Feature Subset Selection Based on ANN Sensitivity Analysis – a Practical Study

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Abstract: - Feature subset selection is a central issue in a vast diversity of problems including classification, function approximation, machine learning and adaptive control. On a wide variety of applications, especially when using real data, input features may be not independent and output variable depends on the relationship among inputs rather than on input values themselves. Feature selection methods that assume independence of attributes will fail on these cases. On the other side, most of alternative approaches are quasi-exhaustive, requiring large CPU processing time. In this paper, an alternative methodology based on sensitivity analysis of trained artificial neural networks (ANN) is analyzed. Results so far attained on illustrative toy examples and on real data support the validity of the developed approach.

Key Words: - feature selection, neural networks, sensitivity, correlation

1 Introduction
Feature subset selection (FSS) consists of identifying a subset of significant attributes, discarding the remaining ones, to represent adequately the system state, initially characterized by a larger set of redundant features. FSS is an essential task when using a wide variety of tools like artificial neural networks (ANN), fuzzy sets, the k-nearest neighbors method or regression trees for classification, function approximation, machine learning and adaptive control.

When the number of attributes is small, exhaustive or quasi-exhaustive search may be used to select the best attributes set in order to accomplish the desired task. But the number of possible combinations grows quickly with the number of attributes – the curse of dimensionality. As higher is the number of attributes the faster and straightforward should be the FSS approach. Typically, FSS methods that assume independence of attributes like correlation analysis, F measure, or information gain, are simple and fast, but may fail on a wide sort of applications. On the other side, there are the FSS time-consuming approaches, using quasi-exhaustive or genetic like search, or needing the repeated training of ANN, while less significant features are discarded one by one [4-7]. Besides, some FSS techniques can only deal with binary inputs and/or outputs.

This paper describes an alternative FSS approach that is simple, straightforward, and, as far as our examples had shown, has no hard application limitations.

2 A Failing Example: Rank Correlation
A common FSS approach adopted frequently is based on computing a input marginal importance measure, i.e., considering each input in isolation. As an example, if rank correlation method is adopted, the following algorithm is followed:

a) Computation of the individual correlation’s factors \((x_i; y)\), where \(x_i\) represents possible feature \(i (i=1,...,\text{number of features})\) and \(y\) is the output;
b) Ranking input variables according to its correlation factor with respect to the output;
c) Selection of the N variables with highest values.

As a matter of illustration of how this algorithm may fail, consider Table 1 where the variables \(x_i\) where randomly generated in the interval \([-2; 2]\). 2000 random patterns were produced. The output variable \(y\) was computed for each input pattern \((x_1,...,x_8)\) using the following equation:

\[
y = \frac{1}{(x_2 - x_4)^2 + 1} + x_6 \sin(2x_8)
\]

(1)

Table 1 – Sample of generated training set

<table>
<thead>
<tr>
<th>x1</th>
<th>x2</th>
<th>x3</th>
<th>x4</th>
<th>x5</th>
<th>x6</th>
<th>x7</th>
<th>x8</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>1.68</td>
<td>-1.46</td>
<td>1.37</td>
<td>-0.56</td>
<td>-0.72</td>
<td>1.49</td>
<td>-0.03</td>
<td>1.72</td>
</tr>
</tbody>
</table>

Equation (1) shows that output variable \(y\) does not depend on \(x_1, x_3, x_4\) and \(x_7\). Table 2 results from the calculus of a correlation analysis and ranking the variables according to the absolute value of the correlation indexes.
### Table 2 – Ranked correlation modules indexes

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Variable</th>
<th>Correlation index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x4</td>
<td>0.043</td>
</tr>
<tr>
<td>2</td>
<td>x1</td>
<td>0.042</td>
</tr>
<tr>
<td>3</td>
<td>x5</td>
<td>0.025</td>
</tr>
<tr>
<td>4</td>
<td>x6</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>x8</td>
<td>0.012</td>
</tr>
<tr>
<td>6</td>
<td>x3</td>
<td>0.008</td>
</tr>
<tr>
<td>7</td>
<td>x7</td>
<td>0.006</td>
</tr>
<tr>
<td>8</td>
<td>x2</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Variables x1 and x5, that were not used in the computation of y, appear in the group of 4 highest correlated with output. These results prove that, at least for a certain type of functions, correlation analysis does not provide correct answers on FSS phase. It is clear, when analyzing function (1), that there are variables intimately coupled (e.g., x2 and x4). FSS methods based on measures that relate individually each input xi with output y may fail.

#### 3 Feature Subset Selection Based on ANN Sensitivity Analysis

Now, let’s experience another approach. Our proposal comprises the following steps:

a) Train an ANN to learn y function, using all possible candidate features. ANN will present an output value O that should be close to y. As before, we suppose we don’t know which variables are important or not;

b) For all training patterns, compute ∂O/∂xi, that is, the derivative of the output with respect to each input i. Later on this paper, we describe a simple algorithm for computing these values;

c) Compute the mean absolute value of derivatives for each input, defining our sensitivity index si:

\[
S_i = \frac{\sum_{p=1}^{\text{nr.of patterns}} |\frac{\partial O}{\partial x_i}|_{p \text{th pattern}}}{\text{nr.of patterns}}
\]  

where p represents the pattern index.

Why should one expect this index give something useful? Suppose we compute the output freezing all input variables except xi and we put the result in a graphic. The same operation is repeated to variable xk. If tangent along the curve f(xk) presents generally a higher slope than for f(xi), then:

\[
\left| \frac{\partial f}{\partial x_b} \right|_{\text{mean}} > \left| \frac{\partial f}{\partial x_a} \right|_{\text{mean}}
\]  

Following the definition of sensitivity given by equation (2), one can state that Sb>Sa. Concluding, if a robust technique might be used for computing these indexes, we’ll have an alternative methodology for FSS.

Computing these derivatives using a trained ANN is in fact quite simple. Suppose Fig. 1 represents the ANN trained to approximate a given function y=f(x), where y and x represent, respectively, output and input vectors. During learning, ANN weights are changed such way its outputs Oj get closer and closer to the targets yj. Each unit of hidden or output layers comprises two functions: a weighted sum and a transfer function:

\[
n_k = \sum_i w_{ki} x_i
\]  

\[
O_k = f_k(n_k)
\]

Transfer function are generally sigmoid type function, like hyperbolic tangent, for hidden layer(s) and linear for output layer. Input units just distribute input values xi.

Fig. 2 shows just one unit for each layer of the ANN. The derivative of the output Oj with respect to Ok, the output of unit k, is given by:

\[
\frac{\partial O_j}{\partial O_k} = f_j' w_{jk}
\]  

where f'j is the derivative of fj at point nj. Using a chain rule:

\[
\frac{\partial O_j}{\partial x_i} = \sum_k f'_k w_{ki} \frac{\partial O_k}{\partial x_k}
\]  

where index k represents all units fed by input xi. If the ANN is a fully connected one, index k refers to all units in the hidden layer. This simple and elegant procedure is similar to the one used for weights adaptation on Backpropagation Algorithm [1,2]. Table 3 shows the si indexes obtained by applying formula (2). Dummy variables (x1, x3, x5 and x7) present the lowest si values showing that proposed methodology was successfully on this example.
Table 3 – ANN sensitivity indexes

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Variable</th>
<th>$s_i$ index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>x8</td>
<td>1.625</td>
</tr>
<tr>
<td>2</td>
<td>x4</td>
<td>0.893</td>
</tr>
<tr>
<td>3</td>
<td>x2</td>
<td>0.879</td>
</tr>
<tr>
<td>4</td>
<td>x6</td>
<td>0.685</td>
</tr>
<tr>
<td>5</td>
<td>x5</td>
<td>0.087</td>
</tr>
<tr>
<td>6</td>
<td>x7</td>
<td>0.035</td>
</tr>
<tr>
<td>7</td>
<td>x1</td>
<td>0.019</td>
</tr>
<tr>
<td>8</td>
<td>x3</td>
<td>0.015</td>
</tr>
</tbody>
</table>

3.1 Some more details about used ANN

The 2000 patterns were normalized to have zero mean and a standard deviation of one. This removes of offset issues and measurement scales. 1500 patterns were used for training and 500 for testing. Training algorithm was the Adaptive Backpropagation [3]. A description of ANN architectures experienced is reported on a further section on this paper.

4 Sensitivity Analysis Under Noise

Results presented before concern to a clean well defined mathematical function $y = f(x_1, .., x_N)$. That is not the case of real world applications. In fact, on practical applications, there’s no “function” and data is affected by noise caused by errors on measuring, failing of data acquisition system, recording problems, and so on. Being so, it is licit to suspect that presented sensitivity calculus approach may fail under these circumstances, that is, when one has just a collection of points to be dealt with to perform a multiregression task.

In the following analysis, random noise has been added to function variables under the succeeding hypothesis:

a) 10% of noise added to output $y$;

b) 20% of noise added to output $y$;

c) 20% of noise added to both inputs $(x_1, .., x_8)$ and output $y$.

As ANN output has been normalized to have zero mean and a standard deviation of 1.0, one may roughly admit that output values are mostly contained in the interval $[-1.0;1.0]$, and consider 10% of noise when random noise is generated in the interval $[-0.1;0.1]$. That’s what was considered on case a). On case b), random noise in the interval $[-0.2;0.2]$ was added to output. Finally on case c), random noise in the interval $[-0.2;0.2]$ was added to all inputs and output. Results obtained are presented on Table 4, were performance refers to the mean absolute percentage error. ANN was trained during 5000 epochs in all cases. Sensitivity indexes are calculated using all patterns of the training set. As can be observed, the ranking of $s_i$ allows, in all cases reported, to select the real features $(x_2, x_4, x_6$ and $x_8)$, with the highest index values. Despite the degradation of ANN performance as noise increases (note more than 20% error on c) case), $s_i$ ranking still provides a correct emplacement of features, with the dummy variables having the lowest values. At the same time, as noise level increases, the differences on $s_i$ values among real features and dummy ones become smaller, which is also an expected result. One may also expect that if noise too high, one might no more obtain a correct ranking of $s_i$. But, in this case, the ANN performance will also be poor. As conclusion, it seems that if ANN is able to perform reasonably, it can also provide the correct $s_i$ ranking, which is a very interesting result.

5 $s_i$ Calculus With Smaller Datasets

The number of patterns (1500) used for training on previous studies with 8 potential input features may be a little too much when compared to some real applications, when the number of training examples may be scarce. So, the analysis were repeated but using this time the second set (the previous test set with 500 patterns). Results obtained are presented on Table 5.

Table 4 – Sensitivity indexes calculus under noise

<table>
<thead>
<tr>
<th>case</th>
<th>Performance (%)</th>
<th>ANN sensitivity indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>training (1500)</td>
<td>test (500)</td>
</tr>
<tr>
<td>a)</td>
<td>4.97</td>
<td>5.73</td>
</tr>
<tr>
<td>b)</td>
<td>9.22</td>
<td>10.31</td>
</tr>
<tr>
<td>c)</td>
<td>23.45</td>
<td>26.90</td>
</tr>
</tbody>
</table>

Table 5 – Sensitivity calculus on a smaller training set

<table>
<thead>
<tr>
<th>epoch</th>
<th>Performance (%)</th>
<th>ANN sensitivity indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>training (500)</td>
<td>test (1500)</td>
</tr>
<tr>
<td>3000</td>
<td>3.84</td>
<td>5.59</td>
</tr>
<tr>
<td>4000</td>
<td>3.54</td>
<td>5.09</td>
</tr>
<tr>
<td>5000</td>
<td>3.46</td>
<td>5.16</td>
</tr>
</tbody>
</table>
occurring. Even so, \( s_i \) ranking still provides the wanted results. If dummy features are eliminated and training is repeated using only the remaining ones, the performance errors attained after 2500 training epochs are 3.11% on training set (500 patterns) and 3.66% on test set (1500 patterns). These results, showing smaller errors on both training and test sets and obtained after fewer training epochs, confirm the advantages of eliminating dummy variables that just introduce noise.

### 6 Dependence on ANN architecture

Previous results refer to an ANN architecture (set initially by chance) of 8-10-6-1: i.e., 8 inputs, 10 units in the first hidden layer, 6 units in the second hidden layer and 1 output unit. The study described on this section aims to infer the possible limitations of ANN architecture choice on the calculus of sensitivity indexes. First, we analyze how ANN architecture simplification may affect the results. After, we go on the opposite direction increasing the number of ANN parameters and interpreting the results. Table 6 presents the performance errors and sensitivity indexes for several ANN architectures. With the exception of ANN 8-2-1, that provide a bad ranking of \( s_i \), all the other perform satisfactory under this issue. The performance error for the 8-2-1 case is about 60%, which is really a poor approximation to the desired function. Next case, 8-4-1, despite the still poor 50% error, already provides a correct ranking of \( s_i \). The next 2 architectures, 8-20-10-1 and 8-50-20-1, provide good results in both performance and \( s_i \) ranking issues. The last case reported, on bottom of Table 6, also concerns to ANN 8-50-20-1, but this time a smaller training set (with 500 patterns) was used. A larger number of epochs was also considered in order to force the growth of the overfitting phenomena – note the 0.3% of error in the training set against 33.2% in the test set. In spite of overfitting, \( s_i \) ranking is still suitable. Note also that derivatives are generally higher because excessive training leads to higher non-linearity mapping surfaces.

### 7 Some More Testing

#### 7.1 Equality relations among inputs

Similar procedure was performed with the following function:

\[
y = \frac{1}{(x_2 + x_4 - x_6 - x_8)^2 + 1} + \sin(x_{10} + x_{12} + x_{14} + x_{16})
\]

(8)

All input variables, except \( x_1, x_{17}, x_{18}, x_{19} \) and \( x_{20} \) were randomly generated within the interval [-2; 2]. The following relations were established:

\[
x_1 = 3x_2 \quad x_{17} = x_4 \quad x_{18} = x_{19} = x_{20} = x_6
\]

(9)

First, a correlation analysis was performed to eliminate correlated variables. This step is necessary when using the proposed FSS approach because sensitivity indexes may be distorted. Note, for instance, that function (8) depends on \( x_6 \) and not on \( x_{18}, x_{19}, \) and \( x_{20} \), and it is impossible just by analyzing data patterns to infer that conclusion. As \( x_{19} = x_{19} = x_{20} = x_6 \), the ANN may use indifferently any one of these inputs, depend a lot, for instance, on \( x_{19} \) and just a bit on \( x_6, x_{18}, \) and \( x_{20} \). So, index \( s_{19} \) we wish to be included in the set of the highest ones, may be distributed by \( s_{19}, s_{18}, s_{19} \) and \( s_{20} \). This fact is probably the main argument against FSS based on sensitivity analysis [10]. However, this problem can be easily overwhelmed by using a simple technique like correlation analysis for discarding linked variables before proceeding with the \( s_i \) calculus. In this case, correlated features \( (x_2, x_{17,20}) \) were discarded. The application of the proposed FSS approach to the remaining variables provides the results summarized on Table 7. \( s_i \) indexes were normalized to give the sum one. This way relative \( s_i \) values that can be perceived as percentages of the whole. A gray shadow marks dummy variables. In this case, after 4000 training epochs, real attributes have already the highest \( s_i \) values, despite of the difference between \( s_i \) (the real feature with lowest \( s_i \)) and \( s_{15} \) (the dummy feature with highest \( s_i \)) is not relevant. However, as long as training proceeds, one can witness the growth of that difference.

### Table 6– Sensitivity calculus on different ANN architectures

<table>
<thead>
<tr>
<th>ANN architecture</th>
<th>Performance (500 epochs) (%)</th>
<th>ANN sensitivity indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training (1500)</td>
<td>Test (500)</td>
<td>( s_1 )</td>
</tr>
<tr>
<td>8-2-1</td>
<td>59.0</td>
<td>62.3</td>
</tr>
<tr>
<td>8-4-1</td>
<td>49.5</td>
<td>51.4</td>
</tr>
<tr>
<td>8-6-1</td>
<td>21.3</td>
<td>22.3</td>
</tr>
<tr>
<td>8-20-10-1</td>
<td>6.1</td>
<td>6.5</td>
</tr>
<tr>
<td>8-50-20-1</td>
<td>3.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Performance (1100 epochs) (%)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Training (500)</td>
<td>Test (1500)</td>
<td>( s_1 )</td>
</tr>
<tr>
<td>8-50-20-1</td>
<td>0.3</td>
<td>33.2</td>
</tr>
</tbody>
</table>
7.3 Friedman series

This example is based on Friedman series [12]:
\[
y = 10\sin(x_1x_2) + 20(x_3 - 0.5)^2 + 10x_4 + 5x_5 + N(0,1)
\]  
(11)

where N(0,1) represents normal distributed noise. Data set has ten input variables \(x_1,\ldots,x_{10}\) but the response only depends on \(x_1,\ldots,x_5\). Performance obtained in this case was 3.1% on the training and 4.4% on test sets. Table 9 shows \(s_i\) indexes obtained. In [13] authors conclude that FSS methods based on sensitivity analysis fail on this series. Table 9 shows that isn’t true: dummy variables have the lowest \(s_i\).

### 8 S\(_i\) Analysis On Logic Functions

Suppose we train an ANN to emulate function (12):
\[
\text{if } ((x_2 < -0.5) \lor (x_4 > 0)) \land ((x_5 < -0.5) \lor (1 < x_5 < 8)) \text{ then } y=1
\]
\[\text{else } y=0\]

function (13)
\[
\text{if } (x_2 < -0.5) \lor (x_4 > 0) \text{ then } x_2 = 1
\]
\[\text{else } x_2 = 0\]

where \(\lor\) stands for logic OR and \(\land\) for AND. If ANN units transfer functions are continuous, the result is a continuous function (the ANN), trying to imitate a no-continuous one. It would still be possible to compute the finite derivatives of the output with respect to inputs. The same approach used for the first example was applied here. Results are shown on Table 10:

### 7.2 Functional relations among inputs

In this study, function to be analyzed is still given by equation (8), but relations among variables are given by the following equations:

\[
\begin{align*}
\frac{x_1}{x_3} &= 3x_2 \\
x_{17} &= x_4 \left( x_2 - 2 \right) \\
x_{18} &= x_4 \sin(x_{10} - x_{12}) \\
x_{19} &= x_20 = x_6
\end{align*}
\]

(10)

This FSS problem is harder than previous one because correlation analysis may not eliminate all functional related variables. In this case, only eliminates variables \(x_2, x_{10}\) and \(x_{20}\). As \(x_1 = 3x_2\), it is equivalent to eliminate either \(x_1\) or \(x_2\), despite of \(x_2\) appear on equation (8) and \(x_1\) not. The remaining 17 variables were used as inputs of a new ANN. Table 8 shows \(s_i\) results obtained after 1500 training epochs. The performance attained was 2.6% on training and 2.8% on test sets.

### 8.1 Data analysis and results

Another study was carry out using the same patterns generated for function (12) but resolving first the inner parenthesis. For instance, if \((x_2 < -0.5)\) then \(x_2 = 1\) else \(x_2 = 0\)
Dummy variables were set to 1 if greater than zero; and zero otherwise. The $s_i$ attained are also shown on Table 10. Some authors, like in [11], argue that derivatives are not suitable for discrete inputs. This is surely true for non-continuous ANN, like when units have a step for transfer function. However, as a continuous ANN is being used, it is mathematically sensible to compute derivatives, even when one is trying to reproduce a non-continuous function.

9 FSS on A Real Data Case

This analysis is based on data of power system from Crete, Greece. Patterns data was generated by changing several system parameters and computing simulation of a security measure [8,9], when power system is shaken by a given disturbance $d_1$. The whole process was repeated for a second disturbance $d_2$. Initial set comprises 60 attributes. A correlation analysis was performed and for each attribute pairs with a correlation index higher than 0.90, a variable was discarded. This procedure eliminates 41 variables. An ANN was trained with the remaining 19 and $s_i$ calculus was performed. $s_i$ indexes were normalized to give the sum one and features with $s_i$ index below 0.02 were discarded. This heuristic has eliminated 12 more variables. A new ANN was trained with the remaining 7 features. Performance was computed and compared to older values reported on [8,9], were 22 attributes were used (Table 11).

<table>
<thead>
<tr>
<th>Table 11 – Results for the Crete case $d_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>old perf.</td>
</tr>
<tr>
<td>train</td>
</tr>
<tr>
<td>0.044</td>
</tr>
</tbody>
</table>

10 Conclusion

A practical study of an alternative FSS approach based on correlation analysis and ANN sensitivity analysis was carry out. The method is simple, straightforward and fairly insensitive to noise and ANN chosen architectures. It requires some time for the initial ANN training but, unlike some concurrent methods, allows the discarding of several (or even all) dummy features at a time. Results attained so far on function approximation studies support the validity of proposed approach and encourage further developments and tests. However, its application to time series should be evaluated because each value is strongly correlated with previous one. As the first step of the method consists of deleting all correlated variables, it means that all variables would be deleted except one. Then, each forecast would be only dependent on previous one, what is, for sure, not consistent with experience. Notice however that on real time series like load forecasting, noise display an important role. In this case, it is important to use several correlated inputs in order to decreased sensitivity to this factor. Nevertheless, further developments on the method should be made to overcome this limitation.

References: