Nonlinear Optimal Power System Voltage Regulator Design

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Abstract: - The design of new optimal nonlinear voltage regulator for power systems is considered. In the proposed approach, the Hamilton-Jacobi-Belman (HJB), a partial differential equation, is solved using Taylor series expansion of its nonlinear terms. The performance of the proposed controller in a single-machine infinite-bus power system is simulated and the advantages of the nonlinear feedback controllers are investigated. The proposed controller action is simulated in different operation points. Simulation results show that the resulting nonlinear controller has greater domain of validity than that of linearized counterpart.

Key-Words: -Nonlinear control, optimal control, power system, voltage regulation, Hamilton-Jacobi-Belman equation.

1 Introduction

Although power system stability may be broadly defined according to different operating conditions [1], an important problem, which is frequently considered, is the problem of transient stability. It concerns the maintenance of synchronism between generators following a severe disturbance. By the excitation control in a generating unit transient stability can be greatly enhanced [2,3,4]. Another important issue of power system control is to maintain steady acceptable voltage under normal operating and disturbed conditions, which is referred as the problem of voltage regulation [5,6,7].

Voltage quality is a very important index of power supply in power system operations both utility equipment and customer equipment are designed to operate within a certain voltage range. Prolonged operation of the equipment at voltage outside the available range could adversely affect its performance and possibly cause it to be damaged. The generator excitation system is the most important means of voltage control in power systems; it should maintain the generator terminal voltage as a constant under normal operation conditions and regulated to its prefault steady value quickly and effectively after a fault occurs. In recent years considerable efforts have been devoted to the enhancement of power system stability, but much less attention has been paid to voltage controller design. In this paper, we will consider the design of optimal nonlinear voltage regulators for single-machine infinite-bus power system. In order to obtain the nonlinear optimal control law, a partial differential equation so-called Hamilton-Jacobi-Belman (HJB) should be solved. It is so difficult to find an exact answer to this equation so we have to use approximate methods. In this work, we will use the Taylor Series expansion of nonlinear terms appeared in the HJB equation. It is well established that the nonlinear feedback control law has greater domain of validity than that of linearized counterpart [8,9]. Simulation of the proposed controller show that using the nonlinear controller, the fault tolerance of the system is increased and the protective relays have more time to act.

2 Model of the system

In this paper, a simplified dynamic model of a power system, namely, a single machine-infinite bus (SMIB) power system is considered [1,4]. This model consists of a single synchronous generator connected through a parallel transmission line to a very large network approximated by an infinite bus (Fig. 1).

The classic third order single-axis dynamic model of the SMIB power system Fig.1 can be written as follows:

Mechanical equations

\[ \delta(t) = \omega(t) - \omega_0 \]  \hspace{1cm} (1)  
\[ \dot{\omega}(t) = -\frac{D}{2H} (\omega(t) - \omega_0) - \frac{D\omega_0}{2H}(P_e(t) - P_m) \] \hspace{1cm} (2)
The mechanical input power $P_m$ is treated as a constant in the excitation controller design, i.e., it is assumed that the governor action is slow enough not to have any significant impact on the machine dynamics.

**Generator electrical dynamics**

$$E_q'(t) = \frac{1}{T_{do}}(E_f(t) - E_q(t))$$

**Electrical equations (Assumed $x_d' = x_q$)**

$$E_q(t) = E_q'(t) + (x_d - x_q')I_d(t)$$

$$E_f(t) = k, u_f(t)$$

$$P_e(t) = \frac{E_q'(t)V_s}{x_{ds}} \sin \delta(t)$$

$$I_d(t) = \frac{E_q'(t) - V_s \cos \delta(t)}{x_{ds}}$$

$$I_q(t) = \frac{V_s}{x_{ds}} \sin \delta(t)$$

$$Q(t) = \frac{E_q'(t)V_s}{x_{ds}} \cos \delta(t) - \frac{V_s^2}{x_{ds}}$$

$$\delta(t) = \text{Power angle of the generator, radians}$$

$$\omega(t) = \text{Rotor speed of the generator, radian/s}$$

$$\omega_s = \text{Synchronous machine speed, radian/s}$$

$$P_m = \text{Mechanical power, p.u.}$$

$$P_e(t) = \text{Active electrical power delivered by the generator, p.u.}$$

$$E_q'(t) = \text{Transient EMF in the quadratic axis of the generator, p.u.}$$

$$E_q(t) = \text{EMF in the quadratic axis of the generator, p.u.}$$

$$E_f(t) = \text{Equivalent EMF in the excitation coil, p.u.}$$

$$V_f(t) = \text{Generator terminal voltage, p.u.}$$

$$u_f(t) = \text{Input of the SCR amplifier of the generator, p.u.}$$

$$V_s = \text{Infinite bus voltage, p.u.}$$

The fault considered in this paper is a symmetrical three-phase short circuit fault, which occurs on the middle of one of the transmission lines. When the fault on the transmission lines is removed, the breakers of the lines are opened.

To achieve a good steady state performance and voltage regulation the generator terminal voltage should be one of the control variables. So, we rewrite the system equations (1)-(3) as the following

$$V_f = \left[ \frac{k}{T_{do}}u_f - \frac{x_{ds}}{T_{do}x_{ds}}E_q + \frac{V_s(x_d - x_q')}{T_{do}x_{ds}} A_1(E_q'x_{ds}^2 + x_qx_f'V_s' A_2) - \frac{x_q'V_s' E_f'(\omega - \omega_s)A_2}{V_s x_{ds}} \right] / V_s x_{ds}$$

$$\dot{\omega} = \frac{D}{2H}(\omega - \omega_s) + \frac{\alpha_0}{2H}P_m - \frac{\alpha o}{2H}E_f'(A_1)$$

$$\dot{E}_q = \frac{k}{T_{do}}u_f - \frac{x_{ds}}{T_{do}x_{ds}}E_q + \frac{V_s(x_d - x_q')}{T_{do}x_{ds}} A_1$$

where

$$A_1 = \frac{V_s^2 x_{ds}^2 - E_q'^2 x_{ds}^2 - V_s^2 x_q'^2}{2x_q'x_{ds}E_q'V_s}$$

$$A_2 = (1 - A_1^2)$$

3 Design of optimal nonlinear feedback controller

Optimal control is the determination of control signals in order to minimize or maximize a definite cost function while fulfilling some constraints. Using dynamic programming and optimality principle results in a nonlinear partial differential equation known as the HJB equation. This equation has the following form [10,11]

Assume a system with the following differential equation:

$$\mathcal{X} = g(\mathcal{X}(t), u(t), t)$$

where $\mathcal{X}$ is the state variables vector and $u$ is the system input vector. The problem of optimal control design is to control the above system such that the following cost function is minimized:

$$J = h(\mathcal{X}(t_f), t_f) + \int_{t_0}^{t_f} g(\mathcal{X}(\tau), u(\tau), \tau) d\tau$$

where $g, h$ are definite functions and $t_o, t_f$ are constants. So, according the HJB equation:

$$J^*(\mathcal{X}(t), t) + H(\mathcal{X}(t), u^*(\mathcal{X}(t), J^*, t), t) = 0$$

where $J^*$ is the minimum of cost function and $u^*$ is the input vector that minimize $H$, and $H$ is Hamilton function that is defined as follows:

![Figure 1](image.png)
Expansion of these possible solutions is using methods to find approximate solution for \( J^* \) form of Taylor expansion of order \( n \). According to (16) \( u^* \) is a function of \( J^* \), so expressing \( J^* \) in the form of Taylor expansion of order \( n \) leads to a controller of order \( (n-1) \). In order to obtain \( J^* \) in the form of Taylor expansion of order \( n \), we will use the method of [9, 12].

1. Using a substitution we define new state variables as the deviation of the state variables from their steady state initial values.
2. If the system differential equation (12) consists of nonlinear terms, they will be replaced by their Taylor’s Series expansion of order \( (n-1) \).
3. \( J^* \) is written as \( n \) ordered polynomial of state variables \( (x_1, x_2, ..., x_m) \). In this form, expressing \( J^* \) is equivalent to a Taylor’s Series expansion of the state variables \( (x_1, x_2, ..., x_m) \), the coefficients of all terms are considered unknown. Due to express \( J^* \) as the Taylor Series expansion completely, all possible terms up to order \( n \) should be included. All possible terms up to order \( i \) for the variables \( (x_1, x_2, ..., x_m) \) can be obtained by expansion of \( (x_1 + x_2 + ... + x_m)^i \) regardless of their coefficients.
4. The Taylor Series expansion of \( J^* \) is given to the HJB equation (15) and the coefficients of different terms is sorted. Then the coefficients of all terms in the form of \( x_1^i x_2^j ... x_m^k \) are set equivalent to zero. Using this method, some nonlinear equations of unknown coefficients in Taylor’s Series expansion of \( J^* \) are obtained.
5. The nonlinear equations obtained this way are solved by numerical methods like Newton-Rafson. So, the value of each coefficient can be calculated. Thus, determining the Taylor’s Series expansion of \( J^* \) will result in \( u^* \).

In here the control variables are

\[
x_1 = V_r - V_m, \quad x_2 = \omega - \alpha_0, \quad x_3 = E_q' - E_q 0
\]

and the cost function is defined as

\[
J = \frac{1}{2} q_1 (V_r - V_m)^2 + \frac{1}{2} q_2 \omega^2 + \frac{1}{2} r u^2
\]

Finally, some nonlinear equations are obtained that can be solved using a symbolic software like MAPLE or MATHEMATICA. In here we have used MAPLE V.

### 4 Simulation results

The simulations have been performed in MATLAB 6. The system parameters and initial conditions are as the following

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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>( D )</td>
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<td>( x_a )</td>
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<tr>
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<td>( x_d )</td>
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<td>( q_2 )</td>
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</tr>
<tr>
<td>( r )</td>
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</tr>
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</table>

Using the proposed procedure in the previous section the following voltage controllers are obtained:

The optimal first order voltage controller:

\[
u^* = 79.3725849 x_1 + 2.018544181 x_2 - 57.8355351 x_3
\]

The optimal second order voltage controller:

\[
u^* = 79.3725836 x_1 + 2.018544241 x_2 - 57.8355351 x_3 - 77.1644356 x_1^2 - 35.9840078 x_2 x_1 - 50.2542583 x_3^2 - 4002697124 x_1^3 + 86.99441103 x_1 x_3 + 20.49521461 x_2 x_3
\]

The optimal third order voltage controller:

\[
u^* = 79.372591 x_1 + 2.018544244 x_2 - 57.8355324 b_3 x_3 - 77.1644354 x_1^2 - 35.9840074 x_2 x_1 - 50.2542588 x_3^2 - 4002697124 x_1^3 + 86.99441201 x_1 x_3 + 20.49521469 x_2 x_3 - 30.4421833 x_1^3 + 72.97194295 x_2 x_1^2 + 52.1987548 x_3 x_1^2 - 497247236 x_1^4 - 25259903 x_2 x_1 + 88.37363305 x_2 x_3 - 29.778551 x_1 x_3^2 + 56.52716415 x_3^3 + 26.4213571 x_3^4
\]

The fault is a symmetrical three-phase short circuit fault with its sequences described as

1. Stage 1: The system is in the prefault steady states;
2. Stage 2: A fault occurs at \( t = t_f \);
3. Stage 3: The fault is removed at \( t = t_f + t_f \);
4. Stage 4: The system is in the post fault state;

In the simulation we will chose \( t_f = 0.1 \) s. In this section the fault is considered with \( t_f = 0.1 \) sec . In Fig. 2 the generator terminal voltage using the first, second and third order controllers are shown. We can see that in this situation the performance of the second order controller is better than the first.
order one and for the third order controller is better than both of them. In fact, the performance of higher order controllers is better than lower order ones. In order to further investigate the performance of each controller we use the energy of terminal voltage vibrations as a criterion to evaluate the quality of control action. The energy of the vibrations can be defined as follows

\[ E_v(t) = \int_{t_0}^{t} (V_i(t) - V_{in})^2 \, dt \]  

With the above criterion we can compare the ability of controllers to damp the vibrations. This comparison is shown in Fig. 3. It can be seen that among these controllers the third order controller has the best performance in controlling the energy of the vibrations.

Another issue about the performance of the nonlinear feedback controllers is the fault tolerance of the system using each controller. In Fig. 4 the terminal voltage for a fault with \( t_f = 0.12 \) sec using the first and second order controllers is shown. It is seen that in this situation the first order controller cannot stabilize the system (dotted line), but the second order controller can stabilize the system (solid line). In Fig. 5 the above fault with \( t_f = 0.18 \) sec is imposed to the system. We can see that in this case the second order controller can not stabilize the system affected by the fault (dotted line), but using the third order controller the power system can be stabilized (solid line). These simulations show that the use of higher order controllers increases the fault tolerance of the power system and the protection relays have more time to act.

In order to investigate further effects of nonlinear feedback controllers to the system response, consider the system with new operating point when the input power is 40% greater than the former, \( P_m = 1.12 \) p.u. In Fig. 6 the closed loop response with the first and third order controllers for the new operating point is shown. It is clear that in this case the system with the second order controller is unstable (dotted line), but the third order controller can stabilize the system (solid line). In fact, the nonlinear control law has greater domain of validity than the linearized counterpart.
Figure 6. Terminal voltage for the fault in new operating point, $P_m = 1.12 \text{p.u.}$, using the first and third order controllers.

5 Conclusion

In this paper, the design of optimal nonlinear state feedback voltage regulator for power systems based on the approximate solution of the $H_{\infty}$ equation was presented. Simulation results showed that higher order controllers have better performance in the sense of controlling the energy of vibrations. Also, using higher order controllers the fault tolerance and domain of validity was increased.

References:


