DE-NOISING OF ECG SIGNAL IN WAVELETS DOMAIN

GÜLAY TOHUMOĞLU- ERGUN ERCELEBİ
Electrical and Electronics Engineering Department
University of Gaziantep
EE Eng. Dept. University of Gaziantep 27310 Gaziantep
TURKEY

Abstract: - De-noising of electrocardiogram signal is studied by the application of wavelet-based approaches. A new thresholding method for linear and non-linear denosing procedure is offered. The threshold value newly defined is assigned by considering the maximum transform coefficient and decomposition level dependably. The new thresholding method is applied to a set of biorthogonal, orthogonal wavelets and an optimized wavelet-packet decomposition. The quantitative comparison of the method with the existing nonlinear denoising method is done in terms of the peak-signal to noise ratio and visual inspection of electrocardiogram signal. The error is measured in the high frequency region QRS complex of the electrocardiogram signal. It is observed that the soft thresholding approach is effective then hard one with the defined threshold.

Keywords: - De-noising, electrocardiogram, thresholding, wavelets and wavelet packets.

1 Introduction

The wavelet transform [1] is a time-scale representation that has successfully found a broad range of applications, particularly in biomedical signal processing. Recently, wavelets have been applied to several problems in electrocardiology as data compression [2], detection of electro-cardiogram (ECG) signal characteristic points [3]. Wavelet transform has been used in removing noise from the signals. The bandpass filter structure of the wavelet [4] decomposition defines the multiresolution analysis and the subbands of the signal which gives the high frequency components at different levels and low frequency components at the last level of the decomposition. The ECG signal having well localized QRS high frequency complex and P and T waves low frequency parts. This is the reason why multiresolution analysis is suitable in the ECG signal decomposition. The wavelet-packets [5] are a generalization of the wavelet transform that allow for arbitrary tree-shaped bandpass filtering. Wavelet packet decomposition can be optimized by considering the characteristics of the signal to be analyzed. De-noising methods of ECG signal involving thresholding selection rule is studied for the stimulated and real signals [6-7]. Recently, Tikkanen [8] studied to de-noise the ECG signal by applying the orthogonal wavelets as Coiflets wavelet and optimized wavelet packet for soft and hard thresholding method. It was concluded that wavelet de-noising approach were more efficient than wavelet packet de-noising and performance of the de-noising in the high frequency part of ECG signal is generally better for soft thresholding than hard thresholding. However, the application of wavelet and wavelet-packet based approaches has not yet been largely studied.

In this paper, two main parts are emphatically studied. Firstly, the thresholding to characterize the signal being analyzed is expressed dependably to the signal properties as wavelet coefficients in the transformed domain not to the number of samples identifying the signal as in the previous works. Secondly, using the new thresholding approach, a different class of wavelets as biorthogonal wavelets and wavelet packets is largely studied in order to de-noise ECG signal simulated by Gaussian and uniformly distributed white noise. The paper is organized as follows: In section 2, A short description of wavelet transform and wavelet packet decomposition. In section 3, it is described the new thresholding and de-noising scheme.In the following section, summary of the experimental results and major findings, and finally, quantitative comparison of the results and conclusions is given.

2 Wavelet Transform

A wavelet is a “small wave” having the oscillating wavelike characteristic and the ability to allow simultaneous time and frequency analysis by the way of a time-frequency localization of the signal. Wavelet systems [9] are generated by dilating and translating a single prototype basic wavelet $\psi(t)$

$$\psi_{a,b}(t) = |a|^{-1/2}\,\psi\left(\frac{t-b}{a}\right) \qquad (1)$$

where the scaling factor a and translation factor b are real ($a \neq 0$). The basic wavelet is stretched by a large value of a to analyze the low frequency components of the signal. A small value of a gives a contracted version of the basic wavelet and thus allows the analysis of high-frequency components. The *mother wavelet* ψ must satisfy $\int \psi(x)\, dx=0$, (i.e.The condition on ψ should be $C_\psi = \int |\omega|^{-1} |\Psi|^2 d\omega < \infty$, where Ψ is the Fourier transform of ψ ; if $\psi(t)$ decays faster than $|t|^{-1}$ for $t \to \infty$, then this condition is equivalent to the one above). The continuous wavelet transform of f(t) with respect to the wavelet $\psi(t)$ is then

$$W_f(a,b) = \langle f(t), \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t)\, \psi_{a,b}(t)\, dt \tag{2}$$

The wavelet transform coefficients are given as inner products of the function being transformed with each of the basis functions.

The inverse continuous wavelet transform is defined [10] as

$$f(t) = \frac{1}{C_\psi} \int_0^{\infty} \int_{-\infty}^{\infty} W_f(a,b)\psi_{a,b}(t)\, db\, \frac{da}{a^2} \tag{3}$$

The second type of wavelet transform is defined as *the wavelet series expansion*. Again, a basic wavelet is scaled by binary scaling and translated by a dyadic translations to form a set of basis functions. For the wavelet expansion, a two-parameter system which is defined for a signal f(t) becomes

$$f(t) = \sum_k \sum_j a_{j,k}\, \psi_{j,k}(t) \tag{4}$$

where the $\psi_{j,k}(t)$ formed from the mother wavelet $\psi(t)$ are the wavelet expansion functions that usually form an orthogonal basis of L^2 (R) defined as

$$\psi_{j,k}(t) = 2^{-j/2}\, \psi(2^{-j}t - k) \tag{5}$$

where both j and k are integer indices, and j determines the dilation while k specifies the translation. The two-dimensional set of coefficients $a_{j,k}$ is called the *discrete wavelet transform (DWT)* of f(t) used in multiresolution analysis constituting an orthonormal (biorthogonal) basis for $L^2(\mathbf{R})$ [Db92] . A more specific form indicating how the $a_{j,k}$'s are calculated by writing inner products as

$$a_{j,k} = \langle \psi_{j,k}(t), f(t) \rangle \tag{6}$$

(Note that $\langle \psi_{j,k}(t), \psi_{l,m}(t) \rangle = \delta_{j,l}\, \delta_{k,m}$ where l and m are integers, $\delta_{j,k}$ is the Kronecker delta function, and <. , .> indicates inner product.) [9-10]. In DWT, f(t) signal decomposition on different scales can be expanded as

$$f(t) = \sum_{j=1}^{K} \sum_{k=-\infty}^{\infty} d_j(k)\, \psi_{j,k}(t) + \sum_{k=-\infty}^{\infty} a_K(k)\, \phi_{K,k}(t) \tag{7}$$

where $\psi_{j,k}(t)$ are discrete analysis wavelet and $\phi_{K,k}(t)$ are discrete scaling functions, $d_j(k)$ are the detailed signal which are wavelet coefficients at scale 2^j , and $a_K(k)$ is the approximated signal which are scaling coefficients at scale 2^K.

The discrete wavelet transform can be implemented by the scaling and wavelet filters

$$h(k) = \frac{1}{\sqrt{2}} \langle \phi(t), \phi(2t-k) \rangle \tag{8}$$

$$g(k) = (-1)^k h(1-k) = \frac{1}{\sqrt{2}} \langle \psi(t), \phi(2t-k) \rangle \tag{9}$$

being quadrature mirror filters (QMF) [1]. Computation by the convolution of the approximate signal at level (j-1) with the coefficients h(k) (g(k)) gives the estimation of the approximate (detailed) signal at level j.

Wavelet packet analysis is a generalization of wavelet analysis offering a richer decomposition procedure. A set of detailed and approximations components of the signal is called wavelet packet decomposition tree. Discrete wavelet decomposition allows searching an optimal decomposition among L trees with the signal length $N=2^L$ where the signal has been decomposed at L levels. Wavelet packet analysis involves the selection of an optimal decomposition tree mostly from the 2^L different subtrees of depth L which is optimized by the minimization of the entropy of the signal to be analyzed. A wavelet packet can be considered as a waveform whose oscillations persists for many cycles but is still finite. In order to apply wavelet packet analysis let us define the scaling function $W_0(t) = \phi(t)$ and the wavelet function $W_1(t) = \psi(t)$. Then it can be written functions $W_m(t)$, $m = 0, 1, 2, \ldots,$ as

$$W_{2m}(t) = 2 \sum_{n=0}^{2N-1} h(n) W_m(2t-n) \tag{10}$$

$$W_{2m+1}(t) = 2 \sum_{n=0}^{2N-1} g(n) W_m(2t-n) \tag{11}$$

The analysis functions [1,8] called wavelet packet atoms are given in an orthogonal case as

$$W_{j,m,n}(t) = 2^{-j/2} W_m(2^{-j} t - n) \quad (12)$$

where j and n are the scale and time-localization parameters respectively, and parameter m gives the roughly the number of ‘cycles’ included in the oscillating waveform. The function $W_{j,\ m,\ n}\ (t)$ analyzes the signal around the position 2^{j}.n at the scale 2^{j} with fixed value of j and the analyzed frequencies are roughly given by n/2N with n=0, 1, …, (2j-1).

3 The New Thresholding and Denoising Scheme

The wavelet transform decorrelates a signal and concentrates its information into a relatively small number of coefficients with large magnitudes containing more energy than the small coefficients. This makes possible to compare these coefficients with a threshold. The threshold value (*th*) is defined as a function of the largest transform coefficient c_i and the depth of the decomposition level L as follows:

$$th = 2^{n-L}, \quad with \quad n = round\left[log_2\left(max\left\{\left|\, c_i \,\right|\right\}\right)\right] \quad (13)$$

It is seen that the signal is whether high frequency or low frequency concentrated. Also it is modified that the threshold level is newly defined for each decomposition level concerned with the heavy noised signals. In literature [11], the threshold is expressed as a function of the signal length for hard or soft thresholding.

In denoising algorithm, the signal is decomposed into the detail components by using wavelet and wavelet packet, then identifying the noise components and reconstructing the signal without those components using the inverse wavelet transform. In the procedure, mainly two assumptions arise, the former says that the noise components can be found within the finest scales and the coarsest scale becomes noise-free. In this case, only the coarse scale transform coefficients are considered and used to reconstruct the signal to treat as a linear denoising approach. The latter one assumes that the noise component appears in all wavelet coefficients and in each scale and using the nonlinear threshold approach that discards the coefficients exceeding the defined threshold [6].

The quantitative comparison of the results is done by the computation of the signal-to-noise ratio (SNR) defined in equation (14) where the $f(i)$ and $f_d(i)$ denotes the noisy and denoised ECG signals,

$$SNR = 20\,log_{10} \frac{\sum_{i=1}^{n} \left| f(i) \right|}{\sum_{i=1}^{n} \left| f(i) - f_d(i) \right|} \quad (14)$$

respectively. Performance evaluation is not only measured as SNR but also visual performance is observed.

4 Experimental Results

In this study, the different number of noise-free ECG signals are used as a 512 Hz sampling frequency with the resolution of 12 bits. They are simulated by adding uniformly distributed white noise with the SNR=–1.1165 dB for the noise free to the noise signal ratio. The noisy ECG signal is studied for 3 period QRS complex seen in Figure 1. Considering biorthogonal and compactly supported wavelet families (bior1.3, bior2.6, bior3.5, bior5.5, bior3.7) for the 3-level and 5-level decompositions with discrete wavelet transform, the performances are very close to each other and they generally give better results for soft thresholding than hard thresholding denoising rule. Additionally, as a orthogonal wavelet Daubachies Db1, Db3, Db3, Db8 is studied. Wavelet packet analysis is still applied for the same set of wavelet family. A sample ECG signal is shown in Figure 1 part (a). A uniformly distributed white noise added ECG signal is depicted in part (b) of Figure 1. It is seen that 5-level decomposition gives better denoising for soft thresholding than hard one for *th* defined in equation (13) and shown for wavelets in Figure 1 (c) and (d) parts, and wavelet packet analysis in Figure 1 parts (e) and (f), respectively. The visual inspection of the denoised signal for the bior2.6 and bior5.5 is better than the rest of the biorthogonal set of wavelets and wavelet packet analysis which are seen in Figure 1 parts (g) and (h). The computed signal-to-noise ratios are approximately 10.5 dB for the used biorthogonal wavelets family.

5 Conclusions

In this work, compactly supported biorthogonal wavelets and wavelet packet-based noise removal is studied for new thresholding using the uniform white noise simulated ECG signal. Basically, wavelets and wavelet packet analysis show different results, however, generally the soft thresholding for the newly defined threshold gives better results than threshold defined in the literature. When the high

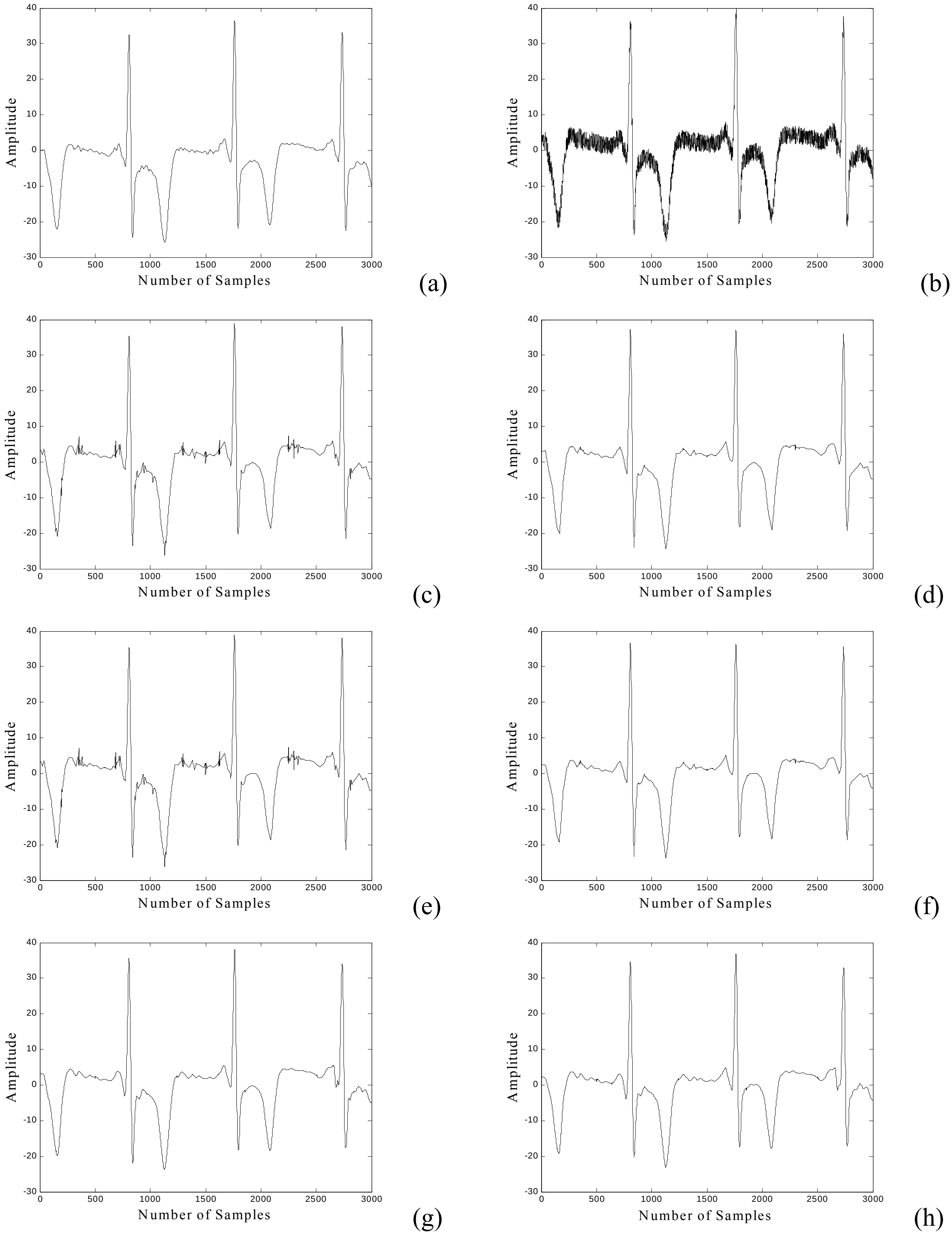

Figure 1 The performance of the noise reductiun method for unfiromly distributed noise: (a) noise free ECG signal, (b) Corrupted ECG signal with uniformly distributed noise, (c) Denoised ECG signal with hard thresholding and bior2.6 filter, (d) Denoised ECG signal with soft thresholding and bior2.6 filter (e) Denoised ECG signal with hard thresholding , wavelet packet and bior2.6 filter (f) Denoised ECG signal with soft thresholding , wavelet packet and bior2.6 filter (g) Denoised ECG signal with hard thresholding and bior5.5 filter (h) Denoised ECG signal with hard thresholding , wavelet packet and bior5.5 filter.

frequency part QRS complex of ECG signal is considered, the measured quantitative error is decreased by the soft thresholding and it has also good visual quality. In general aspect, this newly defined threshold gives better approach for the soft and hard thresholding procedure of the ECG signal denoising.

This study is continuing on the modification of the denoising algortithm in order to experience noisy abnormal ECG signals.

References:

[1] I. Daubechies, *Ten Lectrues on Wavelets*, CBMS- NSF Regional Conference Series in App. MathVol. 61, SIAM, 1992.

[2] J. A. Crowe, N. M. Gibson, M. S. Woolfson, and M. G. Somekh, “Wavelet transform as a potential tool for ECG analysis and compression,” J. Biomed. Eng., vol. 14, 1992, pp.1462-1469.

[3] C. Li, C. Zheng, and C. Tai, “Detection of ECG characteristic points using wavelet transform,” IEEE Trans. Biomed. Eng. Vol.42, No. 1, 1995, pp. 21-28.

[4] Mallat S., “A Theory for Multiresolution Signal Decomposition. The Wavelet Decomposition”, IEEE Trans. PAMI Vol. 11, pp. 674-693, 1989.

[5] R. R. Coifman and Y. Meyer, “Orthonormal wave packet bases,” Math. Dept., Yale Univ., New Haven, CT, Tech. Rep., 1990.

[6] D. Donoho, “ De-noising by soft thresholding,” IEEE Trans. Inform. Theory, Vol. 41, 1995, pp. 612-627.

[7] M. Hilton, R. Ogden, “Data analytic wavelet threshold selection in 2-D signal denoising,” IEEE Trans. Signal Proc. Vol. 45, 1997, pp. 496-500.

[8] P.E. Tikkanen, “Nonlinear wavelet and wavelet packet denoising of electrocardiogram signal,” Biol. Cybern. Vol.80, 1999, pp.259-267.

[9] O. Rioul and M. Vetterli, “ Wavelets and Signal Processing,” IEEE Signal Processing Magazine, Vol. 8, 1991, pp. 85-107.

[10] K. R. Castleman, *Digital Image Processing*, Prentice-Hall Inc., Englewood Cliffs, New Jersey, 1996.

[11] D. Donoho, “Nonlinear wavelet methods for recovery of signals, densities,and spectra from indirect and noisy data,” Proc. Symp. Appl. Math. Vol. 47, 1993, pp.173-205.