Design of digital FIR filters with arbitrary magnitude using the Least squares error

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ABSTRACT

A new analytical method for designing digital FIR filters is presented which equates the filter transfer function using Least Square approximation. this method in comparison with other analytical approaches of FIR filter make a better approximation for passband region .examples are given to illustrate the utility of the technique.

1. INTRODUCTION

there are many approaches to the design of FIR filters. these filters can be designed by using Fourier series method in conjunction with Window functions [1]. They can also be designed using frequency sampling techniques[2] and using approximation filter to a polynomial frequency (e.g. Remez Exchange and etc.). in this paper we present an analytical method for the design of FIR filters using the properties of least squares error approximation.

2. ALGORITHM DESCRIBTION

Magnitude of a digital filter is a curve in ω frequency .The approximation approaches try to approximate this

curve with a polynomial in frequency. In the least squares approach have been defined an error vector in the form $\mathbf{e}$=**U-B C** where **Y** is a n×m matrix that are counstracted using the desired magnitude ,**B** is a vector in ω values and **C** is filter's coefficients. This approach try to minimize the error vector length [4].

if ω=[0 ω_1 ω_2 … ω_n π] and m=[m_0 m_1 m_2 ....m_n m_{n+1}] are normalized frequency vector and desired filter magnitude ,at first we create ω′=[0 ω'_1 ω'_2 ……. ω'_n π] as a new vector by using ω′=2 cos(ω) relationship .The **B** and **Y** are defined as the following form:

$$\mathbf{B} = \begin{bmatrix} 1 \ldots w_1 \ldots w_1^2 \ldots\ldots\ldots w_1^m \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 1 \ldots w_n \ldots w_n^2 \ldots\ldots\ldots w_n^m \end{bmatrix}$$

$$Y_{i\,j} = (m_i)^{j}$$

where i,j are integer and 0<i<m ,0<j<n.

we find the final result when length of error vector is be minimum ,In this case $\mathbf{B}^T\mathbf{e}=0$ and **C** vector will be found using $\mathbf{B}^T\mathbf{Y}=\mathbf{B}^T\mathbf{B}\mathbf{C}$ equation. The Filter transfer function in ω′ frequency is on the following form

$$H_1(e^{jw'}) = \sum_{k=0}^{N} c_k (\cos(w'))^k$$

By replacing

$$\cos(kw') = \frac{1+z^{-2k}}{2z^{-k}}$$

and $z=e^{jw}$ in it ,The final filter find at the below form

$$H(e^{jw}) = \frac{\sum_{k=0}^{2N} c'_k \frac{z^{-k}}{2^k}}{z^{-n}}$$

where $c'_{k+N+1} = c'_k$ and c' coefficients are found from c coefficients.

3. RESULT

Fig .1 and Fig.2 show this method in comparison with other analytical FIR filter designs.

4. CONCLUSION

A new analytical technique for designing digital FIR filters is presented by using Least Squares approach (this approximating method already was used to design an IIR filters)This method gives better approximation and minimize error in passband region. It can be used for application which need pricion magnitude in passband.

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Fig .1

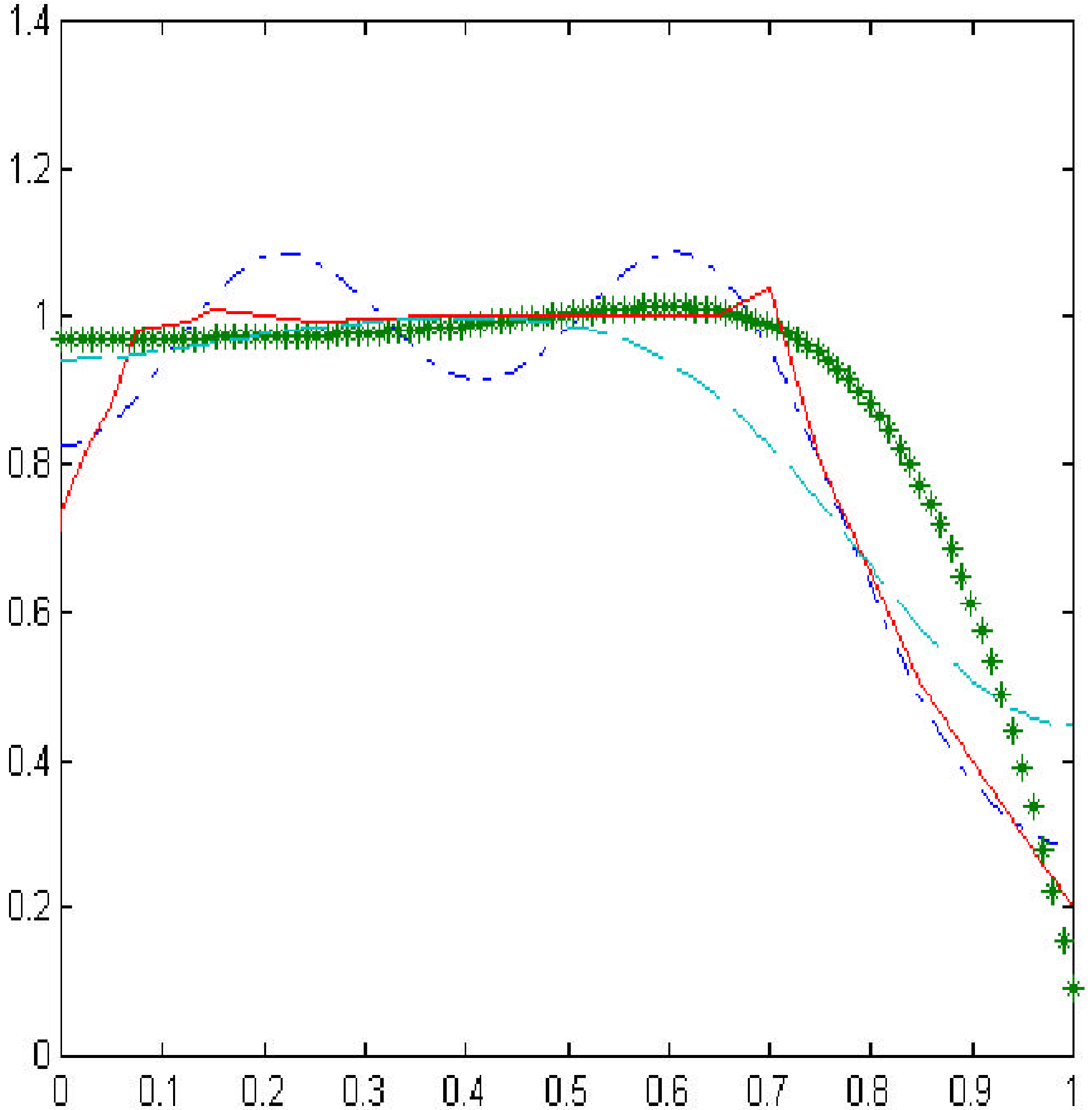

Fig .2