Efficient Digital Correlation in Subbands

ANNA Z. BARANIECKI
Department of Electrical and Computer Engineering
George Mason University
Fairfax, VA 22030
USA

Abstract: A method is described to facilitate the implementation of subband correlation of wavelet representations. The technique is especially applicable for comparing images or image fragments with respect to all translations of a full image, for simultaneous compression and cross-correlation to estimate time of arrival of MSK signals, or to achieve the maximum spectral resolution at a minimum cost for wideband signals in radio interferometer.

Correlation alias components are smaller for longer filters. The direct realization of subband correlation of wavelet representations becomes computationally intensive with the use of long filter orders. The implementation described in this paper allows a reduction of correlation complexity in subbands of wavelet representation.

Key-Words: correlation in subbands, efficient algorithms, wavelet transform, multiresolution analysis.

1. Introduction

The motivation for this work stems from the rapid growth of application areas of Discrete Wavelet Transform (DWT) or its generalization from two bands to M bands, M-DWT [1]-[6].

The M-DWT can be used to decompose a signal represented by \( c_{o,m} \) into successive layers of coarser resolution \( c_{i,m} \) (smoothed down sampled approximation) plus a set of \((M-1)\) detail signals at each layer, \( d'_{i,n} \), also at coarser resolution. This decomposition may be achieved using a cascaded analysis stages of perfect reconstruction (PR) filter bank. Figure 1 shows two stages of M-DWT decomposition and reconstruction respectively. \( H_0 (z) \) is the z-transform of coefficient set \( \{h[k]\} \) and \( H_i (z) \) satisfy well-known orthonormality conditions [1]-[3].

Equations 1 and 2 define convolution followed by subsampling by a factor \( M \). The transfer functions \( H_0 (z) \) and \( H_i (z) \) satisfy

\[
\begin{align*}
    c_{i,n} &= \sum_k h[k - M n] c_{o,k}, \quad i = 1, \ldots, M-1. \\
    d'_{i,n} &= \sum_k g[k - M n] c_{o,k}, i = 1, \ldots, n - 1
\end{align*}
\]
The efficiency of the algorithm proposed in this paper stems from transform domain realization of convolution, which for cascaded filters and decimators reduce the number of computations, and simple products of transform and conjugate transform values generate desired correlations in subbands.

### 2. Correlation

Multiresolution analysis allows computation of normalized correlation coefficients of an image fragment with respect to all translations of a full image. In [4] it is shown how to calculate correlation coefficients using multiresolution analysis. Images are mapped into wavelet representations and the
correlation theorem allows subsamples of correlations of two signal domains to be obtained from a sum of correlations of subbands of wavelet representations of those signals.

Applicability of correlation in subbands for wideband signals in radio interferometer is described in [5] and allows achievement of maximum spectral resolution at minimum cost. In this paper we illustrate correlation in subbands of wavelet representation for simultaneous compression and cross-correlation to estimate time of arrival of Minimum Shift Keying (MSK) signals. To simplify the discussion, throughout this paper, all input data are assumed to be one-dimensional. The extension to two dimensions is straightforward and obvious.

Let $x$ and $v$ be a column vectors of length $N$ and $x \cdot v$ be the correlation whose $i$'th component is

$$ (x \cdot v)_i = \sum_{k=0}^{N-1} x_{i+k \mod N} v_k $$

for $i=1,\ldots,N$.

The correlation theorem is the correlation form of the well known convolution theorem, where the inverse transformation of the product of transform of $x$ and complex conjugate of transform of $v$ (or vice-versa) yields circular correlation. The transform should have the cyclic convolution property, i.e. its kernel should be an $N$'th root of unity. The Fourier Transform and a variety of Number Theoretic Transforms satisfy this condition. Efficient implementation of correlation in wavelet domain is described in section 4.

3. Transform Domain Realization of Wavelet Transform

The M-DWT can be realized using a cascaded filter bank structure. The transform domain realization can be used for each convolution. To compute the filter output using a transform such as Fast Fourier Transform (FFT) or Number Theoretic Transforms (NTT) that obey the cyclic convolution property (CCP), input data is split into non-overlapping or overlapping blocks of length $K$. Then for the filter of length $L$, the transform length $N$ is selected longer than $K+L-1$ to prevent wrap-around effect due to circular convolution property of the transform, $N$-point transform of each input block is multiplied with the $N$-point transform of the filter response. After applying inverse transformation to the result of each multiplication operation, two consecutive blocks can be combined using the overlap-add or overlap-save operation, depending on the scheme used to obtain the input blocks. In Figure 2, we show an implementation of a 2-stage DWT using transform $T$ (FFT or NTT). The block that has an inverse transform (IT) computation followed by the decimation and then by transform $T$ can be replaced by simple summation operation that computes the FFT values of the decimated sequence. This is illustrated in Fig. 3. Alternatively, a cascade of polyphase approach and transform domain algorithm that we described in detail in [6] could be used. Detailed comparison of computational complexities is also provided in [6].

4. Transform Domain Implementation of Correlation in Wavelet Domain

To obtain correlation coefficients in appropriate subbands, the values of either $Y_i$ or $W_i$ should be conjugated and point-by-point product obtained to generate the transform of correlation coefficients. This is illustrated in Fig.4. Total number of computations for the computation of correlation is in the order of transform length at a given stage or resolution. Total correlation can be obtained according to [4] as a sum of correlations in subbands for image applications. For correlating wideband signals, or as illustrated in this paper MSK signals, inverse transformation generates desired correlation in subbands.
To estimate the time of arrival of MSK signals, three level decomposition was tested for Shannon entropy and the resulting best tree is as shown in Fig. 5. The experiment walked down the boxed nodes to test the performance of the algorithm on subbands.

The magnitude of the correlation of complete signal and subband correlations after level 1 and 2 are shown in Figure 6. The subbands produced similar correlation peaks, at the same time allowing compression.
5. Conclusion

This paper presented efficient algorithms to implement a correlation of wavelet representations in subbands. Representation of filter banks in transform domain leads to very efficient computation of correlation coefficients, with the number of multiplications being in the order of transform length at a given stage or resolution. The transform used can be FFT or cascade of polyphase and transform domain representation. To minimize alias components in computing correlation in subbands, longer filters have to be used. A direct approach to computation of wavelet coefficients is significantly less efficient than the method described in this paper. Applicability of this approach to MSK signals has been illustrated in this paper and allows simultaneous compression and computation of correlation. Other applications are in image processing, or correlating wideband signals with the objective of a good resolution at a minimum cost.

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References:


