Modeling of Electromagnetic Band Gap Structure Devices Tuned By Ferrite Cylinders

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Abstract: This paper presents a novel approach to alternate the electromagnetic properties of the photonic band gap, where a ferrite defect is introduced, and in addition, a DC magnetic field is applied to alternate the EBG's electromagnetic property. Numerous devices are examined including a coupler and a Y-branch filter. The numerical experiments demonstrate that the devices electromagnetic characteristics can be easily adjusted by varying the DC magnetic field intensity. The simulations are performed by using the scattering matrix method.

1 Introduction

Electromagnetic band gap (or photonic band gap) structures are periodic structures, which possess a stop band in their electromagnetic transmission characteristics. EBG are of great interest due to its wide applications.

Scattering matrix method has been employed in the analysis of PBG circuits [5]. It is a semi-analytic method and can give both filed distributions and transmission spectra against a static field excitation. Compared with FDTD method, scattering matrix can save computation time [3]

By applying DC magnetic field along the axis of ferrite cylinders, their scattering properties will be affected. The scattering properties also change with the changing of the magnetic filed intensity. Therefore, we can adjust the scattering properties by adjusting the applied DC magnetic field intensity. Many authors have studied the scattering properties of single ferrite cylinder [1] [2], ferrite cylinder array, however, is rarely studied. By generating ferrite defects in dielectric EBG structures, we can design some useful devices. Compared with devices consisted purely of dielectric cylinders, those with ferrite defects can be adjusted to desired scattering properties by adjusting the applied DC magnetic field intensity.

In this paper, we use scattering matrix method to simulate 2-D EBG structure with ferrite cylinders. In section 2, the scattering matrix is reviewed briefly. Several couplers with ferrite cylinders are described in section 3. In section 4, a Y-branch filter with ferrite cylinders is described and discussed.

2 Problem Formulation

The incident and scattering field of plane wave when scattered (TM) by a circular cylinder can be expressed in local coordinates as follows:

$$E_z^{inc} = \sum_{n=-\infty}^{\infty} a_n J_n(k\rho) e^{in\phi} \quad (1)$$

$$E_z^{sca} = \sum_{n=-\infty}^{\infty} f_n H_n^{(2)}(k\rho) e^{in\phi} \quad (2)$$

where E_z^{inc} and E_z^{sca} are incident electric field and scattering electric field respectively, a_n and f_n are incident coefficient and scattering coefficient, (ρ,ϕ) is the cylindrical coordinates originated at the centre of each cylinder, $J_n(x)$ and $H_n^2(x)$ are the first kind Bessel function and second kind Hankel function, k is the wave number.

When considering a set of N parallel cylinders, the electric filed outside cylinders can be expressed as:

$$E_z = E_z^{inc}(\rho_j) + \sum_{\substack{j=1 \\ (j\neq i)}}^{N} \sum_{n=-\infty}^{\infty} f_{jn} H_n^{(2)}(k\rho_j) e^{in\phi_j} \quad (3)$$

where $i = 1, 2, \cdots, N$ and coefficient f_{jn} is unknowns. The incident wave of each cylinder consists of two parts, one is from the source outside all cylinders (plane wave in this paper), and another is from the scattering wave of other cylinders. The scattering coefficient of $i-th$ cylinder can be expressed as:

$$f_i = T_i(a_i + \sum_{i \neq j} \alpha_{ij} f_j) \quad i, j = 1, 2, \cdots, N \tag{4}$$

where a_i is the incident wave coefficients; T_i and f_i stand for the transition matrix and expansion coefficient vector of the scattering wave from *i-th* cylinder respectively; α_{ij} represents the coupling of *j-th* cylinder to *i-th* cylinder, whose elements can be obtained from addition theorem of cylindrical harmonics [4]. Equation (4) can be written as following matrix format:

$$\begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_N \end{bmatrix} = \begin{bmatrix} \mathbf{T}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{T}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{T}_N \end{bmatrix} \left\{ \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_N \end{bmatrix} + \begin{bmatrix} 0 & \alpha_{12} & \cdots & \alpha_{1N} \\ \alpha_{21} & 0 & \cdots & \alpha_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{N1} & \alpha_{N2} & \cdots & 0 \end{bmatrix} \begin{bmatrix} \mathbf{f}_1 \\ \mathbf{f}_2 \\ \vdots \\ \mathbf{f}_N \end{bmatrix} \right\} \tag{5}$$

where

$$a_{jn} = e^{-ik_0 r_j (\cos\theta \cos\varphi_j + \sin\theta \sin\varphi_j)} e^{-in(\frac{\pi}{2}+\theta)} \tag{6}$$

$$\alpha_{ij}(n,m) = H^{(2)}_{n-m}(k\rho')e^{-i(n-m)\phi'} \tag{7}$$

For a circular dielectric rod, the T-matrix is a diagonal one which can be obtained conveniently as:

$$T_n = \frac{\eta J_n(k_1 a) J_n'(k_2 a) - n_c J_n'(k_1 a) J_n(k_2 a)}{n_c H_n^{(2)'}(k_1 a) H_n^{(2)}(k_2 a) - \eta H_n^{(2)}(k_1 a) H_n^{(2)'}(k_2 a)} \tag{8}$$

$$\eta = 1(TE), n_c^2(TM) \qquad n_c = \sqrt{\varepsilon_2 / \varepsilon_1}$$

where ε_1 and ε_2 are permittivity of outside and inside the cylinder respectively.

We can get the value of magnetic field by taking the gradient of electric field, and then the poynting vector can be obtained.

Considering a ferrite cylinder, it has the tensor permeability as follows:

$$\overline{\mu} = \begin{pmatrix} \mu & -ju' & \\ ju' & \mu & \\ & & 1 \end{pmatrix} \tag{9}$$

Here [1]

$$\mu = \mu_0 \frac{\gamma^2 \mu_0 \mathrm{H}_0 B_z - \omega^2}{\gamma^2 \mu_0{}^2 \mathrm{H}_0{}^2 - \omega^2} \tag{10}$$

$$\mu' = \mu_0 \frac{\omega \gamma \mu_0 M_z}{\gamma^2 \mu_0{}^2 \mathrm{H}_0{}^2 - \omega^2} \tag{11}$$

Where, μ_0 is the permeability in the air, and ω is the angular frequency of electromagnetic waves, H_0 is the DC magnetic field, and M_z is the DC magnetization, and the value of H_0/M_z which is usually constant for specific material known as magnetic susceptibility. $B_z = \mu_0(H_0 + M_z)$, γ is the gyro magnetic ratio.

In this case

$$T_n = -\frac{\dfrac{D_n(k_2 a)}{J_n(k_2 a)} - \dfrac{J_n'(k_1 a)}{J_n(k_1 a)}}{\dfrac{D_n(k_2 a)}{J_n(k_2 a)} - \dfrac{H_n^{(2)'}(k_1 a)}{H_n^{(2)}(k_1 a)}} \frac{J_n(k_1 a)}{H_n^{(2)}(k_1 a)} \tag{12}$$

where

$$D_n(k_2 a) = \frac{\mu_0}{\mu_{eff}} \frac{k_2}{k_1} [J_n'(k_2 a) + \frac{\mu'}{\mu} \frac{n}{k_2 a} J_n(k_2 a)] \tag{13}$$

$$k_2 = \omega^2 \mu_{eff} \cdot \varepsilon \tag{14}$$

$$\mu_{eff} = \frac{\mu^2 - \mu'^2}{\mu} \tag{15}$$

When $M_z = 0$, the cylinder becomes a dielectric cylinder.

3 Electromagnetic Characteristics of the Couplers with Ferrite Defects

3.1 A Tunable coupler

The EBG structure of tunable coupler with ferrite cylinders as shown in Fig.1, where the cylinder radius $r_0 = 0.6$, $a = 4$, $d = 2$, $l = 7$, and $\varepsilon_r = 8.41$. A DC magnetic field is applied. In this section, incident wave is from the bottom of this structure, and is TM polarization.

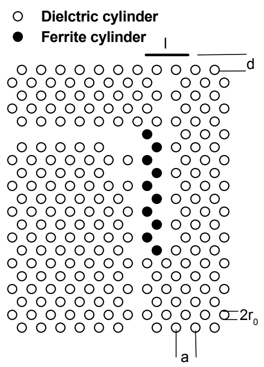


Fig.1 The geometry of alternate Coupler

We put some ferrite cylinders between the two discontinuities (Fig.1) while other cylinders are dielectric cylinders. Then when a wave propagates from bottom, the structure becomes an adjustable coupler. Simulation results are showed in Fig.2, compared with dielectric cylinder coupler, couplers with ferrite cylinder have narrower transmission band, and with the increasing of applied magnetic field intensity, it becomes narrower and narrower.

Fig.3 shows the Electric field distribution at $\lambda = 9.0936$, which verifies the rightness of transmission characteristics showed in Fig.2.

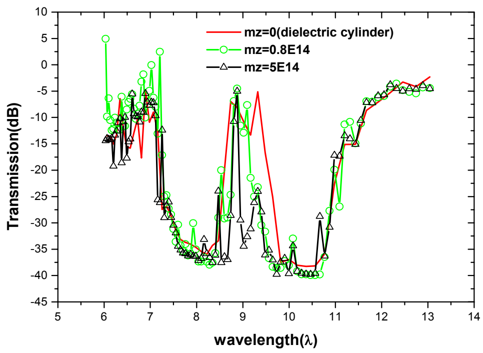
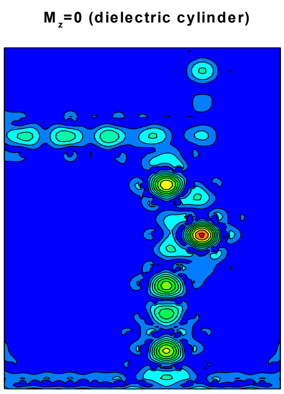
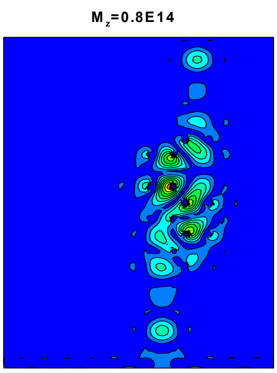


Fig.2 Transmission characteristics of coupler with varying applied magnetic field intensities



(a)



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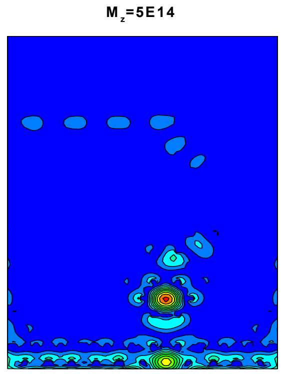


(c)

Fig.3 Electric field distribution with different applied DC magnetic field intensity at $\lambda = 9.0936$, $\gamma = 10$

(a) $M_z = 0$ (dielectric cylinder)(b) $M_z = 0.2E14$ (c) $M_z = 5E14$

The most obvious advantage of this coupler is that its pass band can be adjusted to become narrower whereas coupler with only dielectric cylinders can only achieve a fixed pass band. More desired properties can be achieved by changing the positions or the number of these ferrite cylinders.

3.2 A coupler with ferrite defects

The geometry of this coupler structure is in Fig.4. $r_0 = 0.6$, $a = 4$, $d = 2$, $l = 7$, and $\varepsilon_r = 8.41$ (same geometry as in Fig.1), ferrite discontinuities are created by replacing some dielectric cylinders with ferrite cylinders. A DC magnetic field is applied. In this section, incident wave is from the bottom of this structure and is TM polarization.

When the same DC magnetic field is applied to both discontinuities, due to the same permeability and permittivity of the two discontinuities, coupling will happened in certain frequency ranges.

Fig.5 proves our prediction. The transmission band shifts to higher frequency range (low wavelength value) with the increasing of the DC magnetic field.

Fig.6 gives comparison of Electric field distribution with different applied DC magnetic field intensity at different wavelengths, which verifies the results of Fig.1 and shows the adjustability of our coupler.

Owing to the fact that DC magnetic field can be changed easily, the frequency band in which coupling happens is adjustable. Therefore, compared with the usual discontinuity couplers [6], it is more flexible. Compared with the coupler in 3.1, this coupler changed the frequency range other than bandwidth, which may be more useful for coupler design and producing. By varying the shape of discontinuities or the number of ferrite cylinders, devices with different pass band can be achieved.

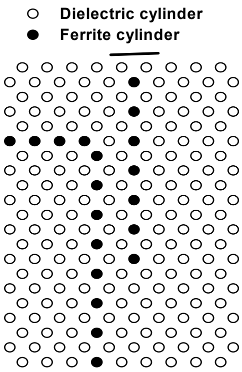


Fig.4 Geometry of coupler with ferrite defects

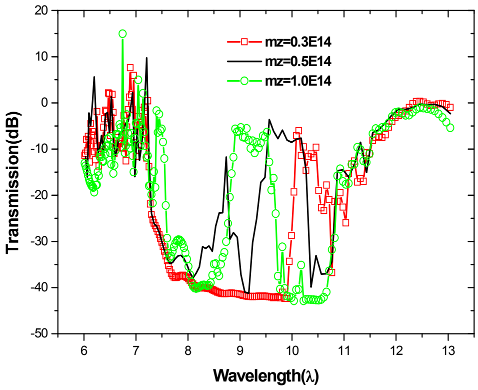
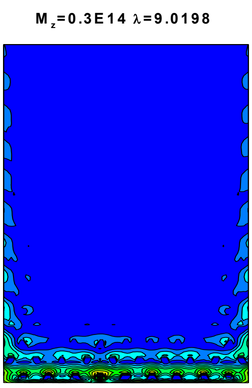
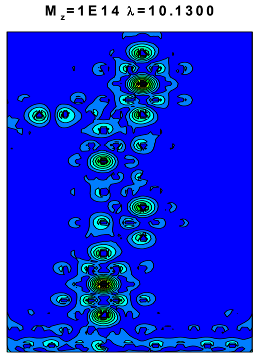


Fig.5 Transmission of coupler with ferrite defects with different applied magnetic field intensities.



(a)



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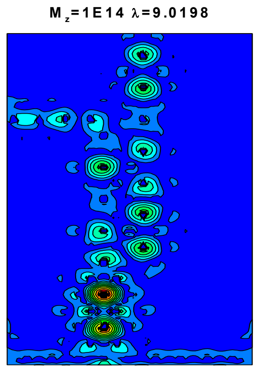

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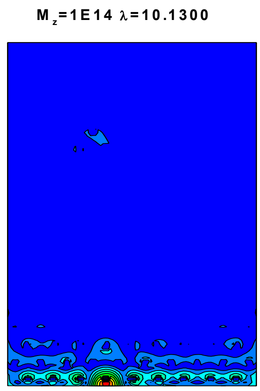


(d)

Fig.6 Electric field distribution with different applied DC magnetic field intensity at γ=10 （a） λ=9.0198 , M_z=0.3E14 (b) λ=9.0198 , M_z=1E14 (c) λ=10.1300 , M_z=0.3E14 (d) λ=10.1300 , M_z=1E14

4 A Y-branch Filter

Fig.7 gives the geometry of the Y-branch filter. $r_0 = 0.6$, $a = 4$, $d = 2$, $l = 7$, and $\varepsilon_r = 8.41$, ferrite discontinuities are created by replacing some dielectric cylinders with ferrite cylinders. A DC magnetic field is applied along the axis of ferrite cylinder. Incident wave is from the bottom of this structure. The polarization is fixed to the TM. Owing to the existence of Ferrite cylinders, the wave can go though the discontinuities in some frequency band.

Fig.8 shows the corresponding transmission spectra when the applied DC magnetic field in left discontinuity $M_z = 0.8E14$, in right discontinuity $M_z = 3E14$. The Transmissions band for left port and right port are different, it is due to the different applied DC magnetic field intensity. By adjusting the applied DC magnetic field intensity, the filter becomes an adjustable filter, which is more flexible compared with conventional filter [3] [6].

Fig.9 shows the electric filed distribution of Y-branch against different wavelength when the applied DC magnetic field in left discontinuity $M_z = 0.8E14$, in right discontinuity $M_z = 3E14$. As predicted in Fig.8, it shows the filtering behaviour.

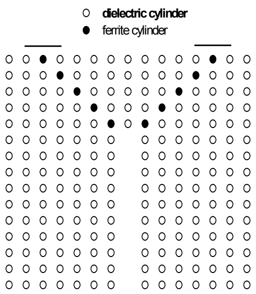


Fig.7 Geometry of ferrite the Y-branch filter

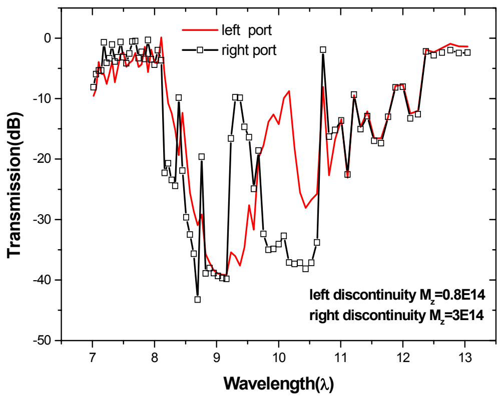


Fig.8 Transmission spectra of the Y-branch structure when left discontinuity M_z=0.8E14, right discontinuity M_z=3E14.

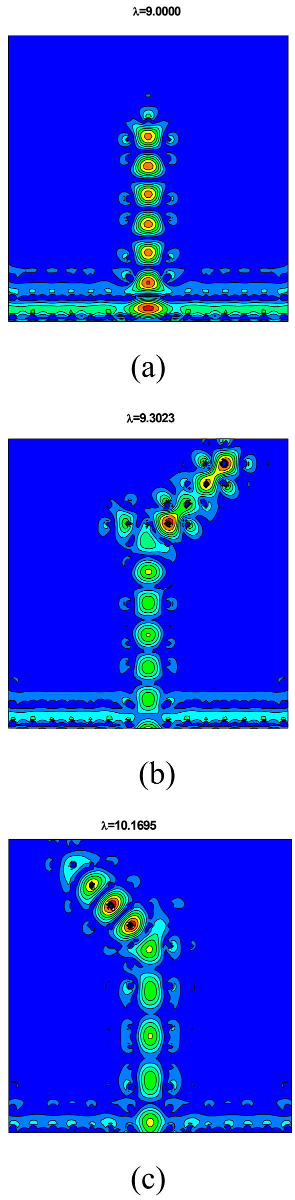
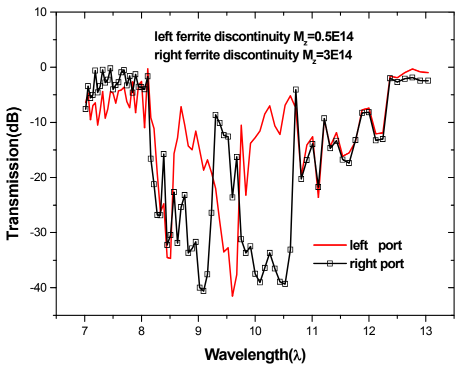
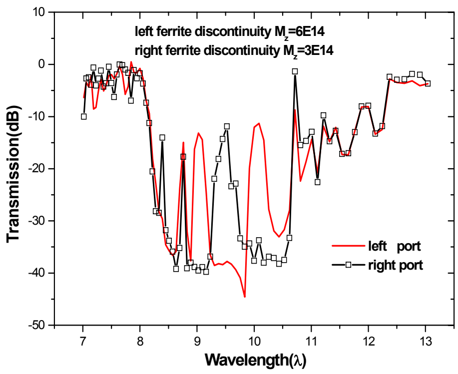


Fig.9 Electric field distribution of Y-branch filter with different wavelength at γ=10 when left discontinuity M_z=0.8E14, right discontinuity M_z=3E14 (a) λ=9.0000 (b) λ=9.3023 (c) λ=10.1695

We know that the main advantage of our filter is flexibility. To achieve a different stop band, the applied DC magnetic intensity is changed. Fig.10 give the transmission spectra when the applied DC magnetic field in left discontinuity $M_z = 0.5E14$, in right discontinuity $M_z = 3E14$ and when the applied DC magnetic field in left discontinuity $M_z = 6E14$, in right discontinuity $M_z = 3E14$ respectively.



(a)



(b)

Fig.10 Transmission spectra of the Y-branch structure when (a) left discontinuity $M_z = 0.5E14$, right discontinuity $M_z = 3E14$ (b) left discontinuity $M_z = 6E14$, right discontinuity $M_z = 3E14$

5 Conclusion

Scattering characteristics of electromagnetic devices with ferrite cylinder defects are studied by using scattering matrix method in this paper. By replacing some dielectric cylinders with ferrite cylinders, the scattering characteristics become adjustable. Two couplers with ferrite cylinders are described. Their transmission frequency range can be adjusted according to requirements. Compared with those devices with only dielectric cylinders, they are more flexible. A Y-branch filter is presented. By adjusting the applied DC magnetic filed intensity of different branches, we can obtain electromagnetic wave with desired frequency bands in desired ports.

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