Abstract: - This paper present an initial Input Shaping implementation for flexible machines with pneumatic actuators. Some pneumatic circuits are designed for basic shapers characteristics. Non linear pneumatic dynamic reduces shaping performances, to overcome this drawback, basic shaper are convolved with unity-magnitude robust shapers. Some methods for implement this robust shapers in real time control systems are proposed.

Key-Words: - Vibration, Input signals.

1 Introduction
The increasing use of pneumatic actuators, light and flexible structures based on thin profiles of aluminum or polystireno, foots with shock absorvers, and dynamic components with large inertias implies a moderate degree of flexibility in a numerous group of machines where deflection and residual vibration at the end of movement takes place. An example of such machine is shown in Fig. 1. A table which gives rigidity to the machine is coupled to the structure for fast assembly with foots with shock absorbers. These foots are vertical dynamic absorbers, nonetheless in several cases forces take place in the horizontal plane and these absorbers imply additional flexibility for the machine in this plane. Usually over or under the table some pneumatic cylinder actuators move a tool header that cuts, curver or perform another task for manufacturing the piece. As cylinders move tool header inertia forces induce vibration in the machine structure. Machine of Fig.1 moves the cars letters one time they have been isolated from its plastic pallet. A vertical pneumatic cylinder lifts it up to a defined height using a venturi device later a pneumatic cylinder moves them horizontal to the right working point where the letters are left. The complete cycle is constituted by the sequential operations:

- Head-tool descend to the joined letters.
- Cut the pallet of the joined letters.
- Lifting the isolated letters.
- Horizontal movement to the right point.
- Leaving of the isolated letters.
- Mounting the letter into an strip.
- Coming back to the start.

A low cost control system based on a PLC is the most popular control solution implemented in these flexible machines. With pneumatic actuators PLC-outputs drive electro-valves that translate the desired movement to physical reference signals according to the command generator PLC-program. To put under pressure the pneumatic circuit at start up in a progressive mode a main valve with progressive opening is usually instaled (SMC EAV2000-3000-4000), but later commands are either a constant positive value, zero, or a constant negative, so in the following, machine pneumatic cylinders will be treated as on-off actuators.

In this application we are concerned with accomplishing tool head motion free of machine structure residual vibration. Several methods that reliably generate commands that do not induce vibration and are time-optimal for a wide variety of flexible systems have been recently proposed [1][2]. All of them are different types of multi-switch bang-bang (MSBB) commands well suited for on-off actuators on linear and non-linear systems. Also kinematics control of pneumatic system by hybrid fuzzy-PID give a reasonable positioning time [3] but alternatively dynamic analysis is not performed and residual vibration reduction at move end is not considered then settling time may be large.

In the following the command generation problem for ON-OFF pneumatic actuators is formulated as an input shaping problem. Recall from [4] that Input Shaping is a technique used in the command generator to reduce residual vibration in mechanical systems with some degree of flexibility. The system
parameters (frequency and damping) are used to design the input shaper, which is an impulse sequence whose amplitudes and time location are functions of those parameters. This sequence is convolved with the input and the convolution result is the new shaped input to the system. Many papers have been published on input shaping since its original presentation in [4]. A method for increase insensitive to modeling errors was presented in [5]. Input shaping was shown to be effective for multiple mode systems [6]. Flexible systems equipped with constant-force actuators were shown to be compatible with input shaping [7][8][9]. Input Shaping was acknowledged as an established technique for controlling flexible structures in [10].

2 Dynamic Analysis

As in every shaping problem related to machine dynamics, the first important step is the assessment of the specific experimental dynamic behaviour and the search for an appropriate machine model that has to be at the same time accurate and manageable. Fig 2 shows a dynamic model for this flexible machine whose table is modelled as a mass $M_1$, feet with shock absorvers as a spring $K_1$ and a damper $b_1$, the horizontal cylinder as a mass $M_c$ and tool head as a mass $M_2$ connected with a spring to the table $M_1$ due to mechanical link with a vertical beam between the cylinder and the table. Note that it is an initial ideal linear model. During movement the non-linearity induced by friction in pneumatic cylinder and the variation of dynamics, due to springiness of air with heavy loads derives in hysteresis ranges of around 50%.

Nonetheless pneumatic actuators are used widely in manufacturing, aerospace, and other industries for material handling, positioning, conveying and other purposes, they are preferred due to low cost and high power, softness in handling (compliance), and being environmentally friendly (using air instead of oil versus hydraulic systems).

Pneumatics dynamics do not conform to traditional linear exact models upon which modern control theory has been developed. In the following we investigate if input shaping on-off commands for these non-linear pneumatic actuators it is possible to improve machine dynamic by reducing structure deflection and residual vibration

From Fig.2, Applying Newton second law to each mass, the mathematic model for the horizontal cylinder that moves tool head load follows:

$$M_2\ddot{w} = p \cdot A - b_2(w - z)$$  \hspace{1cm} (1)

where $p$ is the air pressure, $A$ is the inner cross-section area of pneumatic cylinder and $b_2$ is the air damping. In air dashpot at the beginning of the movement, the piston will quickly travel a finite distance with relative little resistance, air spring, this occurs until the pressure builds or drops, depending of damping direction, this implies a gradual ramping up of damping [11] until an approximate stationary value of around ($b_2\approx1.5$ N.s/m).

$$M_{y} \ddot{y} = -b_1(y - z) - K_1y - b_1\dot{y}$$  \hspace{1cm} (2)

Thus,

$$M_{z} \ddot{z} = -b_2(z - w) - K_2(z - y)$$  \hspace{1cm} (3)
Defining state space variables as follows:

\[
x_1 = y; x_2 = \dot{y}; x_3 = z; x_4 = \dot{z}; x_5 = w; x_6 = w
\]  

(4)

Then from (1), (2) and (3), we get

\[
x_2 = \frac{-(K_1 + K_2)}{M_1} x_1 - \frac{b_1}{M_1} x_2 + \frac{K_2}{M_1} x_3
\]

(5)

\[
x_4 = \frac{K_2}{M_c} x_1 - \frac{K_2}{M_c} x_3 - \frac{b_2}{M_c} x_4 + \frac{b_2}{M_c} x_6
\]

(6)

\[
x_6 = \frac{b_2}{M_c} x_4 - \frac{b_2}{M_c} x_6 + A \cdot P
\]

(7)

Hence the state space equations are:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
-(K_2 + K_1) & -\frac{b_1}{M_1} & K_2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{b_2}{M_c}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
1
\end{bmatrix} \frac{Ap}{M_2}
\]

(8)

\[
y = [1 0 0 0 0 0] \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6
\end{bmatrix}
\]

The associated eigenvalue problem implies solving the polynomial equation \( |\lambda I - A| = 0 \).

\[
\lambda^6 + \left( \frac{b_1}{M_1} + \frac{b_2}{M_c} \right) \lambda^5 + \left( \frac{K_2}{M_1} + \frac{K_2}{M_c} \right) \lambda^4 + \left( \frac{b_2}{M_c} + \frac{b_1}{M_1} \right) \lambda^3 + \left( \frac{K_1}{M_1} + \frac{K_2}{M_1} + \frac{K_2}{M_c} + \frac{K_2}{M_c} \right) \lambda^2 + \left( \frac{b_2 K_1}{M_c} + \frac{b_2 K_1}{M_1} + \frac{K_2 b_1}{M_1 M_c} + \frac{K_2 b_1}{M_1 M_c} + \frac{K_2 b_1}{M_1 M_c} + \frac{K_2 b_1}{M_1 M_c} \right) \lambda + \left( \frac{b_2 K_1 K_2}{M_1 M_c M_2} \right) \lambda = 0
\]

(9)

Mathematica 4.0 was used to compute symbolic solutions of this polynomial equation, but quite complex expressions arise, so in the following a numerical approach is first performed. From the mechanical design, dynamic parameters have been estimated as shown in Table 1:

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>M₁(P-1)</td>
<td>450x785x20</td>
<td>2.8</td>
<td>19.78 Kg</td>
</tr>
<tr>
<td>M₁(P-2)</td>
<td>1200x800x20</td>
<td>2.8</td>
<td>53.76 Kg</td>
</tr>
<tr>
<td>M₂ Tool-head</td>
<td></td>
<td>2.8</td>
<td>6 Kg</td>
</tr>
<tr>
<td>K₁ Foot Spring</td>
<td></td>
<td>2.7 (10^4) N/m</td>
<td></td>
</tr>
<tr>
<td>K₂ Beam Spring</td>
<td></td>
<td>10³ N/m</td>
<td></td>
</tr>
<tr>
<td>b₁ Dashpot</td>
<td></td>
<td>24 N.s/m</td>
<td></td>
</tr>
<tr>
<td>b₂ Air damping</td>
<td></td>
<td>1.5 N.s/m</td>
<td></td>
</tr>
<tr>
<td>Slew dist</td>
<td></td>
<td>200 mm</td>
<td></td>
</tr>
<tr>
<td>A Inner cross section area of cylinder</td>
<td></td>
<td>25π cm²</td>
<td></td>
</tr>
<tr>
<td>P Air pressure</td>
<td></td>
<td>6 Kgr/cm²</td>
<td></td>
</tr>
</tbody>
</table>

Given the mechanical coupling of the cylinder to the table with a beam with cross section 80 x 80 mm and length 306.5 mm \( K_2 \) value is calculated as follows,

\[
K_2 = \frac{3EJ}{L^3} = 10^8 \text{[N/m]} \]

(10)

Where:

\[
E = 0.7 106 \text{Kg/cm}^2 \text{ (aluminium).} \\
J: \text{a } b^3 / 3 = 1.365 10^{-5} \text{m}^4. \\
L: 306.5 \text{mm.}
\]

The solution of the corresponding polynomial equation using numerical computation gives the eigenvalues:

\[
\begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3 \\
\lambda_4 \\
\lambda_5 \\
\lambda_6
\end{bmatrix} = \begin{bmatrix}
-0.74 + 10067j \\
-0.74 - 10067j \\
-0.17 + 19.10j \\
-0.17 - 19.10j \\
0.25 \\
0
\end{bmatrix}
\]

(11)

The model shown in Fig. 2 represents a mechanical system with one underdamped high mode at 1603 Hz and the second underdamped mode with 3.04 Hz vibration frequency according to (11). This last one was verified experimentally.

### 2 Input Shaping Tool Head Pneumatic Actuator Commands.

We perform a search in the FESTO data sheet for the DFM-50-200PPV-A horizontal cylinder installed in the machine working at 5.5 [bar] with a load of 6 [Kgr]. Table-2 and Fig.3 shows dynamic cylinder...
response for a slew distance of 200 [mm] according to manufacturer tests.

**Table 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positioning time</td>
<td>0.79 [s]</td>
</tr>
<tr>
<td>Average velocity</td>
<td>0.26 [m/s]</td>
</tr>
<tr>
<td>Stroke velocity at move end</td>
<td>0.33 [m/s]</td>
</tr>
<tr>
<td>Max. velocity</td>
<td>0.43 [m/s]</td>
</tr>
<tr>
<td>Dissipated impact energy</td>
<td>0.38 [J]</td>
</tr>
<tr>
<td>Air speed</td>
<td>17.3 [m/s]</td>
</tr>
<tr>
<td>Flux regulation</td>
<td>20%</td>
</tr>
</tbody>
</table>

Flux regulation is the most important influence parameter. Fig. 3ab shows response when flux regulation value is 20% at both sides of the cylinder admission and unload and Fig. 3b for a 100% flux regulation. As can be seen the velocity and acceleration profiles are quite different in both cases. While in the first one the velocity is almost constant along the movement and acceleration has two low pulses at the beginning and move end, in the second case the velocity is not constant and its profile is near to a triangular profile with some non-linearities.

Fig. 3a-b: Pneumatic cylinder dynamic response, Conditions: (a) 20%, (b) 100% Flux Regulation.

To accomplish this goal the pneumatic circuit initially installed in the machine as shows in Fig. 4, has three electro-valves for each movement sense and two flow control valves (F_C_A1 and F_C_A2) connections that act as pneumatic damping for the horizontal cylinder. With this circuit two velocity levels are available when cylinder flux regulation (F_C_1 and F_C_2) is maintained under the 50% and an approximate linear response is expected as stated above in Fig. 3. With such constrain, the velocity profile initially designed for the horizontal cylinder motion has those two velocity levels. Applying Input Shaping such a profile can be attained as the result of convolving the step reference signal for the electrovalve described above with an input shaper with zero vibration constraint at mechanical system fundamental frequency $f_n=3.04$ and $d=0.17$. This shaper contain two impulses,

$$\begin{bmatrix}
    t_1 \\
   A_i
  \end{bmatrix} = \begin{bmatrix}
    0 & 0.167 \\
    0.6323 & 0.3677
  \end{bmatrix}$$ (12)

That have been computed by making the percentage of residual vibration $V_{tol}$ equal to zero.

$$V_{tol} = e^{-\delta \omega} \sqrt{C(\omega)^2 + S(\omega)^2}$$ (13)

Where,

$$C(\omega) = \sum_{i=1}^{n} A_i e^{\delta \omega t_i} \cdot \cos(\omega \sqrt{1 - \delta^2})$$ (14)

$$S(\omega) = \sum_{i=1}^{n} A_i e^{\delta \omega t_i} \cdot \sin(\omega \sqrt{1 - \delta^2})$$ (15)

To minimize time delay the first impulse is placed at time zero $t_1=0$, furthermore, the impulse amplitudes must sum to one and be positive. Four equations must be satisfied, an Input Shaper with two impulses give four unknows (two amplitudes and two time locations). Fig. 5 shows the ZV-Shaped command reference profile.

Note that shaper is implemented in the command generator (PLC) by setting time locations with a temporize interrupt routine, and the corresponding amplitudes by setting pneumatic hardware flow control valves (F_C_A1, F_C_A2) to the corresponding levels according to shaper amplitudes.

As stated in introduction section during movement the non-linearity induced by friction in the pneumatic cylinder and the variation of dynamics, due to springiness of air implies that at the beginning of the movement, the piston will quickly travel a finite distance with relative little resistance, air spring, this occurs until the pressure builds or drops, depending of damping direction, then we have a gradual ramping up of damping until an approximate stationary value is attained [11]. This implies that in the actual
implementation of the ZV-Shaper incorrect impulses amplitudes are applied, and a robustness lost take place. To demostrate this effect the amplitude of residual vibration has been ploted as a function of modeled frequency error Fig.6. The results of Fig.6 demostrate that when the ZV-shaper impulse amplitudes are in inverse rate ($ZV_{A_2-A_1}$) vibration is only reduced in a low percentage (45%) at modeled frequency. Also giving the two impluses amplitudes the same value ($ZV_{A_1-A_1}$), the average residual vibration keeps only under the 30%.

To improve shaper robustness the ZV-Shaper can be convolved with robust Input Shapers, without increase move time significantly.

This small periods of over pressure could not be accomplished in this application because the machine has a non-contact protection device based in a safety light grid fitted in the access area, it is used as a hand protection to secure danger cutting zones on machine. Valves electrically actuated are shut down and motion is stopped if a minimum of one light beam is interrupted. If this is the case structure machine vibration may be bigger than with the ZV shaped force profile if an overpressure phase is on line at shutdown time. Elimination of the overpressure altogether is desirable and has motivated the development of the unit magnitude input shapers (UM-input-shapers) that are convolved with the ZV-Shaper to improve robustness in this application.

The first option consist in allow the input shaper to contain negative impulses [13]. But this shapers lead to shaped command profiles which exceed the magnitude of the unshaped command for small periods of time.

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At the end the velocity commands of Fig 3. consist of ramps and constants these correspond to acceleration commands that are near to rounding steps changes:

$$u_i(t) = u_i(t)$$

$$T_{i-1} < t < T_i$$

$$i = 1, 2, ..., r$$

(16)

where $|u_i(t)| \leq 1$, we assume the actuator limits are at $-1$ and $+1$ for simplicity, $T_i$ is the time where the command transitions from $u_i$ to $u_{i+1}$, and $r$ is the number of step changes in the command. When commanding the flexible machine each transition causes vibration. If the magnitude change and timing of the transitions are carefully selected, there may be very little or no vibration at the end of the command. A unit magnitude input shaper shapes each of the transitions in (16) to reduce or eliminate residual vibration at the end of the command. To satisfy the zero vibration and derivative constraints,
the shaper must contain five impulses, extra-insensitive constraints achieve more insensitivity by relaxing the constraint of zero vibration at the damped modeled frequency. If the residual vibration at modeled frequency, \( \omega \), is limited to some small value, \( V_{\text{tol}} \), instead of zero, the UM-EI-Shaper must contain seven impulses,

\[
\begin{bmatrix}
  t_1 \\
  A_j
\end{bmatrix} =
\begin{bmatrix}
  0 & 0.0427 & 0.1309 & 0.2219 & 0.2607 & 0.3869 & 0.3963 \\
  1 & -1 & 1 & -1 & 1 & -1 & 1
\end{bmatrix}
\] (14)

This UM-EI-Shaper was designing for mechanical parameters \( f = 3.04 \) [Hz] and \( d = 0.17 \) using tables given in [12]. Convolving this shaper with the ZV-Shaper initially designed convolution result has fourteen impulses,

\[
\begin{bmatrix}
  t_1 \\
  A_j
\end{bmatrix} =
\begin{bmatrix}
  0 & 0.044 & 0.132 & 0.168 & 0.211 & 0.223 & 0.262 \\
  0.63 & -0.63 & 0.63 & 0.37 & -0.37 & -0.63 & 0.63
\end{bmatrix} +
\begin{bmatrix}
  0.299 & 0.388 & 0.390 & 0.397 & 0.429 & 0.555 & 0.564 \\
  0.37 & -0.63 & -0.63 & 0.63 & 0.37 & -0.37 & 0.36
\end{bmatrix}
\] (15)

the shaped reference signal switches between four values \([0.37 \ 0.63 \ 1]\) that are reached by translating the corresponding commands to the valves electrically actuated in order to change the flux control valve at the correct time. As can be seen the price that is necessary to pay to improve robustness is a quite valuable programming effort. This shaped command was installed in a Siemens S7-224 PLC using a temporised interrupt routine modeled with a Petri network Fig.8, where transitions are solved when the milliseconds counter reach shaping times according to the shaped reference signal.

A simple pneumatic system is presented that allows a mechanical engineer implement a ZV-shaper in low cost flexible machines with pneumatic actuators. Performances of ZV-shaper are improved by convolving this shaper with robust unit-magnitude input shapers. A method for implement this shaper in real time control systems PLC based is proposed.

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References:

Fig 8.- Petri-Network and Interrupt Routine template for UM-EI-ZV-Shaper implementation. 4 Conclusion