Improvement of a Measurement Plotting System Performance with Fractional Order Hold adjusted by Neural Networks

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Abstract. -A connectionist method for autotuning the free parameter of a Fractional order hold (FROH) in order to improve the stability properties of the resulting discrete-time zeros is proposed. Such a technique employs multilayer perceptrons to approximate the mapping between the sampling period/continuous-time parameters of the plant and the optimal values of the FROH parameter. The neural networks are designed to adapt on-line to changing system structures, parameter values and sampling periods. To achieve this objective, the network weighting coefficients are determined during an off-line training phase. In this training phase, the optimal values of the FROH parameter are obtained by applying the classical generalised root locus procedure. Simulation results are presented to illustrate the properties of the complete regression system.

Key-words: Digital systems, Intelligent control, Neural networks, FROH, Zeros.

1 Introduction

In the purely discrete-time LQR context, the stability properties of the discrete-time zeros do not influence the closed-loop stability but, as shown in [1], discretization zeros can lead to intersample ripple in some cases. For most of H2-norm synthesis problems, see e. g. [2], and for LQR optimal sampled-data control systems [1], the problem of stability of the discretization zeros is completely irrelevant. However, several techniques for control system design are based on the cancellation of process zeros by the controller. Unfortunately, such methods can not be applied when the process has unstable zeros, see e. g. [3] and references therein. Consequently, special attention is being paid to the study of the zeros of sampled-systems. This subject was studied by Åmström et al. [4] (whose work has been recently extended by Blachuta [5]) for the case of Zero Order Hold (ZOH). Hagiwara et al. [6] carried out a comparative study demonstrating that a First Order Hold (FOH) provides no advantage over a ZOH as far as stability of the zeros of resulting sampled-systems is concerned. Passino et al. [7] proposed the Fractional Order Hold (FROH) as an alternative to the ZOH. Finally, in the very motivating work by Ishitobi [8], the stability properties of the limiting zeros (i. e. T→+0) of continuous-time plants with a relative degree up to relative degree p=5 were analysed. It was concluded that the FROH with -1<β<0 produces discrete-time models with all the zeros being stable for a wider class of plants than the zero order hold.

2 Improving the stability properties of the FROH zeros

Suppose that the state space equation of a nth-order time-invariant SISO (single-input single-output) controllable and observable system is expressed as

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + bu(t) \\
y(t) &= cx(t)
\end{align*}
\]

where \( u(t) \) and \( y(t) \) are the input and the output scalars, and \( x(t) \) is the state vector. In order to design a digital control scheme, we are interested in the discrete-time system composed of a hold device, the linear continuous-time system and a sampler in series. When the fractional-order hold (FROH) signal reconstruction device is used, the input is given by

\[
u(t) = u(KT) + \beta \left[ \frac{u(KT) - u((K - 1)T)}{T} \right](t - KT)
\]

where \( KT \leq t < (K + 1)T \)

where \( K \) is a non-negative integer, \( T \) is the sampling period and \( \beta \) is the device adjustable gain.
Then, the sampled system is (see, e. g., [7][8]):

\[
\begin{bmatrix}
\Phi & \beta \gamma \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x(K+1) \\
x(K)
\end{bmatrix} + \gamma = \begin{bmatrix}
\Phi & \beta \gamma \\
0 & 0
\end{bmatrix} \begin{bmatrix}
x(k) \\
x(k)
\end{bmatrix} u(K) + \begin{bmatrix}
y(k) \\
0
\end{bmatrix}
\]

where

\[
\Phi = e^{\Phi T}, \quad \gamma = \int_0^T e^{\Phi T} \gamma d\tau
\]

and 0 is an nth-row vector with all the components being zero. Thus, the sampled transfer function is

\[
G_{\beta}(z) = \frac{N_{\beta}(z)}{D_{\beta}(z)} = \begin{bmatrix}
1 & \phi & \gamma
\end{bmatrix}^{-\beta} \begin{bmatrix}
1 & -\gamma
\end{bmatrix}
\]

In the following, the ZOH and the FOH will be considered as particular cases of the FROH when \(\beta=0\) and \(\beta=1\), respectively. In doing so,

\[
N_{\beta}(z) = \det \begin{bmatrix}
z - \phi & -\gamma \\
\gamma & \gamma
\end{bmatrix}
\]

where \(N_{\beta}(z)\) and \(N_{\gamma}(z)\) are the numerators of the sampled transfer functions with ZOH and FOH, respectively. In addition, notice that, according to eqn. 5, the location of the discrete-time poles does not depend on \(\beta\). Nevertheless, a pole-zero cancellation in \(z=0\) occurs when \(\beta=0\) (ZOH). As a result of this cancellation, an order reduction effect will appear in the \(G_{\beta}(z)\) in respect of \(G_{p}(z)\) for any \(\beta\neq0\).

The generalised root-locus approach will be used to study the location of the FROH zeros on the complex plane. The roots of \(N_{\beta}(z)\) are the zeros of \(G_{\beta}(z)\). On the other hand, substituting eqns. 6 and 7 into eqn. 5, \(N_{\beta}(z)=0\) can be written as a generalised root locus with the parameter \(\beta\) being the generalised gain

\[
1 + \beta \left[ \frac{N_{\gamma}(z) - \gamma N_{\beta}(z)}{z N_{\gamma}(z)} \right] = 0
\]

It has been shown in [9] that, for any continuous-time plant and for a sufficiently small sampling period, it is always possible to obtain FROH discrete-time zeros that are more stable than the ZOH and FOH ones by means of an appropriate choice of negative values of the parameter \(\beta\). In addition, the FROH discretization of zero-free continuous-time plants with relative degree two was analysed and a straightforward method was developed in order to obtain the optimum value of the parameter \(\beta\), that is, the value that provides the FROH zeros as stable as possible for a particular plant and sampling period. However, there exist two alternative approaches for determining the optimum value of \(\beta\) (for an application example, see [10]) in order to discretize continuous-time plants of any relative order with sufficiently small sampling periods:

(a) The most general approach consists of obtaining the roots of eqn. 8 as a function of parameter \(\beta\). Then, the value of \(\beta\) is adjusted in order to minimise the largest root magnitude by using sound numerical recipes.

(b) The second approach consists of sketching the complementary generalised root locus. Then, the value of \(\beta\) corresponding to the most stable location of the discrete zeros can be obtained by studying this root locus.

The second approach is, fairly, the most advantageous. Nevertheless, since the roots loci given by eqn. 8 depend on the continuous-time plant and the sampling period, it is necessary to study each case one by one. Therefore, methods that enable the on-line automatic tuning of the FROH parameter \(\beta\) could be of practical importance.

3 Connectionist approach to FROH tuning

The significance of the artificial neural networks in pattern classification problems has been well-established [11]. A connectionist procedure to obtain the optimum value of the FROH parameter \(\beta\) -as far as the stability properties of the discrete-time zeros is concerned- is proposed in this paper. The method is based on the determination of the optimum value of \(\beta\) for a wide range of continuous-time plants by using the procedures presented in the previous section. Having obtained the optimum FROH parameters, multilayer perceptrons are used in order to store the mappings between these parameters and the corresponding continuous-time plants plus sampling period. Since the mappings achieved by neural networks will be approximations, errors are to be expected and measured. However, the resulting networks must provide a quasi-optimum value of \(\beta\) when the well-suited description of the continuous-time plant and sampling period is applied to their
inputs. The methods for obtaining the optimum value of the FROH parameter described in section 2 are restricted to zero-free continuous-time plants when the sampling period is finite and sufficiently small. We choose zero-free continuous-time systems of second and third orders for the purpose of training the networks. Note that a good performance cannot be expected of the multilayer perceptrons for systems outside the range presented in the training data. Thus, a variety of damping ratios, natural frequencies and sampling periods was chosen, ensuring that a sufficiently wide range of plant parameterisations resulted.

3.1. Training set for second-order continuous-time plants.

In particular, this training set was composed of 330 examples. We choose 55 plants with damping ratios $\delta \in [-2, 2]$ and natural frequencies $\omega_n \in [0.1, 3]$. Each plant was discretized by using 6 different sampling periods $T \in [0.001, 1.5]$ and the corresponding optimum FROH parameter was obtained by applying the classical root locus method to the expression 8. Each example was composed by three elements, that is, the parameters of the characteristic polynomial and the sampling period. Thus, the networks will have three inputs and the optimum FROH parameter is their only output.

3.2. Training set for second and third-order continuous-time plants.

Now, the training set was composed by 660 examples, 330 from the previous set and 330 obtained by adding a third pole to the second-order plants. The third pole was real and located in $z = -1/a$, where $1/a = \delta \omega_n$ in order to obtain a third-order system response. Consequently, the examples will have four inputs and one desired output, obtained again by applying the generalised root locus approach.

3.3. Topologies.

We tried four topologies of multilayer perceptrons, two for each training set. In the first case, for second-order plants, one (Network A) had one hidden layer with twenty non-linear neurons and the other (Network B) had two hidden layers, each with ten non-linear neurons. In the second case, for second and third-order plants, we used one network (Network C) with one hidden layer of twenty-five non-linear neurons and the other one (network D) with two hidden layers of ten neurons. The nonlinearities used in the neurons of the hidden layers were tan-sigmoid functions.

3.4. Training algorithm.

A non-linear least-square routine using a Levenberg-Marquardt method [12] was used to optimise the non-linear parameters of the network. This algorithm appears to be the fastest method for training moderate-size (up to several hundred weights) feedforward neural networks.

3.5. Improving generalisation.

A regularisation method was used in order to improve the generalisation of the networks. This method involves modifying the performance function by adding a term, modulated by a free parameter (performance ratio), that consists of the mean of the sum of squares of the network parameters. In this way, the new performance function will force the network response to be smoother and less likely to overfit. The performance ratio is adjusted by using the Bayesian framework described in [13] –see also [14]-.

3.6. Data pre-processing.

The mean and standard deviation of the training sets were normalised in order to scale the network inputs and targets. In doing so, the data will have zero mean and unity standard deviation. Finally, a principal component analysis of the training set was perform in order to eliminate the highly redundant data and, in this way, to reduce the dimension of the inputs. In particular, the components which contribute less than 0.1 % to the total variation in the data set were discarded (a very conservative choice). Evidently, the validation and test data subsets were prepared in the same way.

4 Results

In order to investigate the networks response in detail, a regression analysis between these responses and the corresponding targets was performed. Three parameters were obtained for measuring the trained networks performances. The first two, $m$ and $b$, correspond to the slope and the $y$-intercept of the best linear regression relating targets to network outputs. The slope would be 1 and the $y$-intercept would be 0 when the network reach a perfect fit (the outputs exactly equal to targets). The third variable obtained, the correlation coefficient $R$, is a measure of how
precisely the variation in the output is explained by the targets. This coefficient is equal to 1, when there is a perfect correlation between targets and outputs. This analysis was applied to the entire data set. However, the network responses to the training and test subsets was studied separately in order to measure the capability of generalisation reached by the trained networks.  

All the routines mentioned above are available in the Neural Network Toolbox incorporated in MATLAB [15] and the following results were obtained in this way. These results are divided into two parts, one being referred to the FROH discretization of second-order plants and the other related to second and third-order plants.

4.1. Second order plants FROH discretization.

Initially, the training and test sets were calculated by using the method described in 3.1. In particular, the sets were obtained from all the possible combinations of the following values for the damping ratio, natural frequency and sampling period:

**Training set T1** (330 examples):
- $\delta$=[1.77 1.33 1 0.66 0.33 0 -0.33 -0.66 -1 -1.33 -1.77];
- $\omega_n$=[0.1 0.5 1 2 3];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Test set TS1** (330 examples):
- $\delta$=[1.33 1 0.66 0.33 0 -0.33 -0.66 -1 -1.33];
- $\omega_n$=[0.1 0.5 1 2 3];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Test set TS2** (330 examples):
- $\delta$=[1.55 1.14 0.77 0.4 0.0 -0.43 -0.87 -1.24 -1.65];
- $\omega_n$=[0.1 0.5 1 2 3];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Test set TS3** (330 examples):
- $\delta$=[1.55 1.14 0.77 0.4 0.0 -0.43 -0.87 -1.24 -1.65];
- $\omega_n$=[0.3 0.7 1 1.4 2 2.2 2.7 2.9];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Network A:** It is a two-layer network, with a tan-sigmoid function in the hidden one and a linear transfer function in the output layer. The network should have three inputs (continuous-time parameters and sampling period), one output (optimal FROH parameter) and twenty neurons on the hidden layer. The results of the training are presented below:

**Training set T1. Network A:**
- m=1; b=-3.3339e-005; R=1.

**Test set TS1. Network A:**
- m=0.9999; b=-3.7497e-005; R=1.000.

**Test set TS2. Network A:**
- m=0.9001; b=-0.0035; R =-0.9997.

**Test set TS3. Network A:**
- m=0.9525; b=-0.0284; R=0.9944.

**Network B:** Now, we have two hidden layers with ten nonlinear neurons on each.

**Training set T1. Network B:**
- m=1.0000; b=-1.2941e-0010; R=1.000.

**Test set TS1. Network B:**
- m=1.0000; b=3.0969e-008; R=1.0000.

**Test set TS2. Network B:**
- m=0.9995; b=1.9308e-004; R =1.0000,

**Test set TS3. Network B:**
- m=0.9930; b=0.0022; R=0.9998.

In this case, the quality of the approximations is very high. In fact, the sum of the squared error reached with the network B is SEE= 5.32e-009. Note that m and R are almost the unity and b is close to zero. Moreover, the capability of generalisation is proven since the network performance is also excellent when the three test subsets are put through the trained network. Finally, the chosen dimension of the network A and B seems to be large enough to provide an adequate fit, the network A uses 92 parameters out of 101 available and the network B 158 out of 161.

4.2. Second and third-order plants FROH discretization.

The training and test sets were calculated following the method described in 3. 2.:

**Training set T2** (660 examples):
- $\delta$=[1.77 1.33 1 0.66 0.33 0 -0.33 -0.66 -1 -1.33 -1.77];
- $\omega_n$=[0.1 0.5 1 2 3];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Test set TS4** (660 examples):
- $\delta$=[1.86 1.49 1.12 0.79 0.45 0.01 -0.27 -0.58 -0.97 -1.27 -1.68];
- $\omega_n$=[0.1 0.5 1 2 3];
- $T$=[0.01 0.05 0.1 0.5 1 1.5];

**Network C:** One hidden layer with twenty-five non-linear neurons.

**Training set T2. Network C:**
- m=0.9997; b=-1.2345e-004; R=0.9999.

**Test set TS4. Network C:**
- m=0.9118; b=-0.0577; R=0.9440.

**Network D:** Two hidden layers with ten non-linear neurons on each one.

**Training set T2. Network D:**
- m=0.9999; b=-2.0912e-005; R=1.0000.

**Test set TS4. Network D:**
- m=0.9061; b=-0.0553; R=0.9619.
Observe that approximating the function is a harder work for the networks C and D (SEE=1.13e-2 for Network C and SEE= 1.92e-3 for Network D), when the discretized plants have second and third-order models, despite of the increment in the number of non-linear neurons on the hidden layers. However, the trained networks performance remains high.

**Remark 1:** Notice that the topologies with two hidden layers had approximated the function with higher accuracy in all the cases.

**Remark 2:** It seems reasonable to expect better results by using different networks for each kind (order) of continuous-time plants.

### 5. Measurement plotting system.

Many physical phenomena are characterised by parameters that are transient or slowly varying. If recorded, these changes can be examined at leisure and stored for future reference or comparison. To accomplish such a recording, a number of electromechanical instruments have been developed, among them the X-Y recorder. In this instrument, the displacement along the X-axis represents a variable of interest or time and the displacement along the Y-axis varies as a function of yet another variable [16]. The HP 7090A plotting system will be used in this application example. Such recorders can be found in many laboratories recording experimental data such as changes in temperature, variations in transducer output levels, and stress versus applied strain, to name just a few.

The purpose of a plotter is to follow accurately the input signal as it varies. We will consider the design of the movement of one axis, since the movement dynamics of both axes are identical. Thus we will strive to control very accurately the position and the movement of the pen as it follows the input signal.

Since we wish to move the pen, we select a dc motor as the actuator. The feedback sensor will be a 500-line optical encoder. By detecting all state changes of the two-channel quadrature output of the encoder, 2000 encoder counts per revolution of the motor shaft can be detected. This yields an encoder resolution of 0.001 inch at the pen tip. The encoder is mounted on the shaft of the motor. Since the encoder provides digital data, it is compared with the input signal by using a digital microprocessor. Next we propose to use the difference signal calculated by the microprocessor as the error signal and then use the microprocessor to calculate the necessary algorithm to obtain the designed compensator. The output of the digital compensator is then, by using a FROH reconstruction device, converted to an analog signal that will drive the actuator.

The feedback position control system (see, e. g. [17]) has a sampling period $T=0.01$ seconds. The transfer function for the motor and pen carriage is

$$G(s) = \frac{1}{s^3 + 1010s^2 + 10000s}$$  \hspace{1cm} (9)

Now, the optimum value for the FROH parameter is obtained by using the previously trained neural networks. Concretely, we make use of the net D due to the higher performance observed. The inputs of the net D are the continuous parameters of the plant plus the sampling period of digital control system. The initial input vector is then,

$$\text{Input vector} = \begin{bmatrix} 1010 \\ 10000 \\ 0 \\ 0.01 \end{bmatrix}$$  \hspace{1cm} (10)

This input vector is pre-processed and the network output is post processed as is described in the section 3.6. The final output of the network D is

$$\beta_{\text{opt}} = -0.4475$$  \hspace{1cm} (11)

Note that this FROH provides a discretization zero magnitude ($|z_{\text{opt}}|=0.6588$) that is 51.29 % smaller than the magnitude corresponding to the ZOH discretization ($|z_{\text{ZOH}}|=1.3527$). Besides, the FROH zeros are inside the stability region –inverse stable plant–, so that they can be cancelled by means of a digital controller. In this way, the performance of the controlled system could be significantly improved [18].

By using the expression (8), the generalised root locus that describes the evolution of the discretization zeros with $\beta$ can be obtained (see figure 1).

![Figure 1. Generalized RL of the discrete zeros.](image-url)

In this case, as figure 1 shows, the optimum value of the parameter $\beta$ corresponds to the breakaway point
located at \( z = -0.6394 \) and marked as \( z_{\text{opt}} \). Therefore, the exact value of \( \beta_{\text{opt}} \) is \( \beta_{\text{opt}} = -0.4251 \). This FROH provides a discretization zero magnitude \( |z_{\text{opt}}| = 0.6394 \) that is 52.73% smaller than the magnitude corresponding to the ZOH discretization \( |z_{\text{ZOH}}| = 1.3527 \).

**Remark 3:** Evidently, the small discrepancies found with respect to the results obtained using the first approach are due to the finite accuracy of the trained network when approximating the stability function of the FROH zeros. However, as we can see, the performance is good enough in order to obtain a well-fitted value of \( \beta \).

6. Conclusions.

A connectionist approach to the automatic tuning of the FROH parameter in order to improve the stability properties of the resulting discretization zeros is proposed. Such method uses the optimal values of the FROH parameter, obtained by applying the classical generalised root locus procedure, as the desired outputs of the neural networks. The inputs to the networks are the sampling period and the continuous-time parameters of the plants to discretize by using the FROH signal reconstruction method. Simulations results show that good tuning, very close to the optimal FROH \( \beta \) parameter values, is reached. In this way, a powerful tool is obtained in order to avoid the individualised study for each continuous-time plant and each sampling period. Furthermore, this allows us to design adaptive discretization systems for improving the stability properties of the discrete-time zeros, which can be applied to time-varying plants and/or multirate sampling control systems.

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**References:**


