Corner Charge Singularity of Conductors using Last-passage Algorithms

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ABSTRACT

In this paper, we introduce a diffusion-based Mone Carlo method, the last-passage algorithms and demonstrate the possibility of evaluating the power-law singularity associated with the corner of a conductor for the computation of capacitance coefficients for very large scale integration (VLSI) interconnects. We illustrate our method in the solution of a benchmark problem, the charge singularity at the corner of the unit square plate. The result is in good agreement with the previous result by Morrison and Lewis. Furthermore, it should be noted that the method for determining corner singularities is completely general; it is not limited to the 2-D rectangular corner geometry.

1 Introduction

A major difficulty computing capacitance coefficients for VLSI interconnects arises from the corner and edge singularities when we apply deterministic methods like finite difference or element methods. To ease the problem, a simple formula taking the charge concentration at any 3-D rectangular corner into account has been given [1].

In this paper, we introduce a Monte Carlo method, last-passage algorithms [2]–[4], for getting the corner singularity of a conducting object. The Monte Carlo method is illustrated in the solution of the charge singularity at the corner of the unit square plate.

2 Last-passage Algorithms

In this section, we review the last-passage algorithms [2] which allow one to efficiently calculate the charge density at a point on a conducting surface by utilizing the backward diffusing paths that initiate at the point.

Using the isomorphism between electrostatic problems and diffusion problems [5], [6], we obtain a formula for the electrostatic potential $V(\mathbf{x}+\epsilon)$, very near the point $\mathbf{x}$ on the conducting surface [7]:

$$V(\mathbf{x}+\epsilon) = \int_{\partial\Omega_y} d^2\mathbf{y}\, g(\mathbf{x}+\epsilon,\mathbf{y})p(\mathbf{y},\infty), \tag{1}$$

Here, $g(\mathbf{x}+\epsilon,\mathbf{y})$ is the Laplacian Green's function associated with Dirichlet boundary conditions on the region $\partial\Omega_y$. In particular, $g(\mathbf{x}+\epsilon,\mathbf{y})$ is the probability density associated with a diffusing particle initiating at the point $\mathbf{x}+\epsilon$ and making the first contact on the surface $\partial\Omega_y$ at the point $\mathbf{y}$. Also, $p(\mathbf{y},\infty)$ is the probability of a diffusing particle initiating at the point $\mathbf{y}$ on the upper first-passage surface and diffusing to infinity without ever returning to the conductor. Thus, Eq. 1 represents the electrostatic potential, $V(\mathbf{x}+\epsilon)$, as the probability density of a diffusing particle initiating at the point $\mathbf{x}+\epsilon$ near a conducting surface, and diffusing without ever contacting the conducting surface again. It should be noted that in Eq. 1 the first factor is analytically simple, but it depends on the quantity ϵ and the second factor is very complicated, but it is independent of ϵ.

Gauss' law gives the surface charge density $\sigma(\mathbf{x})$ on a conductor in terms of the formula [8]:

$$\sigma(\mathbf{x}) = -\frac{1}{4\pi}\frac{d}{d\epsilon}\bigg|_{\epsilon=0} V(\mathbf{x}+\epsilon). \tag{2}$$

Inserting Eq. 1 for $V(\mathbf{x}+\epsilon)$, this becomes

$$\sigma(\mathbf{x}) = \frac{1}{4\pi}\int_{\partial\Omega_y} d^2\mathbf{y}\, G(\mathbf{x},\mathbf{y})p(\mathbf{y},\infty), \tag{3}$$

where

$$G(\mathbf{x},\mathbf{y}) = \frac{d}{d\epsilon}\bigg|_{\epsilon=0} g(\mathbf{x}+\epsilon,\mathbf{y}). \tag{4}$$

The function $G(\mathbf{x},\mathbf{y})$ is the Laplacian Green's function for a point dipole centered on the conducting surface at point $\mathbf{x}$ and normal to the surface.

For a flat conducting surface, this dipole Green's function is readily shown to be given by

$$G(\mathbf{x},\mathbf{y}) = \frac{3}{2\pi}\frac{\cos\theta}{a^3}, \tag{5}$$

where θ is the angle between the vectors x and y, and a is the radius of the absorbing sphere. Substituting Eq. 5 into Eq. 3 gives the formula:

$$\sigma(\mathbf{x}) = \frac{3}{8\pi^2}\int_{\partial\Omega_y} dS\frac{\cos\theta}{a^3}p(\mathbf{y},\infty). \tag{6}$$

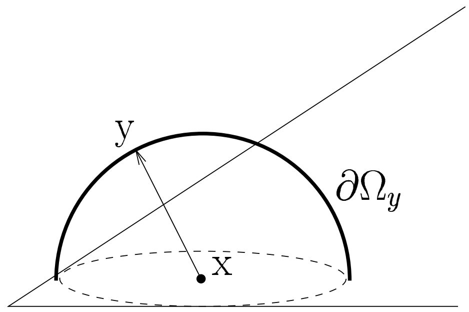

Figure 1: This schematic view illustrates that a diffusing particle is initiated at the point y on the surface of a hemisphere surrounding the point $\mathbf{x}$ on a square plate to obtain the charge density at $\mathbf{x}$.

3 Numerical Example

In this section, we demonstrate the last-passage method for getting the corner charge singularity exponent of the unit square plate and compare with the previous result by Morrison and Lewis [9].

The last-passage method obtains the charge density at the point $\mathbf{x}$ as follows (see Fig. 1): N diffusing particles are initiated at points selected randomly with density $\cos\theta\, dS$ on the surface of a hemisphere surrounding the point $\mathbf{x}$. They diffuse until they either hit the conductor or diffuse to infinity.

Application of this method gives for the corner singularity of the square plate the exponent, 0.7031, and in good agreement with the singular purturbation result, 0.7034 [9]. The exponent is obtained from the linear regression of the charge densities along the diagonal of the square plate with 10^9 trajectories for each data point (see Fig. 2).

4 Summary and Discussions

In this paper, we demonstrate our last-passage method in the solution of a benchmark problem, the charge singularity at the corner of the unit square plate. The result is in good agreement with the singular perturbation result by Morrison and Lewis [9]. We should note that our method is not limited to the 2-D rectangular corner geometry. Our method can be used to ease the difficulty treating the corner charge singularities when we apply deterministic methods like finite difference or element methods for the computation of capacitance coefficients for VLSI interconnects.

ACKNOWLEDGMENTS

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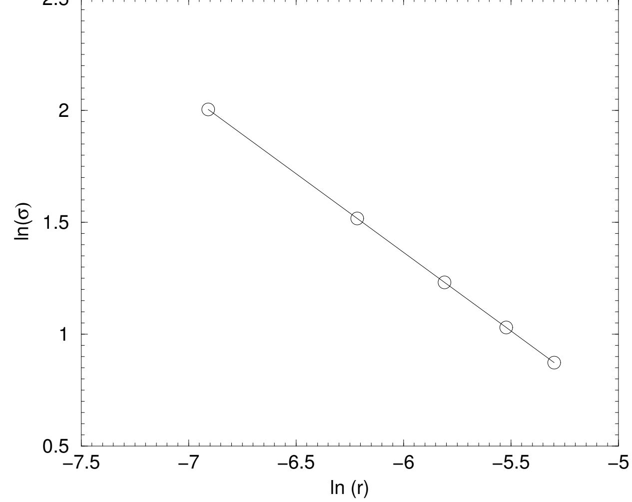

Figure 2: Corner exponent of the unit square plate with 10^9 trajectories for each data point: $r = \sqrt{x^2 + y^2}$ is the diagonal distance from the corner and the linear regression slope is -0.7031 with correlation coefficient -0.9999999.

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