Fuzzy Sliding Mode Controller Design with Variable Sliding Surface

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Abstract: In this paper, a fuzzy sliding mode controller (FSMC) with the variable sliding surface instead of the fixed sliding surface and adjustable membership functions are proposed for the control of second order system. The Taguchi’s method and the Takagi-Sugeno (TS) fuzzy model are applied to regulate the membership functions and modify the time-varying parameters, respectively. Through these modifies, we can obtain a suitable membership functions without trial-and-error and improve the oscillation phenomenon of control law during the reaching phase. Finally, an inverted pendulum system is proposed to simulate and contrast with the performances by using the conventional FSMC.

Key-Words - Taguchi’s Method; Takagi-Sugeno fuzzy model; variable sliding surface

1 Introduction
The fuzzy sets theory was introduced by Zadeh in 1965 [1]. Since then, several research efforts have led to rapid developments in fuzzy set theory and its applications [2]. Fuzzy logic controller (FLC), in the application of fuzzy sets theory [3, 4], has succeeded in many control problems that the conventional control theories have difficulties to deal with. Although these fuzzy control systems have achieved many practical successes in many fields, quite frequently the design of the FLC depends to a large extent on the expert’s knowledge and the huge amount of fuzzy rules making the analysis complex. The other problem in the design of FLC is that the suitable membership functions should be given by time-consuming trial-and-error procedure. For these reasons, to construct the fuzzy controller effectively and efficiently becomes an active research topic in FLC design.

To overcome the first drawback, the design of a FLC with the fuzzy sliding surface is proposed. The sliding-mode control approach is one of the robust control methods to handle systems with model uncertainties [1]. Systematic design procedures for sliding-mode controller are well known and available in the literature [5]. In many fuzzy control systems, the rules are composed by using the error and the rate of error [6-8], but we combine these two types of information into one type. Thus, we can reduce the dimension of the input space of the FLC and the number of fuzzy rules. The design of the FSMC also can improve the undesired chattering phenomenon due to the high-frequency switching near the sliding surface in the sliding mode control.

To overcome the second drawback and simplify the design procedure of the FSMC, the design procedures, Taguchi’s method, is applied. First, we give each design factor three values by designer, respectively. Then, we simulate nine experiments each takes different combination of the values of all factors. At last, we find out some values, each for one design factor, which can lead the best performance under these selectable values by a series of analytical procedures of Taguchi’s method. When the dimension of the input of the controller is huge, the time-consuming trial-and-error procedure in getting the suitable membership functions will be avoided.

On the other hand, when we observe the figure of response of control law, we can find
that the control law oscillates heavily during the reaching phase. This is because the state is far from the sliding surface $s$ at the beginning of our simulation, the control law may be too large to let the state stabilize to the boundary layer $\Phi$ of the sliding surface. For overcoming this problem, a FSMC with variable sliding surface is proposed even there have parameter variation and disturbance in the system. At the same time, the TS fuzzy model [9] is applied to the regulation of the variable sliding surface.

2 The design of fuzzy sliding mode controller with VSS

Now, we will introduce the design of a FSMC with variable sliding surface (VSS) in the following.

2.1 The time-varying parameters $\lambda(t)$ and $\beta(t)$ of variable sliding surface

The difference between the variable sliding surface and the fixed sliding surface is that the parameter $\lambda$ in variable sliding surface is time-varying. The function of variable sliding surface is described as follows:

$$s(X;t) = \frac{d}{dt} + \dot{e}(t) + e = \dot{x} - x_d.$$  \hfill (1a)

and

$$s(X;t) = \ddot{e} - \dot{\lambda}(t)$$  \hfill (1b)

when the state is in region (II or IV) and region (I or III) of Fig. 1, respectively. Then, if the VSS described as (1) satisfy the following stable conditions [10], the control system will satisfy the Lyapunov theorem [11] and the system trajectory will converge to the boundary layer of the VSS.

$\lambda > 0$ and $1 + \lambda^2 > \dot{\lambda}$

if $\lambda < \lambda_f$, then $\dot{\lambda} > 0$ ($\lambda_f$: the final value.)

if $\lambda > \lambda_f$, then $\dot{\lambda} < 0$  \hfill (2)

if $\lambda = \lambda_f$, then $\dot{\lambda} = 0$

and

$\beta > 0$ and $\dot{\beta} < 0$

$\beta < 0$ and $\dot{\beta} > 0$  \hfill (3)

if $\beta = \beta_f$ then $\dot{\beta} = 0$ ($\beta_f$: the final value )

The parameters $\lambda(t)$ and $\beta(t)$ of the sliding surface that satisfies the conditions in (2) and (3) will be designed.

2.2 The design of parameters $\lambda(t)$ and $\beta(t)$ by using the TS fuzzy model

It is observed that the functions of $\lambda(t)$ and $\beta(t)$ are nonlinear functions and can not easily be obtained in a closed form. To overcome this situation, TS fuzzy model [11-12] is used to infer the nonlinear functions $\lambda(t)$ and $\beta(t)$ by employing the advantages of local linearization of TS model. In the following, the procedure will be introduced.

The TS fuzzy model is of the following form:

$L^i: If \ x_i \ is \ A_i^i \ and \ \cdots \ \ and \ x_m \ is \ A_m^i$ Then \ $y^i = a_0^i + a_1^i x_1 + \cdots + a_m^i x_m$ \hfill (4)

where $L^i$ ($i = 1, 2, \cdots, n$) denotes the $i$th fuzzy implication, $n$ is the number of fuzzy implications, $x_1, \cdots, x_m$ are input variables, $y^i$ is the output from the $i$th fuzzy implication, $a_0^i, \cdots, a_m^i$ are consequent parameters, $A_1^i, \cdots, A_m^i$ are fuzzy membership functions. That is, the considered rules whose IF part is fuzzy but whose THEN part is crisp. The output is a linear combination of input variables. For a real-valued input vector $X = [x_1, \cdots, x_m]^T$, the defuzzified output $y$ of TS fuzzy system is a weighted average of the $y^i$s:

$$y = \sum_{i=1}^{n} w^i y^i / \sum_{i=1}^{n} w^i$$  \hfill (5)

where the weight $w^i$ implies the overall truth value of the premise of rule $L^i$ for the input and is calculated as

$$w^i = \prod_{j=1}^{m} A_{ij}^i (x_{ij}^0)$$  \hfill (6)

After the determination of time-varying parameters $\lambda(t)$ and $\beta(t)$ by some fuzzy rules, the VSS with time-varying parameters $\lambda(t)$ and $\beta(t)$ can be obtained by (1). Then, we will introduce the design of FSMC with VSS in next section.
2.3 Design of FSMC with variable sliding surface

This paper focuses on a class of second order nonlinear system represented by
\[ \dot{x}(t) = f(X; t) + b(X; t)u(t) + d(t) \]  
\[ |f(X; t) - \tilde{f}(X; t)| \leq F(X; t) \]  
\[ |d(t)| \leq D(X; t) \]  
where \( X \) is the state vector, \( u(t) \) is the control input, \( d(t) \) is the disturbance, and the dynamics \( f(X; t) \) (possible nonlinear or time-varying) is not known exactly, but estimated as \( \tilde{f}(X; t) \). The estimation error on \( f(X; t) \) and the disturbance \( d(t) \) are bounded by the known function \( F(X; t) \) and \( D(X; t) \), respectively.

Define the tracking error \( e = x - x_d \) and the tracking error vector \( E = X - X_d = [\tilde{x}, \dot{x}] \), where \( x_d \) is the desired trajectory. Then our control object is to make the error dynamics of the system to be globally stable by designing a proper FSMC and a sliding surface defined as (1) can determine a fuzzy set \( \tilde{F}_s \) in \( X \), each rule \( R^i \) is the state vector,  \( \tilde{s} \) is the fuzzy sliding surface defined by a linguistic expression. For the second order system, we use the following fuzzy sets of \( F_s \), \( l = 1, 2, \ldots, n \), to partition the universe of discourse of \( s \).

\[ T(\tilde{s}) = \{NB, NM, ZR, PM, PB\} \]
\[ = \{\tilde{F}_s^1, \tilde{F}_s^2, \tilde{F}_s^3, \tilde{F}_s^4, \tilde{F}_s^5\} \]  
(10)
where \( T(\tilde{s}) \) is the term set of \( \tilde{s} \), and \( NB, NM, ZR, PM \) and \( PB \) are labels of fuzzy sets, which are negative big, negative medium, zero, positive medium, and positive big, respectively. For the control output, \( u \), its term set and labels of the fuzzy sets are defined similarly by:

\[ T(\tilde{u}) = \{SMALLER, SMALL, MEDIUM, BIG, BIGGER\} \]
\[ = \{\tilde{F}_u^1, \tilde{F}_u^2, \tilde{F}_u^3, \tilde{F}_u^4, \tilde{F}_u^5\} \]  
(11)
From the above two term sets, we can build the following fuzzy rules according to the distance between the output and the desired trajectory:

\[ R^1: \text{If } s \text{ is } NB, \text{ then } u_f \text{ is } BIGGER \]
\[ R^2: \text{If } s \text{ is } NM, \text{ then } u_f \text{ is } BIG \]
\[ R^3: \text{If } s \text{ is } ZR, \text{ then } u_f \text{ is } MEDIUM \]  
(12)
\[ R^4: \text{If } s \text{ is } PM, \text{ then } u_f \text{ is } SMALL \]
\[ R^5: \text{If } s \text{ is } PB, \text{ then } u_f \text{ is } SMALLER \]

Let \( X \) and \( Y \) be the input and output space of the fuzzy rules, respectively. For any arbitrary fuzzy set \( \tilde{F}_s^j \) in \( X \), each rule \( R^i \) can determine a fuzzy set \( \tilde{F}^j_x \circ R^i \) in \( Y \). Use the sup-min compositional rule of inference and suppose \( \tilde{F}_s \) as a fuzzy singleton, then
\[ \mu_{\tilde{F}_s \circ R^i}(u_f) = \min[\mu_{\tilde{F}_s}(\alpha), \mu_{\tilde{F}_s}(u_f)] \]  
(13)
and the deduced membership function \( \tilde{F}_u^d \), of the consequences of all rules is
\[ \mu_{\tilde{F}_u^d}(u_f) = \max[\mu_{\tilde{F}_s \circ R^i}(u_f), \ldots, \mu_{\tilde{F}_s \circ R^5}(u_f)] \]  
(14)

the output \( \mu_{\tilde{F}_u^d}(u_f) \) is a fuzzified output. For the defuzzifier, we use the center-of-area defuzzifier to find the crisp output such as
\[ u = \frac{\int u_f \cdot \mu_{\tilde{F}_u^d}(u_f) du_f}{\int \mu_{\tilde{F}_u^d}(u_f) du_f} \]  
(15)

Then, a FSMC with variable sliding surface which satisfies the reaching condition will be designed, the reaching condition is
\[ s \leq -\eta |s| \quad \text{for } \eta > 0 \]  
(16)
and \( u \) has the following form
\[ u = \hat{u} - k(X, t) \cdot \text{sig}(s/\Phi), \]  
(17)
where
\[ k(X, t) \geq \{b^{-1}|F(X; t) + D(X; t) + \eta|, \quad \eta > 0 \]  
(18)

\[ \text{sig}(\Phi) = \begin{cases} 
-1, & \text{if } \phi < -1 \\
-1 \cdot (2 \phi + 3)(3 \phi + 1), & \text{if } -1 \leq \phi < -1/2 \\
2 \cdot 4 \phi^2 + 6 \phi + 1, & \text{if } -1/2 \leq \phi < 0 \\
-1 \cdot \phi(2 \phi + 3), & \text{if } -1 \leq \phi < -1 \\
2 \cdot 4 \phi^2 + 2 \phi - 1, & \text{if } 0 \leq \phi < 1 \\
1 \cdot (2 \phi - 3)(3 \phi - 1), & \text{if } 1/2 \leq \phi < 1 \\
2 \cdot 4 \phi^2 - 6 \phi + 1, & \text{if } 1/2 \leq \phi < 1 \\
1, & \text{if } \phi \geq 1 
\end{cases} \]  
(19)
and
\[ \hat{u} = (b)^{-1}[-\hat{f} + \tilde{x}_d(t) - \lambda(t)\hat{e} - \hat{\lambda}(t)e], \quad (20) \]
\[ \hat{u} = (b)^{-1}[-\hat{f} + \tilde{x}_d(t) + \hat{\beta}(t)], \quad (21) \]
when the state is in region (II or IV) and region (I or III) of Fig. 1, respectively. The parameters \( \lambda(t) \) in (1a) and \( \hat{\beta}(t) \) in (1b) are derived by section 2.2, respectively.

2.4 The adjustment of membership functions by using the Taguchi’s method

In this section, we will describe that what the adjustable factors of membership functions are and how we adjust the membership functions of the input \( s \) and output \( u \) of our FSMC without trial-and-error for better performance by Taguchi’s method [13-15]. As described in section 2.3, the five fuzzy terms for the sliding surface \( s \) and control law \( u \) are defined in (10) and (11), respectively. Because we hope the boundary layer of sliding surface \( s \) is adjustable, a scaling factor \( \tilde{\delta} \) will be used to adjust the fuzzy set of the sliding surface \( s \). Therefore, the fuzzy sliding surface of a second order system for normalized fuzzy sets is shown in Fig. If the symmetry of fuzzy terms corresponding to \( s \) and \( u \) is assumed, the remaining design factors of the FSMC are the values of \( A, B, C, D, \) and \( \tilde{\delta} \) as shown in Fig. 2-Fig. 4. Next, the adjustment of the membership functions by Taguchi’s method will be introduced.

The Taguchi’s method is a kind of experimental design method that to find a set of values of the design factors of membership functions which obtain the performance what we want by using the Taguchi’s method. In the following, the adjustment method of membership functions is introduced.

1. Give one design parameter to the design factor \( C \) for deciding the range of the discourse of the membership function of control law \( u \) by designer. Then, give another design factors each for three parameters, say \( a_i, b_i, d_i, \) and \( \tilde{\delta}_i, i:1,2,3 \) with respect to \( A, B, D \) and \( \tilde{\delta} \) and arrange the parameters like L9 Table [14]. The simulation can be done once at each combination of design factors. In general,
\[ \frac{C}{3} < B < C, \quad 0 < A < B_{\min}, \]
\[ 0 < D < 1, \quad D_{\max} < \tilde{\delta} < 1 \]

2. After the analysis by using the Taguchi’s method[10], we can find the best condition of the membership functions and the design parameters of best condition are the best values of our design factors which can obtain the best performance from the selectable values proposed by designer.

3 Simulation

In this section, we apply the proposed FSMC derived to an inverted pendulum system [3] on the presence of viscous friction and disturbance.
\[ \ddot{x} + q_1\dot{x} + q_2\cos(x) = u + d(t) \] (23)
where \( x \) denotes the angle of the pendulum from the vertical axis. The mass properties of the pendulum and the amount of viscous friction are not known. The actual parameter values are \( q_1 = 0.5 + 0.2\sin(t), \quad q_2 = 3.2 \) while the disturbance term is \( d(t) = \sin(3t) \). It is assumed that these parameters can be written as
\[ q_1 = \hat{q}_1 + \Delta q_1, \quad q_2 = \hat{q}_2 + \Delta q_2. \]
We estimate \( \hat{q}_1 = 0.5, \hat{q}_2 = 3.5 \) and the parameter uncertainty are bounded:
\[ |\Delta q_1| = 0.2\sin(t) \leq 0.2, \quad |\Delta q_2| = 0.3. \]
Then, we can obtain
\[ f(X; t) = -\hat{q}_1\dot{x} - \hat{q}_2\cos(x) = -0.5\dot{x} - 3.5\cos(x), \]
and
\[ |\Delta f(X; t)| = |\Delta q_1\dot{x} + \Delta q_2\cos(x)| \leq 0.2|\dot{x}| + 0.3 = F(X; t) \]
\[ |d(t)| = |\sin(3t)| \leq 1 = D(X, t). \]
Hence, the proper values of control law \( u \) are obtained from (20) and (21) as:
\[ \hat{u} = [0.5\dot{x} + 3.5\cos(x) + \dot{x}_d + \hat{\beta}(t)] \]
\[ u = \hat{u} - [(0.2|\dot{x}| + 0.3) + 1 + 2]\text{sig}(s) \]
and
\[ \hat{u} = [0.5\dot{x} + 3.5\cos(x) + \dot{x}_d - \lambda(t)(\dot{x} - \dot{x}_d) - \hat{\lambda}(t)(x - x_d)] \]
\[ u = \hat{u} - [(0.2|\dot{x}| + 0.3) + 1 + 2]\text{sig}(s) \]
when the initial position of the state is in region (I or III) and region (II or IV) of Fig. 1, respectively. In our simulation, $b = 1$, $\Phi = 1$, $\eta = 2$, for the desired trajectory $x_d(t) = u_d(t)$ (unit-step function). We start the simulation with $x = 0$ rad, $\dot{x} = -0.5$ rad/sec and take the membership function values as Table 1. The results of $S/N$ ratio and values of $\Delta \eta$ are also shown in Table 1. After the analysis of membership functions described in section 2.4, the best condition is obtained as $A = 0.8$, $B = 0.9$, $D = 0.6$, $\delta = 0.8$. The simulation results by using our proposed FSMC and by using the conventional FSMC are shown in Fig. 5 and Fig. 6, respectively. Comparing with these figures, we can find out the rising time, the delay time, and the settling time are shorter than the conventional FSMC. Otherwise, the oscillation of control force $u$ during the reaching phase by using our proposed FSMC is also improved than the conventional FSMC.

4 Conclusion

In this paper, a FSMC with adjustable membership functions and variable sliding surface is proposed for the control of second order system. The Taguchi’s method can be applied to adjust the membership functions of sliding surface $s$ and control law $u$ systemic without trial and error. Otherwise, the motivation of the modification in the fixed sliding surface to variable sliding surface is that we want to overcome the oscillation heavily of control law during the reaching phase. Comparing the simulation results, we can say that the proposed FSMC owns the advantages that adjusts the membership functions without trial-and-error and more stable in the reaching phase.

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References


Fig.1 Phase partition of the $e - \dot{e}$ plane

Fig.2 Fuzzy sliding surface for a second system

Fig.3 Fuzzy partition of the universe of discourse of sliding surface

Fig.4 Fuzzy partition of the universe of discourse of control law

Fig.5 System responses of inverted pendulum system by using our proposed FSMC.
(a) state response (b) control force

Fig.6 System responses of inverted pendulum system by using the conventional FSMC.
(a) state response (b) control force

Table 1: The L9 table for the simulation of inverted pendulum system

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<th>NO.</th>
<th>A</th>
<th>B</th>
<th>D</th>
<th>$\delta$</th>
<th>S/N</th>
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