Detection and Estimation of Spectral Change

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Abstract: - Spectral change estimation is important in many applications, because changes in spectra often correspond to changes in important physical parameters. Most attempts at detecting and estimating spectral change simply differ- ence conventional spectra; this approach is flawed, however, because conventional spectra implicitly assume the observed signal is stationary over the measurement interval. This paper proposes a spectral change estimator which makes no such assumptions about stationarity. It is effective for signals with rapidly changing spectral components, even in the presence of heavy noise.

Key-Words: - spectral change, polynomial phase components, cubic phase function, additive noise, spectral change estimator, smoothing techniques, non-stationary, detection, estimation

1 Introduction

Spectral analysis occupies a key place in scientific research because it is the basis for a large number of important applications. These applications include power system disturbance monitoring, radar, telemetry, communications, etc. Spectral analysis as outlined originally by Fourier dictates that the signal be decomposed into its sinusoidal components. This type of analysis is sensible in linear stationary systems, for which the eigenfunctions (i.e. the natural signals to propagate) are the sinusoids. It is far less sensible, however, in systems which are non-stationary or non-linear.

Because there are many non-linear and non-stationary systems researchers have been attempting to extend Fourier analysis beyond the standard sinusoidal component paradigm for decades. Many of these attempted extensions have involved modeling the observed signal as a sum of arbitrary polynomial phase components rather than sinusoidal (linear phase) ones [1], [2], [3]. There have, however, been substantial roadblocks along the road to success with this kind of approach. While the extension to higher order phase polynomial components based on maximum likelihood (ML) principles is straightforward in principle, its direct implementation requires a multi-dimensional search which is intractable in most practical scenarios. Alternative solutions have been reported, but these are either very computationally intensive [1] or work only at high SNR [2], [3].

Additionally, recent research indicates that for many practical systems the spectral changes can be of more use than the spectra themselves. It was wrongly believed for many years, for example, that the human auditory system could not detect frequency components above 20kHz. This misconception existed because of a failure to realise that humans respond to changing sounds quite differently to the way they respond to non-changing sounds [4]. Humans can only hear up to 20kHz if they are listening to sounds which are spectrally unchanging. They can hear well above 20kHz, however, if they are listening to sounds whose spectra are rapidly varying. Parallel findings exist in the field of vehicle aerodynamics. Wind noise can be a major source of annoyance in automobiles and car manufacturers naturally want to design cars which minimise this annoyance. It has been found that the annoyance factor in humans is highly dependent on how much the noise changes, i.e. on how much spectral
change there is. Similarly, in disturbance monitoring of power systems, the detection of spectral changes is very critical because such changes can indicate a departure from a stable norm. They can also indicate operating point changes. There is thus a need to develop a strategy for quantitatively characterising spectral changes. The following sections discuss one such strategy.

2 Spectral Change Estimation

In conventional spectral analysis of a real signal, \( z(n) \), one typically forms the auto-correlation function, \( R(m) \), and decomposes this function into sinusoidal or linear phase components. Mathematically this is achieved according to:

\[
P(\omega) = \sum_{m} R(m)e^{-j\omega m}
\]

\[
= \sum_{m} E\{z(n+m)z(n-m)e^{-j\omega m}\}, \quad |n| \leq (N-1)/2
\]

where \( E\{\cdot\} \) denotes the expected value. The aim of a spectral change operator is to detect not frequency (i.e. the rate of change of phase), but the rate of change of frequency. In this paper it is proposed to use the following new definition for the spectral change estimator:

\[
P'(\Omega) = \sum_{m} R(m)e^{-j\Omega m^2}
\]

\[
= \sum_{m=0}^{(N-1)/2} E\{z(n+m)z(n-m)e^{-j\Omega m^2}\}, \quad (4)
\]

\[
= E\{CP(n,\Omega)\}, \quad |n| \leq (N-1)/2, \quad (5)
\]

where the definition for \( CP(n,\Omega) \) is evident from the above. The spectral change estimator is seen from (6) to be the expected value of the function, \( CP(n,\Omega) \), which is also referred to as the “cubic phase (CP) function”. This name is given because, interestingly, the CP function yields highly concentrated spectral change representations for signals whose phase is linear, quadratic or cubic. (This is in contrast to conventional spectral representations which only yield strong energy concentrations for linear phase signals). The CP function is essentially a “time-frequency rate representation”, with \( n \) being the time index, and \( \Omega \) being the frequency rate index. The function is obviously quite simple in structure, and yet has some remarkable properties. Some of these properties are listed below.

1) The expected value of the CP function neatly separates the stationary (or spectrally constant) and non-stationary (spectrally changing) parts of the signal. The information relating to the stationary portion of the signal is contained in \( E\{CP(n,\Omega)\} |_{\Omega = 0} \), while the information pertaining to the non-stationary signal portion is in \( E\{CP(n,\Omega)\} |_{\Omega \neq 0} \). Further information on the character of the non-stationary part of the signal is revealed in the way energy is distributed across \( E\{CP(n,\Omega)\} |_{\Omega \neq 0} \). That is, one can see how energy is distributed across the “time-frequency rate” plane.

2) For signals consisting of either sinusoidal, quadratic or cubic phase components, the CP function is a natural analysis tool - it enables optimal parameter estimation of such signals at comparatively low SNRs [6], providing it is accompanied by some very simple post-processing. The parameter estimation techniques can also be extended to achieve optimal parameter estimation for higher order polynomial phase signals.

3) The discrete-time Fourier transform with respect to \( n \) of \( E\{CP(n,\Omega)\} |_{\Omega = 0} \) yields the spectrum of all stationary signal components.

\( CP(n,\Omega) \) is a bilinear function of the signal, \( z(t) \), and as such, is plagued with cross-terms if multiple components are present. Note, however, that it is the expectation of the CP function (rather than the function itself) which constitutes the spectral change estimator in (6). To render the expected value of the CP function effective in the presence of multiple components (or in very heavy noise) it is pertinent to do some averaging. Ideally ensemble averaging would be used, but since an ensemble of realisations is often not available, one normally has to resort to time and/or frequency rate averaging. This averaging or “smoothing” should preferably be optimal, and as such, should be data dependent. Determination of the best forms of smoothing needed to implement the expectation is critical. One approach to the smoothing is presented in the following paragraph.
2.1 Smoothing techniques for the spectral change estimator

At low SNR, the random components of the slices of the CP function at distinct time positions are quasi independent. i.e. the random components of \( CP(n_1, \Omega) \) and \( CP(n_2, \Omega) \) are very close to being independent for \( n_1 \neq n_2 \). Noise reduction (and cross-term suppression) will occur if multiple slices are intelligently (i.e. coherently) averaged or smoothed. Appropriately chosen “data-dependent abbreviated weighted line integrations” realise this coherent averaging, and thereby enable highly efficient and highly effective noise and cross-term suppression. These abbreviated weighted line integrations are formed according to:

\[
MCP(n, \Omega) = \max_{\alpha_1, \ldots, \alpha_3} \left[ \sum_{n=0}^{n+K} CP(n, \Omega) e^{-j(\alpha_1 n + \Omega/2 n^2 + \alpha_3 n^3)} \right],
\]

where \( K \) is an integer. As a rough rule of thumb it has been found that if one wants to operate down to an SNR threshold of \( S_T \) (where \( S_T \) is less than 1) then one needs to intelligently fuse \( 1/S_T \) slices. That is, \( K \) needs to be approximately \( 0.5/S_T \). If, for example, one wants to operate down to an SNR threshold of \( 1/10 \), then 10 CP function slices need to be computed and fused according to (7). In the case of cubic phase signals, the SNR thresholds obtained with this type of approach appear to be almost as low as those obtained via ML estimation/detection of cubic phase signals, but with much less computation (typically two orders of magnitude less!). Figure 1 shows a smoothed spectral change representation for two narrowband spectral change components, one whose frequency is changing at 0.01 rad/sec, and the other whose frequency is changing at -0.003 rad/sec. The sampling rate was unity and there was significant additive noise on the signal (-10dB relative to each of the two components). The signal was 515 samples long and smoothing was performed by fusing 14 CP function slices. The figure shows two clear spectral change lines corresponding to the two different rates of spectral change of the components. These two spectral change lines are clear despite the very low SNR.

3 Conclusion

A spectral change estimator has been introduced. It has been found to discriminate differing spectral change rates, even in heavy noise.

References:


Fig.1 Spectral change representation for two time-varying narrowband components in -10dB noise.