ago [7]-[8]. This process was named as generalized optimization and it consists of the Kirchhoff law ignoring for some parts of the optimization process. The special cost function is minimized instead of the circuit equation solution. In practical aspects this idea was developed for the microwave circuit optimization [9] and for the synthesis of high-performance analog circuits [10]-[11]. Nevertheless all of these ideas are needed to generalize.

Another way can be proposed for the design problem reformulation. This approach consists of generalization of the total design problem to obtain a set of different design strategies inside the same optimization procedure [12]-[13]. In this case the time-optimal design strategy can be formulated and this strategy can be proposed as a theoretical basis for the time-optimal design algorithm searching. Some preliminary promising results of quasi optimal strategies were obtained in [13]. On the other hand an additional acceleration effect of the design process has been discovered recently by the analysis of various design strategies with the different initial points [14]. This effect can be proposed as one of the principal elements of the time-optimal algorithm construction. The main step of this construction is
the definition of the control vector optimal switch points during the design process. This problem is discussed in the present paper by means of the property study of the special Lyapunov function of the design process.

2 Problem Formulation

The design process for any analog system design has been generalized on the basis of the control theory approach as shown in [12]. In this case the design process is defined by means of the optimization procedure (1) and by the analysis of the system (2):

\[ X^{s+1} = X^s + t_s \cdot H^s \]  \hspace{1cm} (1)

\[ (1 - u_j^s) g_j (X) = 0, \quad j = 1, 2, \ldots, M \]  \hspace{1cm} (2)

where \( M \) is the number of dependent parameters and \( H \) is the vector of directional movement. In this case the vector \( H \) depends not only on the optimization procedure and objective function structure but from the vector of special control functions \( U = (u_1, u_2, \ldots, u_M) \) that control the design process, where \( u_j \in \Omega; \Omega = \{0;1\} \). In this case a new generalized objective function is needed to define as \( F(X) = C(X) + \psi(X) \) with a special additional penalty function \( \psi(X) = \frac{1}{e} \sum_{j=1}^{M} u_j \cdot g_j^2(X) \). All control variables \( u_j \) are the functions of the current point of the design process. The total number of the different design trajectories produced inside the same optimization procedure is practically infinite. The problem of the optimal design strategy search is formulated now as the typical problem for the functional minimization of the control theory.

The numerical results for the different electronic circuits shown that the optimal control vector \( U_{opt} \) can be found and can reduce the total computer time significantly [13]. The optimal trajectory is differed from the traditional design strategy \( (u_j = 0, \forall j=1,2, M) \) and differed from the modified traditional design strategy \( (u_j = 1, \forall j=1,2, M) \) as shown in [15]. Comparing this result with [9]-[11] we can conclude that idea, which was realized in [9]-[11] is not optimal from the computer time point of view.

The discovery of the additional acceleration effect of the design process [14] allows defining the essential features of the optimal algorithm. The optimal algorithm consists of one or several trajectory jumps from quasi modified traditional strategy to quasi traditional strategy. Acceleration effect leads to large computer time gain. For instance the time gain for two transistor cells amplifier is near 100 times and for three transistor cell amplifier more than 400 times as shown in previous papers [12]-[13]. On the other hand the construction of the optimal algorithm on basis of the acceleration effect turns on unknown switch point positions for the control functions \( u_j \).

3 Initial Point Optimal Selection

3.1 Two-dimensional problem

The problem of the initial point selection for the design process is one of the main problems of the time-optimal algorithm construction. The analysis of the acceleration effect for the simplest electronic circuit of the Fig. 1 was provided in [14].

The nonlinear element has the next dependency: \( R_s = r_0 + bV_i^2 \). The vector \( X \) of the state variables has two components \( X = (x_1, x_2) \), where \( x_i \) is the independent parameter \( (x_1 \equiv r_2) \) and \( x_2 \equiv V_1 \). The objective function is defined by the formula \( C(X) = (x_2 - k_v)^2 \), where \( k_v \) has a fixed value. The optimization procedure in accordance with the new design methodology is defined by the equations:

\[ x_i^{s+1} = x_i^s + t_s \cdot f_i(X, U), \quad i = 1, 2 \]  \hspace{1cm} (3)

where the right hand side \( f_i(X, U) \) for the gradient method can be defined as:

\[ f_1(X, U) = -\frac{\delta}{\delta x_1} F(X, U), \]

\[ f_2(X, U) = -u_1 \frac{\delta}{\delta x_2} F(X, U) + \frac{(1-u_1)}{t_s} \frac{\delta}{\delta x_2} x_2^2 + n(x) \]  \hspace{1cm} (4)

![Fig. 1. Topology of a simplest electronic circuit](image)


The vector of the control variables \( U \) consists on one coordinate \( u_i \) only. The equation (2) is transformed now to the next form:

\[
(1 - u_i) g_1(X) = 0
\]

(5)

\( F(X,U) \) is the generalized objective function, \( \eta_2(X) \) is the implicit function \( x_2^{i+1} = \eta_2(X) \) and it gives the value of the parameter \( x_2 \) from the equation (5). The vector of the control variables \( U \) consists on one coordinate \( u_i \) only for this example.

The idea of the system design problem definition as the problem of functional minimization of the control theory does not have dependency from the optimization method and can be embedded into any optimization procedure as shown in [12].

We can select the initial point of the design process with the negative coordinate \( x_2 \) as shown in [14]. In this case the acceleration process is realized. We analyze the characteristics of the acceleration effect to decide what value of the coordinate \( x_2 \) is better. The family of the design curves for the circuit on Fig. 1, which corresponds to the modified traditional design strategy \( (u_i = 1) \) and the negative initial value of the second coordinate \( (x_2 < 0) \) of the vector \( X \) is shown in Fig. 2 for the 2-D phase space.

Fig. 2. Trajectories of the modified traditional strategy for the different start points with the negative coordinate \( x_2 \).

These curves have different start points but the same final point F. The start points were selected on the circle arc and have the different initial coordinates. The special curve SF, which is marked by thick line, is the separating curve. This curve separates the trajectories that are the candidates for the acceleration effect achievement (all curves that lie under the curve SF), and the trajectories that can not produce the acceleration effect (curves that lie over the curve SF). It is clear that the projections of the final point F to all curves of the first group define the switch point of the optimal trajectory, which produces the acceleration effect. All curves of the first group (1-7) approach to the final point F from the left side, and all curves of the second group (9-16) approach to the final point from the right side. The comparison of the relative computer time for all curves of the Fig. 2 is shown in Fig. 3 as the function of the curve number \( n \). The separating curve SF has the minimal computer time among all of the trajectories. At the same time this curve can not be used as the basis for the time-optimal trajectory construction because the projection of the point F to this curve is the same point F, but the movement slows down considerably near this point. Only the curves that lie under the curve SF can be serve as the first part of the time-optimal trajectory with the following jump to the point F. The relative computer time \( \tau \) of the optimal trajectories with acceleration effect (on the basis of the curves 1-7, Fig. 2) is shown in Fig. 4 as the function of the curve number \( n \). The curves 9-16 can be optimized too but the time reduction about 10-15% only takes place.

Fig. 3. Relative computer time \( \tau \) as the function of the curve number \( n \).

Fig. 4. Relative computer time \( \tau \) of the optimal trajectories with acceleration effect as the function of the curve number \( n \).
Fig. 4 shows that the total computer time increases when the start point approaches to the curve $SF$, and on the contrary, the more acceleration can be obtained if the start point lies far from the curve $SF$ (from curve 7 to curve 1). So, the start point selection with at least one negative initial coordinate of the vector $X$ and the value of this coordinate that gives the start point position under the separating line are the sufficient conditions for the acceleration effect appearance.

3.2 N-dimensional problem

All above mentioned conclusions are correct for the $N$-dimensional problem too. We need to analyze the different projections of the $N$-dimensional curve in this case. The $N$-dimensional problem solution gets complicated by a large number of the different admissible trajectories and a large number of the different trajectory projections. In this case we need to choose the most perspective trajectories and analyze them. It can be done by some approximate methods of the control theory [16]. In present paper it was done by the careful analysis of all possibilities. The total set of the various trajectories can be divided in two different subsets. The first subset consists of the trajectories that are similar to the traditional design strategy trajectory. The second subset consists of the trajectories that are similar to the modified traditional strategy trajectories. In this case the trajectories of the second group serve as the candidates for the first part of the optimal trajectory and the first group trajectories serve as the candidates to the jump produce. Two of these main steps together with the following different trajectory adjustment make up the essence of the optimal algorithm construction. We can decide from the experience that not all of the feasible projections are important to the acceleration effect obtained. First of all the admittance-voltage two-dimensional projections are more important. Variables that are included in the objective function formula have a greater importance among all of these projections. By this preliminary selection we can reduce the number of the more perspective candidates for the time-optimal algorithm elaboration. This problem final solution will be based on the optimal algorithm intrinsic structure. However, the results obtained here serve as the next step on the way of this problem solution. Now it is clear that the optimal algorithm must include the special conditions for the start point selection to the acceleration effect reach. On the other hand the problem of the switch point optimal position determination of the control vector is discussed in the next section.

4 Switch Point Definition

On the basis of the analysis in previous section we can conclude that the time-optimal algorithm has one or some switch points where the switching realize from like modified traditional strategy to like traditional strategy with an additional adjusting. At least one negative component of the start value of the vector $X$ is needed for the optimal trajectory obtained. The main problem of the time-optimal algorithm construction is the unknown sequence of the switch points during the design process. We need to define a special criterion that permits realizing the optimal or quasi-optimal algorithm by means of the optimal switch points searching. In this paper we propose to use a Lyapunov function of the design process for the optimal algorithm structure revelation, in particular for the optimal switch points searching. There is a freedom of the Lyapunov function choice because of a non-unique form of this function. Let define the Lyapunov function of the design process as:

$$V(Y,U) = \sum_i \left( \frac{\partial F(Y,U)}{\partial x_i} \right)^2$$

where $F(Y,U)$ is the generalized objective function of the optimization procedure. This form holds all of the necessary characteristics of the standard Lyapunov function definition. It is supposed that the vector $Y$ is defined as the difference between two vectors $X$ and $A$, where $A$ is the stationary point of the design process (the final point). First of all the function (6) can be used for the stability analysis of the design process. In this context this function is used for the analysis of the design trajectories behavior with the different switching points. We can define now the system design process as a transition process that provides the stationary point during some time. The problem of the time-optimal design algorithm construction is the problem of the transition process searching with the minimal transition time. There is a well-known idea [17]-[18] to minimize the transition process time by means of the special choice of the right hand part of the principal system of equations, in our case these are the functions $f_i(X,U)$. By this conception it is necessary to change the functions $f_i(X,U)$ by means of the control vector $U$ selection to obtain the maximum speed of the Lyapunov function decreasing (the maximum of $-dV/dt$) at each point of the process. Unfortunately the direct using of this idea does not serve well for the time-optimal design
algorithm construction. It occurs because the change of the design strategy produces not only continuous design trajectories (when we change the strategy $u_i = 0$ to the strategy $u_i = 1$ for the circuit in Fig. 1 for instance) but non-continuous trajectories too (the changing from $u_i = 1$ to $u_i = 0$). Non-continues trajectories had never appeared in control theory for the objects that are described by differential equations, but this is the ordinary case for the design process on the basis of the described design theory. In this case we need to correct the idea to maximize $-dV/dt$ at each point of the design process. We define another principle: it is necessary to obtain the maximum speed of the Lyapunov function decreasing for that trajectory part which lies after the switch point. In this case the trajectories with the different switch points are compared to obtain the maximum value of $-dV/dt$. This idea is realized technically by some probes comparing with the different switch points. Than the best probe can be selected that provides the maximum value of $-dV/dt$ after the switching. Numerical results prove this idea. The two-cell transistor amplifier circuit is shown in Fig. 5.

![Fig. 5. Circuit topology for two-cell transistor amplifier.](image)

As shown in [15] the optimal design strategy for this amplifier has the time gain near 100 comparing with the traditional design strategy. Analysis of the Lyapunov function time derivative gave the next results. The behavior of the function $dV/dt$ for this circuit for three neighbor switch points 1, 2 and 3 that correspond to the five consecutive integration steps before (a), (b) in (c) and after (d), (e) the optimal point is shown in Fig. 6. The optimal switch point corresponds to the curve 3 of Fig. 6 (b), or curve 2 of Fig. 6 (c), or curve 1 of Fig. 6 (d). It is clear that this point corresponds to the maximum negative value of function $dV/dt$ and at the same time corresponds to the minimum value of the total design steps. In this case we suppose that the optimal position of the switch point is found. This

![Fig. 6 Time derivative of Lyapunov function for three switch points 1,2,3 of consecutive integration steps before (a), (b) in (c) and after (d), (e) the optimal point.](image)
switch point or some switch points serve as the basis to the time-optimal design algorithm construction. It is clear that we are forced to lose the computer time to do some probes and to look for the optimal position of the switch points. It means that we can never obtain the time gain, which characterizes the veritable optimal strategy. It was determined from the analyzed examples that the time loses can have the same order as the optimal algorithm own computer time. So, the maximum real time gain is equal to 50 for the circuit in Fig. 5. This result twice worse than the theoretic prediction, but this gain is significant too.

5 Conclusion

The traditional design approach to the analog system design is not time-optimal. The problem of the time-optimal algorithm construction can be solved as the functional optimization problem of the control theory. Three main ideas can be define to the optimal algorithm construction: the acceleration effect of the design process, the optimal selection of the design algorithm start point and the optimal switch point determination of the control vector for the best design trajectory realization. The initial point selection permits to obtain the additional acceleration effect of the design process with a great probability. This effect reduces the total computer time additionally and serves as the basis for the optimal or quasi optimal algorithm construction. The optimal position of the necessary switch points can be obtained on the basis of the analysis of the design process Lyapunov function. The minimization of the time derivative of this function serves as the principal criterion to the optimal switch point definition. Thus the combination of three above mentioned ideas serves as the principal mechanism for the quasi optimal algorithm construction.

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