Methods and Techniques in Operation Research

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Abstract: In this paper, a brief summary of the heuristic methods, single-stage optimization methods, time-phased optimization methods, artificial intelligence (AI) techniques and iterative improvement methods are presented. Finally, some of the important characteristics of network programming methods and their strengths and weaknesses are identified and compared.

Key-words Heuristic Methods, Single-Stage Optimization Methods, Time-Phased Optimization Methods, Artificial Intelligence Techniques, Iterative Improvement Methods.

1 Introduction

The main classes of tools and techniques that are commonly used in operation research are heuristic methods; single-stage optimization methods; time-phased optimization methods; artificial intelligence techniques and iterative improvement methods.

Heuristic methods define as "the study of the methods and rules of discovery and invention". Heuristics improve the efficiency of a search process acting like a tour guide. Although they point in interesting directions, they can lead into dead ends. Using good heuristics, one can hope to get good (but not necessarily optimal) solutions to complex problems.

Heuristics are employed in two basic situations:

i. when a problem does not have an exact solution because of inherent ambiguities in the problem statement or available data, medical diagnosis is an example of this;

ii. when a problem has an exact solution, but the computational cost of finding it may be prohibitive, such as in chess and production scheduling (combinatorial problems).

Unfortunately, like all rules of discovery and invention, heuristics are fallible. A heuristic is only an informed guess at the next step to be taken in solving a problem. They are often based on experience or intuition (common sense). When there is only limited information heuristic search is often the only practical answer.

Heuristic search problems are often not easily described in a form that leads to immediate mathematical derivations of an optimal solution. Often heuristics are developed by trial and error, in conjunction with a number of reasonable approximations, simplifications, reasonable guesses, or domain-specific problem knowledge.

Mathematical programming, and especially linear programming is one of best developed and most used branches of OR. It concerns the optimal allocation of limited resources among competing activities, under a set of constraints imposed by the nature of the problem being studied. These constraints can reflect financial, technological, marketing, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical representation aimed at programming or planning the best possible allocation of scarce resources. When the mathematical representation uses linear functions exclusively, it has a linear programming model.

Single-stage optimization methods can be used for determining the optimal network expansion from one stage to the next. But they do not give the timing of the expansion. The mathematical programming techniques used in single-state optimization methods include linear programming; integer programming and non-linear programming.

A time-phased optimization method can include inflation and interest rates, etc. in the comparison of various network expansion plans. Both integer programming and dynamic programming optimization methods have been used to solve the time-phased network expansion models. Integer programming has been applied by dividing a given time horizon into numerous annual sub-periods. Consequently, the objective function in terms of present worth of a cost function is minimized in order to determine the capacity, location, and timing of new facilities subject to defined constraints.

Artificial Intelligence (AI) in its simplest form was first researched as long ago as the early 1950s. AI is a way of making a computer behave 'intelligently'. This can be accomplished by studying how people think when they are trying to make decisions and solve problems, breaking those thought processes down into basic steps, and designing a computer program that solves problems using those same steps. AI thereby provides a simple, structured approach to designing complex decision making programs. The goal of an AI system is to analyze human behavior in the fields of perception, comprehension, and decision making in the ultimate hope of reproducing the behavior on a machine, namely a computer.

An iterative improvement method is a search method that starts with an initial solution and tries to improve this solution by 'local modification'.

2 Heuristic methods

Heuristic methods are based on intuitive analysis so they are relatively close to the way that engineers think. They can give a good design scheme based on experience and analysis. However, they are not strict mathematical optimization methods. In operation research, the heuristic approach finds wide application because of its straightforwardness, flexibility, speed of computation, easy involvement of personnel in decision making and ability to obtain a comparatively good solution that meets practical engineering requirements. The characteristics of the heuristic methods are simple method and logic; user interaction and families of feasible, near optimal plans. Whereas the contrasting characteristics of the mathematical programming methods are no user interaction; fixed method by program formulation; detailed logic or restriction set definition and single 'global' solution.

In contrast to mathematical methods, heuristic methods can be considered to be custom-made. Some of them help to simulate the way a system planner employs analytical tools such as load-flow programs and reliability analysis involving simulations of the planning process through automated design logic.

In general, a characteristic of heuristic techniques is that strictly speaking an optimal solution is not sought, instead the goal is a 'good' solution. Whilst this may be seen as an advantage from the practical point of view, it is a distinct disadvantage if there are good alternative techniques that target the optimal solution [1-3].

3 Mathematical Programming

Linear programming has been used successfully in the solution of problems concerned with the assignment of personnel, distribution and transportation, power engineering, banking, education, petroleum, social problems, etc. Three primary reasons for its wide use are:

i. a large variety of problems in diverse fields can be represented or at least

approximated as linear programming models;
ii. efficient techniques for solving linear programming problems are available;
iii. sensitivity analysis can be handled through linear programming models.

Integer programming (IP) deals with the solution of programming problems in which some or all of the variables can assume non-negative integer values only. An integer program is called mixed or pure, depending on whether some or all of the variables are restricted to integer values. If in the absence of the integrality conditions the objective and constraint functions are linear, the resulting model is called an integer linear program.

In this review, two categories of IP methods are reviewed here:
i. search methods;
ii. cutting methods.

The most important search method is the branch and bound technique which applies directly to both the pure and mixed problems. The general idea of the method is first to solve the problem as a continuous model.

Cutting methods, which are developed primarily for integer linear problems, start with the continuous optimum. By systematically adding special 'secondary' constraints, which essentially represent necessary conditions for integrality, the continuous solution space is gradually modified until its continuous optimum extreme point satisfies the integer conditions. The name 'cutting methods' stems from the fact that the added 'secondary' constraints effectively cut (or eliminate) certain parts of the solution space that do not contain feasible integer points.

Cutting planes does not partition the feasible region into sub-divisions, as in branch and bound approaches, but instead works with a single linear program, which is refined by adding new constraints until the new constraints solution is found.

Non-linear programming problems come in many different shapes and forms. Unlike the Simplex Method for linear programming, there exists no single algorithm that will solve all of them. Instead, algorithms have been developed for various individual special types of non-linear programming problems.

Non-linear programming has been applied to many important problems such as the product mix problem with price elasticity, the transportation problem with volume discounts on shipping costs, portfolio selection with risky securities, distribution location, transmission network planning, etc. Two examples of non-linear programming applied to single-state network programming are:
i. the gradient search method;
ii. quadratic programming.

The mathematical programming technique used in the time-phased optimization method is Dynamic Programming (DP). Dynamic programming is a computational technique best suited to the optimization of sequential or multi-stage decision making problems. Dynamic programming converts such multi-stage decision problems into a series of single-stage decision problems, each with one or a few decision variables. Then, starting with the first stage, each stage is optimized over possible alternative feasible decisions within the stage, while taking into consideration the cumulative effect of the optimum decisions made in the previous stages. The ultimate solution of the problem is then generated from among the available stage optima.

Like any other optimization technique, dynamic programming requires that the problem be represented by a mathematical model, formulated in the light of clearly defined decision variables, parameters, and constraints, as well as an established measure of effectiveness. However, unlike other optimization techniques of mathematical programming, there is no unique algorithm for the solution of dynamic programming problems.

The following two examples of dynamic programming that can be applied to network programming are:
i. the backward induction process;
ii. the forward induction process.

4 Artificial Intelligence

In this review, one major category of AI techniques is reviewed - expert systems (ES). Some authors regard ES as being different from AI techniques, but this distinction is not an issue here. Also, as it will be explained later, genetic algorithms (GA's) can be considered to be a form of machine learning which in turn is a category of AI techniques [4].

With the development of expert system theory and techniques, some new expert system approaches to the network programming have been proposed in recent years [5-9]. The main advantage of the expert system approach lies in its ability to simulate the experience of planning experts in a formal way. However, knowledge acquisition is always a very difficult task in applying this method. Moreover, maintenance of the large knowledge base is very difficult. Expert systems are not appropriate for solving combinatorial search problems, rather they are more useful for analysis of models and their solutions.

5 Iterative Improvement Methods

The iterative improvement methods used for network programming are principally tabu search, simulated annealing and genetic algorithms.

Tabu search (TS) was developed by Glover [10-11]. TS has emerged as a new, highly efficient, search paradigm for finding quality solutions to combinatorial problems. It is characterized by gathering knowledge during the search, and subsequently profiting from this knowledge. The attractiveness of the technique comes from its ability to escape local optimality.

TS has now become an established optimization approach that is rapidly spreading to many new fields. For example, successful applications of TS have been reported recently in solving some power system problems, such as hydro-thermal scheduling [12], fault section estimation [13], alarm processing [14], and transmission network planning [15].

The simulated annealing (SA) technique was first introduced by Kirkpatrick [16]. This idea was based on the Metropolis Algorithm [17]. Annealing is the physical process of heating up a solid, followed by cooling it down until it crystallizes into a state with a perfect lattice. During this process, the free energy of the solid is minimized. Practice shows that the cooling must be done carefully in order not to get trapped in locally optimal lattice structures with crystal imperfections.

Combinatorial optimization can be defined by a similar process. This process can be formulated as the problem of finding, among a potentially very large number of solutions, a solution with minimal cost. Now, by establishing a correspondence between the cost function and the free energy, and between the solutions and the physical states, it can introduce a solution method in the field of combinatorial optimization based on a simulation of the physical annealing process.

A genetic algorithm (GA) is a simulation procedure based on a theoretical model originally proposed by Holland [18] and his research team at the University of Michigan during the late 1960s and early 70s. This model is called an algorithm because it is a computational model.

6 Comparison of the Operation Research Methods

Heuristic methods can be considered to be custom-made which is in contrast to mathematical methods. The heuristic methods of network programming are mainly characterized by expanding a network step by step without considering the interaction between decision variables. Therefore, they cannot guarantee an optimal solution, and that is their main disadvantage. A further disadvantage in that the heuristic method is typically application specific. In contrast, their advantages include simplicity and user interaction. In general, a characteristic of heuristic techniques is that strictly speaking an optimal solution is not sought, instead the goal is a ‘good' solution.

Whilst this may be seen as an advantage from the practical point of view, it is a distinct disadvantage if there are good alternative techniques that target the optimal solution.

With mathematical optimization methods (linear programming, integer programming, zero-one, etc.) the weaknesses are most require a large number of decision variables; most require long computation times; most allow no user interaction; models are fixed by program formulation; considerable effort and good mathematical knowledge is usually required for adaptation to specific applications and most integer programming problems are such that the amount of work needed to produce a guaranteed optimal solution increases exponentially with the problem size.

But one of the main advantages of the mathematical programming methods is that an optimal solution is guaranteed.

An expert system differs from a conventional computer program in two essential ways. An expert system, at any time, can explain its behavior to the human expert and receive new pieces of information from the human expert without any new programming being required. There is no doubt that expert systems have a major role to play in transmission planning. They could perform valuable service to human planners by:

i. preserving knowledge;
ii. concentrating knowledge;
iii. formally representing knowledge;
iv. acting as a training tool;
v. generating alternative plans more rapidly;
vi. permitting the user to test plans;
vii. integrating various knowledge sources, simulations and data bases.

But creating a knowledge base for an ES is difficult if an application requires a more complicated conflict resolution mechanism, or when the rules are difficult to obtain. Also, they are laborious and expensive to maintain. Expert systems are not appropriate for solving combinatorial search problems, although they are more useful for the analysis of models and their solutions.

Tabu search and simulated annealing have been applied successfully to a great variety of problems. Their key advantages are their generality and ease of applicability. Another advantage is their ability to escape local optima. The effectiveness of tabu search depends heavily on the way that the tabu list is defined and manipulated. This is a disadvantage of TS. In simulated annealing, the cooling scheme plays an important part. If the temperature is reduced too rapidly, it may not increase the probability of finding better solutions. On the other hand, the slower the temperature is reduced, the longer the time that the algorithm will take to achieve the final solution.

GA's are based in concept on natural genetic and evolutionary mechanisms working on populations of solutions in contrast to other search techniques that work on a single solution. An important aspect of GA's is that although they do not require any prior knowledge or any space limitations such as smoothness, convexity or unimodality of the function to be optimized, they exhibit very good performance in most applications. They only require an evaluation function to assign a quality value (fitness value) to every solution produced. Another interesting feature is that they are inherently parallel (solutions are individuals and unrelated with each other), therefore their implementation on parallel machines would reduce the CPU time required significantly. GA's are suitable for traversing large search spaces since they can do this relatively rapidly and because the mutation operator diverts the method away from local optima, which will tend to become more common as the search space increases in size. Other key advantages of GA's are their generality and ease of applicability. Empirical analysis of the performance of GA's and the effects of GA's parameters on this performance in addition to the aforementioned advantages shows that GA's are a feasible, robust and practical engineering tool and are considered further in this paper for network programming in response to the weaknesses seen in the mathematical methods that are conventionally applied to network programming.

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