A Monotone Iterative Method for Bipolar Junction Transistor Circuit Simulation

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Abstract: - In this paper, we solve bipolar junction transistor electric circuit ordinary differential equation with the decoupling and monotone iterative methods. With the monotone iterative technique, we prove each decoupled and transformed circuit equation converges monotonically. The proposed method here provides an alternative in the numerical solution of semiconductor circuit model in time domain.

Key Words: - Nonlinear Circuit Model, Semiconductor Device, ODE, Monotone Iterative Method

I. Introduction

Numerical methods for the fundamental electric circuit equations of bipolar junction transistors (BJTs) provide an efficient alternative in the development of microelectronics [1]-[7]. The equivalent circuit equations of heterojunction bipolar transistor (HBT) including the Gummel-Poon model consist of a set of coupled nonlinear ordinary differential equations (ODEs) has been of great studied for high frequency semiconductor device and circuit simulation in the past years [4]-[10]. A widely approach to solve these equations efficiently is to decoupled them firstly with the decoupling method (so-called the waveform relaxation (WR) method) [11]. Each decoupled equation is then discretized and solved with the Newton's iteration (NI) method subsequently [12]. The Newton's method is a conventional method for the solution of system of nonlinear algebraic equations and converges quadratically when the initial guess is in the neighborhood of the exact solution. However, to simulate the high frequency semiconductor devices in time domain with an accurate initial guess is a quite difficult task and has involved many engineering works. For example, the famous HSPICE circuit simulator [13] applied the NI method in the solution of nonlinear system. Unfortunately, for some high frequency simulations, the SPICE encounters the convergence problems and particular initial guesses are required.

The monotone iterative (MI) method is a classical constructive technique for the solutions of differential equations and useful for the numerical solutions of physical and engineering models [14]-[23]. The MI method has been successfully developed and applied to semiconductor device simulation by us earlier [17]-[22]. In this work, we apply this method to simulate HBT characteristics with exploiting the basic nonlinear property in the equivalent circuit model. By considering the Kirchhoff's current law for each node, the circuit governing equations are formulated in terms of the nodal voltages $(V_C, V_E, V_B, V_{BX}, V_{CX}, V_{EX})$. The circuit model is decoupled into several independent ODEs with the WR decoupling scheme. The basic idea of the decoupled method for circuit simulation is similar to the well-known Gummel's decoupling method for device simulation; it is that the circuit equations are solved sequentially [11, 24]. In the circuit model, the first equation is solved for $V_C^{(g+1)}$ given the previous states $V_X^{(g)}, X = E, B, BX, CX$, and EX, respectively. and $v^{(g)}$. For the second equation is solved for $V_E^{(g+1)}$ given $V_X^{(g)}, X = C, B, BX, CX$, and EX, respectively. We have the same procedure for other ODEs. Each decoupled ODE is transformed and solved with Runge-Kutta (RK) method and the monotone iterative method. We prove this approach converges monotonically for all decoupled circuit equations. It means that we can solve the circuit ODEs with arbitrary initial guesses. Numerical results for various devices have been reported to demonstrate the robustness of the method in our earlier work [4, 5, 6, 7].

This paper is organized as follows. In Sec. 2, we state the ODE model. Sec. 3 is the monotone iterative method for the decoupled ODEs. For each decoupled equation, we prove the convergence property by using monotone iterative method. A computational procedure is also introduced in this section. Sec. 4 draws the conclusions and suggests future works.

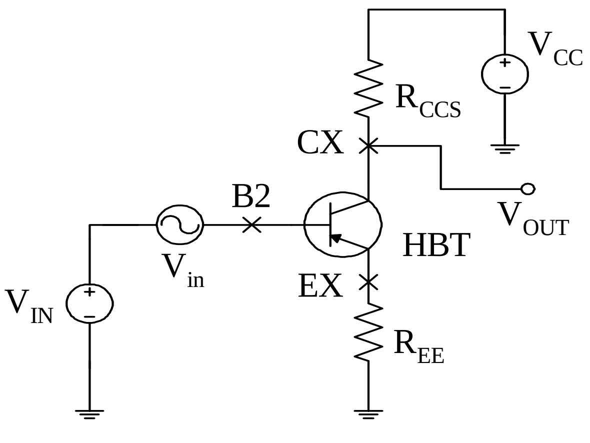

Fig. 1. A typical BJT circuit in the modeling and simulation.

II. An HBT Nonlinear Equivalent Circuit Model

As shown in Fig. 1, based on the node current flow conservation (the well-known Kirchhoff's current law) and utilize the Gummel-Poon large signal equivalent circuit model (Fig. 2) for the HBT device [12, 25, 26], the complete simulation model can be formulated with nodal equations. The system of node equations for time dependent HBT circuit is a set of nonlinear coupled ODEs. At nodes C, E, and B we have the following differential equations (1)-(3), respectively.

$$
\begin{gathered}
C_{JCX}(\tfrac{dV_{BX}}{dt} - \tfrac{dV_C}{dt}) + C_{DR}(\tfrac{dV_B}{dt} - \tfrac{dV_C}{dt}) + \\
C_{JCI}(\tfrac{dV_B}{dt} - \tfrac{dV_C}{dt}) + I_2 + I_{BL2} - \\
\frac{I_{CT}}{\frac{q_1}{2} + \sqrt{(\frac{q_1}{2})^2 + q_2}} + \frac{V_{CX} - V_C}{R_C} = 0,
\end{gathered} \tag{1}
$$

$$
\begin{gathered}
C_{DF}(\tfrac{dV_B}{dt} - \tfrac{dV_E}{dt}) + C_{JE}(\tfrac{dV_B}{dt} - \tfrac{dV_E}{dt}) + \\
I_1 + I_{BL1} + \frac{I_{CT}}{\frac{q_1}{2} + \sqrt{(\frac{q_1}{2})^2 + q_2}} + \frac{V_{EX} - V_E}{R_C} = 0,
\end{gathered} \tag{2}
$$

$$
\begin{gathered}
C_{DR}(\tfrac{dV_B}{dt} - \tfrac{dV_C}{dt}) + C_{JCI}(\tfrac{dV_B}{dt} - \tfrac{dV_C}{dt}) + \\
C_{DF}(\tfrac{dV_B}{dt} - \tfrac{dV_E}{dt}) + C_{JE}(\tfrac{dV_B}{dt} - \tfrac{dV_E}{dt}) + \\
I_1 + I_{BL1} + I_2 + I_{BL2} + \frac{V_B - V_{BX}}{RB} = 0,
\end{gathered} \tag{3}
$$

Similarly, at nodes BX, CX, and EX we formulate, respectively, the equations as follows:

$$
\begin{gathered}
C_{JCX}(\tfrac{dV_C}{dt} - \tfrac{dV_{BX}}{dt}) + \frac{V_B - V_{BX}}{R_B} + \\
\frac{(V_{IN} + V_{in}) - V_{BX}}{R_{B2}} = 0,
\end{gathered} \tag{4}
$$

$$
\frac{V_C - V_{CX}}{R_C} + \frac{V_{CC} - V_{CX}}{R_{CCS}} = 0, \tag{5}
$$

$$
\frac{V_E - V_{EX}}{R_E} - \frac{V_{EX}}{R_{EE}} = 0. \tag{6}
$$

The Eqs. (1)-(4) are the ODEs, and the Eqs. (5) and (6) are the algebraic equations. These equations are subject to proper initial values at time $t = 0$ for all unknowns to be solved. All currents I and capacitances C above are nonlinear functions of unknown variables. These nonlinear terms are

$$
I_1 = \frac{I_S}{B_F}[e^{\frac{V_B - V_E}{N_F V_T}} - 1], \tag{7}
$$

$$I_2 = \frac{I_S}{B_R}[e^{\frac{V_B - V_C}{N_R V_T}} - 1], \quad (8)$$

$$I_{BL1} = I_{SE}[e^{\frac{V_B - V_E}{N_E V_T}} - 1], \quad (9)$$

$$I_{BL2} = I_{SC}[e^{\frac{V_B - V_C}{N_C V_T}} - 1], \quad (10)$$

$$I_{CT} = -I_S[e^{\frac{V_B - V_C}{N_R V_T}} - e^{\frac{V_B - V_E}{N_F V_T}}], \quad (11)$$

$$C_{DR} = \frac{\partial}{\partial(V_B - V_C)}[T_R I_S e^{\frac{V_B - V_C}{N_R V_T}}], \quad (12)$$

$$C_{DF} = \frac{\partial}{\partial(V_B - V_E)}\{\frac{\tau_F I_S[e^{\frac{V_B - V_E}{NF \cdot V_T}} - 1]}{\frac{q_1}{2} + \sqrt{(\frac{q_1}{2})^2 + q_2}}\}, \quad (13)$$

$$C_{JE} = \begin{cases} C_{JEO}(1 - \frac{V_B - V_E}{V_{JE}})^{-M_{JE}}, \\ \quad \text{if } (V_B - V_E) \leq F_C V_{JE}, \\ C_{JEO}(1 - F_C)^{-M_{JE}} \times \\ [1 - F_C(1 + M_{JE}) + \frac{M_{JE}}{V_{JE}}(V_B - V_E)], \\ \quad \text{if } (V_B - V_E) > F_C V_{JE}, \end{cases} \quad (14)$$

$$C_{JCX} = \begin{cases} (1 - X_{CJC})C_{JCO}(1 - \frac{V_{BX} - V_C}{V_{JC}})^{-M_{JC}}, \\ \quad \text{if } (V_{BX} - V_C) \leq F_C V_{JC}, \\ (1 - X_{CJC})C_{JCO}(1 - F_C)^{-M_{JC}} \times \\ [1 - F_C(1 + M_{JC}) + \frac{M_{JC}}{V_{JC}}(V_{BX} - V_C)], \\ \quad \text{if } (V_{BX} - V_C) > F_C V_{JC}, \end{cases} \quad (15)$$

$$C_{JCI} = \begin{cases} X_{CJC}C_{JCO}(1 - \frac{V_B - V_C}{V_{JC}})^{-M_{JC}}, \\ \quad \text{if } (V_B - V_C) \leq F_C V_{JC}, \\ X_{CJC}C_{JCO}(1 - F_C)^{-M_{JC}} \times \\ [1 - F_C(1 + M_{JC}) + \frac{M_{JC}}{V_{JC}}(V_B - V_C)], \\ \quad \text{if } (V_B - V_C) > F_C V_{JC}, \end{cases} \quad (16)$$

where $V_T = \frac{kT}{q}$ is thermal voltage; q_1, q_2, and τ_F are as follows:

$$q_1 = 1 + \frac{V_B - V_E}{V_{AR}} + \frac{V_B - V_C}{V_{AF}}, \quad (17)$$

$$q_2 = \frac{I_S}{I_{KF}} \cdot [e^{\frac{V_B - V_E}{N_F V_T}} - 1] + \frac{I_S}{I_{KR}}[e^{\frac{V_B - V_C}{N_R V_T}} - 1], \quad (18)$$

$$\tau_F = T_F[1 + X_{TF}(\frac{I_{bf}}{I_{bf} + I_{TF}})^2 e^{\frac{V_B - V_C}{1.44 V T_F}}]. \quad (19)$$

There are 6 coupled ODEs with the nonlinear current and capacitance models have to be solved and the unknowns to be calculated in the system of ODEs are V_C, V_E, V_B, V_{BX}, V_{CX}, and V_{EX}, respectively. We note that the system consists of strongly coupled nonlinear ODEs, due to the exponential dependence of current and capacitance models. The model parameters above for the HBT model can be found in [1, 25, 26].

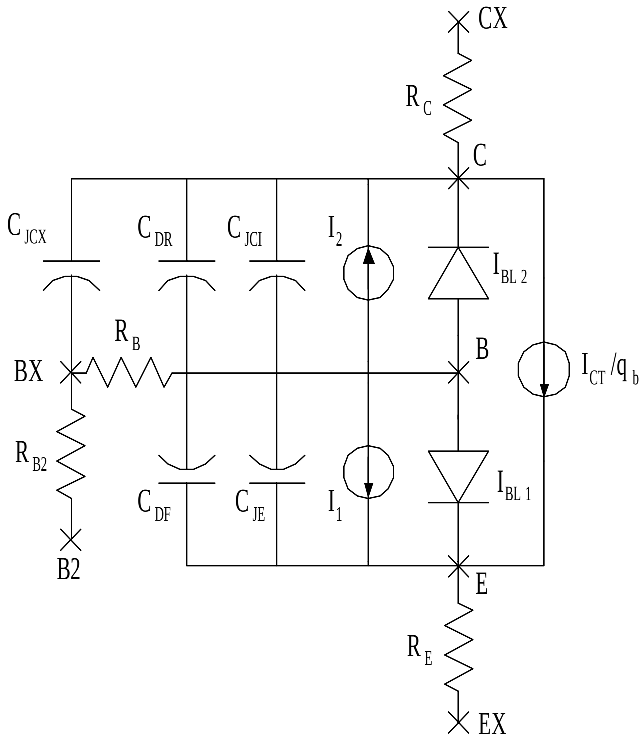

Fig. 2. A schematic diagram of the Gummel-Poon model for BJT circuit simulation.

III. Monotone Iterative Method for Decoupled ODEs

We propose here a decoupled and globally convergent simulation technique to solve the system ODEs in the large-scale time domain directly. Firstly, under the steady state condition, we find the DC solution as the starting point to compute other time dependent solutions. For a specified time period T, to solve these nonlinear ODEs in the time domain, the computational scheme consists of following steps: (i) Let an initial time step t be given. (ii) Use the decoupling method to decouple all Eqs. (1)-(6). (iii) Each decoupled ODE is solved sequentially with the MI and RK methods. (iv) Convergence test for each MI loop. (v) Convergence test for overall outer loop. (vi) If the specified stopping criterion is reached for the outer loop, then go to step (vii), else update the newer results and back to step (iii). (vii) If $t < T$, $t = t + \Delta t$ and repeat the steps (iii)-(vi) until the time step meets the specified time period T.

For a given specified time step t and the previous calculated results, the decoupling algorithm solves the circuit equations sequentially, for instance, the V_C in Eq. (1) is solved for given the previous results $(V_E, V_B, V_{BX}, V_{CX}, V_{EX})$. The V_E in Eq. (2) is solved for newer given V_C and $(V_B, V_{BX}, V_{CX}, V_{EX})$. The V_B in Eq. (3) is solved for newer given (V_C, V_E) and (V_{BX}, V_{CX}, V_{EX}). We have similar procedure for other unknowns. Each decoupled ODE is solved with the MI algorithm. To clarify the MI algorithm for the numerical solution of the decoupled nonlinear ODEs, we write the above decoupled ODEs as the following form

$$\begin{array}{c} \frac{dV_X^{(g)}}{dt} = f(V_X^{(g)}, t) \\ V_X^{(g)}(0) = V_{X_0}^{(g)} \end{array} , \qquad (20)$$

where $V_X^{(g)}$ is the unknowns to be solved, g is the decoupling index $g = 0, 1, 2, \ldots$. We note that the f is the collection of the nonlinear functions and $f \in C[I \times \mathbb{R}, \mathbb{R}]$ and $I = [0, T]$. For a fixed index g and X, because the upper and lower solutions, $\overline{V}_X^{(g)}$ and $\underline{V}_X^{(g)}$, exist in the circuit and $\overline{V}_X^{(g)} \geqslant \underline{V}_X^{(g)}$, we can prove the solution existence in the set $\Omega = \{(t, V_X^{(g)}) \mid \overline{V}_X^{(g)} \geqslant V_X^{(g)} \geqslant \underline{V}_X^{(g)}, \forall t \in I\}$ for each decoupled circuit ODE.

Theorem 1 *Let $\overline{V}_X^{(g)}$ and $\underline{V}_X^{(g)}$ are the upper and lower solutions of Eq. (20) in $C^1[I \times \mathbb{R}, \mathbb{R}]$ such that $\overline{V}_X^{(g)} \geqslant \underline{V}_X^{(g)}$ in the time interval I and $f \in C[I \times \mathbb{R}, \mathbb{R}]$. Then there exists a solution $V_X^{(g)}$ of Eq. (20) such that $\overline{V}_X^{(g)} \geqslant \underline{V}_X^{(g)}$ in the time interval I.*

Proof. It is a direct result with the continuous property of f, here the comparison theorem is applied [27]. ■

Remark 1 *We note that for each decoupled ODE, the nonlinear function f is nonincreasing function of the unknown $V_X^{(g)}$ and the upper and lower solutions $\overline{V}_X^{(g)}(0)$ and $\underline{V}_X^{(g)}(0)$ of Eq. (20) in I can be found. We can further prove there exists a unique solution $V_X^{(g)}$ of Eq. (20) in I and $\overline{V}_X^{(g)}(0) \geqslant V_X^{(g)} \geqslant \underline{V}_X^{(g)}(0)$.*

We see that the Theorem 1 provides an existence result of the problem, and we now describe a monotone constructive method for the computer simulation of the circuit ODEs. The constructed sequences will converge to the solution of Eq. (20) for all decoupled ODEs in the circuit simulation. In this condition, instead of original nonlinear ODE to be solved, a transformed ODE is solved with such as the RK method. Now we state the main result for the solution of each decoupled HBT circuit ODEs.

Theorem 2 *Let the $f \in C[I \times \mathbb{R}, \mathbb{R}]$, $\overline{V}_X^{(g)}(0)$ and $\underline{V}_X^{(g)}(0)$ are the upper and lower solutions of Eq. (20) in I. Because $f(t, V_X^{(g)}) - f(t, \widetilde{V}_X^{(g)}) \geq -\lambda(V_X^{(g)} - \widetilde{V}_X^{(g)})$, $\overline{V}_X^{(g)}(0) \geq V_X^{(g)} \geq \widetilde{V}_X^{(g)} \geq \underline{V}_X^{(g)}(0)$ and $\lambda \geq 0$, sequences $\overline{V}_{X_n}^{(g)} \overset{unif.}{\longrightarrow} \overline{V}_X^{(g)}$ and $\underline{V}_{X_n}^{(g)} \overset{unif.}{\longrightarrow} \underline{V}_X^{(g)}$ as $n \longrightarrow \infty$ monotonically in I.*

Proof. For $\mathcal{V} \in C[I \times \mathbb{R}, \mathbb{R}]$ such that $\overline{V}_X^{(g)} \geqslant \mathcal{V} \geqslant \underline{V}_X^{(g)}$ we consider the following transformed ODE equation for the fixed g and X

$$\begin{array}{c} \frac{dV_X}{dt} = f(t, \mathcal{V}) - \lambda(V_X^{(g)} - \mathcal{V}) \\ V_X^{(g)}(0) = V_{X_0}^{(g)} \end{array} , \qquad (21)$$

then $\forall \mathcal{V}$, $\exists!$ $V_X^{(g)}$ of Eq. (20) in I. Define $\Theta\mathcal{V} = V_X^{(g)}$ and we can prove: (i) $\Theta\overline{V}_X^{(g)}(0) \geq \overline{V}_X^{(g)}(0)$ and $\Theta\underline{V}_X^{(g)}(0) \leq \underline{V}_X^{(g)}(0)$; (ii) Θ is a monotone operator in $[\overline{V}_X^{(g)}(0), \underline{V}_X^{(g)}(0)] \equiv [V_X^{(g)} \in C[I, \mathbb{R}] \mid \overline{V}_X^{(g)}(0) \geq V_X^{(g)} \geq \underline{V}_X^{(g)}(0)]$.

Now we construct two sequences by using the mapping Θ: $\Theta\overline{V}_{X_n}^{(g)} = \overline{V}_{X_{n+1}}^{(g)}$ and $\Theta\underline{V}_{X_n}^{(g)} = \underline{V}_{X_{n+1}}^{(g)}$ and by above observations, the following relation holds

$$\overline{V}_{X_0}^{(g)} \geq \cdots \geq \overline{V}_{X_n}^{(g)} \geq \underline{V}_{X_n}^{(g)} \geq \cdots \geq \underline{V}_{X_0}^{(g)}$$

in I. Hence $\overline{V}_{X_n}^{(g)} \stackrel{unif.}{\longrightarrow} \overline{V}_{X}^{(g)}$ and $\underline{V}_{X_n}^{(g)} \stackrel{unif.}{\longrightarrow} \underline{V}_{X}^{(g)}$ as $n \longrightarrow \infty$ monotonically in I. Furthermore the $\overline{V}_{X_n}^{(g)}$ and $\underline{V}_{X_n}^{(g)}$ satisfy

$$\begin{cases} \frac{d\underline{V}_{X_{n+1}}^{(g)}}{dt} = f(t, \underline{V}_{X_n}^{(g)}) - \lambda(\underline{V}_{X_{n+1}}^{(g)} - \underline{V}_{X_n}^{(g)}) \\ \underline{V}_{X_n}^{(g)}(0) = V_{X_0}^{(g)} \end{cases}$$

and

$$\begin{cases} \frac{d\overline{V}_{X_{n+1}}^{(g)}}{dt} = f(t, \overline{V}_{X_n}^{(g)}) - \lambda(\overline{V}_{X_{n+1}}^{(g)} - \overline{V}_{X_n}^{(g)}) \\ \overline{V}_{X_n}^{(g)}(0) = V_{X_0}^{(g)} \end{cases},$$

respectively. Thus $\overline{V}_{X}^{(g)}$ and $\underline{V}_{X}^{(g)}$ are the solutions of Eq. (20). ■

Theorem 3 *For decoupled ODEs. the nonlinear function f is nonincreasing in $V_X^{(g)}$ and $f(t, V_{X_1}^{(g)}) - f(t, V_{X_2}^{(g)}) \geq -\lambda(V_{X_1}^{(g)} - V_{X_2}^{(g)})$, $\forall V_{X_1}^{(g)} \geq V_{X_2}^{(g)}$. Thus $\left\{\overline{V}_{X_n}^{(g)}\right\}_{n=1}^{\infty}$ and $\left\{\underline{V}_{X_n}^{(g)}\right\}_{n=1}^{\infty}$ converge uniformly and monotonically to the unique solution $V_X^{(g)}$ of Eq. (20).*

Proof. By using Theorem 2 and note the nonincreasing property of f, the result is followed directly. ■

IV. Conclusion

A novel circuit simulation method based on the waveform relaxation and monotone iterative methods has been proposed. With monotone iterative technique, we have proved each decoupled circuit ODE converges monotonically. The proposed method here is an alternative in the numerical solution of electric circuit equations. Computational results have been reported in our earlier work [4, 5, 6, 7, 17, 18, 19, 20, 21, 22] to demonstrate the robustness of the method. This method not only provides a computational technique for the time domain solution of circuit ODEs but also can be generalized for circuit simulation including more transistors. Furthermore, this method can be parallelized and systematically extended to simulate such as radio frequency circuit and system-on-a-chip large-scale VLSI circuit.

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