APPLICATION OF A TWO-DIMENSIONAL RUNGE-KUTTA TIME STEPPING METHOD FOR NUCLEATING FLOWS OF STEAM IN TURBINE BLADING

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Abstract.- This paper describes the results obtained from a time marching solver based on Jameson’s Scheme to study the nucleating two dimensional flows of steam through blade cascades. The main flow is considered non-viscous and the nucleating and wetness are introduced in the Euler solver following the quasi-stream lines formed by the solution. H grids were employed modified to fit normal to the suction side of the blade cascade.

Key-Words: blade cascades, steam, wetness, nucleating flows, time marching.

1. Introduction

During the course of expansion of steam in turbines, the state path crosses the saturation line and the fluid first supercools and the nucleates to become a two phase mixture. The formation of the liquid phase create problems which lead to losses of the performance in the wet stages of turbines. A number of investigations into the behaviour of flowing wet steam have been reported, e.g., Filippov, et.al. [1], Walters [2], F. Bakhtar [3,4], Kiryukhin, et.al. [5], Kirillov [6] and Troyanovski [7] amongst many others.

With the growing increase of speed and memory of computers, numerical techniques to predict and study the behaviour of flows have become an important tool. During the past two decades a number of theoretical investigations in nucleating flows can be found, e.g., Bakhtar [8,9,10], Skillings [11], Young [12].Guha, et.al [13]. Most of the numerical techniques employed in the study of nucleating flows have benefited from the numerical techniques developed for the flow over airfoils. Of these, one that has provided very good results in calculating flow over wings and airplane fuselages [14,15] is the four stage Runge-Kutta method developed by Jameson, et.al. [16]. This technique has been used for calculations of flow in turbomachinery amongst others by Holmes [17] and Subramanian [18], and later including viscous terms by Arnone [19] and Bassi, et. al. [20] amongst others. The comparisons presented by these workers show very good agreement with experimental observations. It was therefore thought that Jameson's technique had the potential for obtaining good results in nucleating flows.

2. The Four Step Runge-Kutta Method

Jameson, Schmidt and Turkel [16] presented a four stage Runge-Kutta time stepping scheme to increase the efficiency of the finite volume time marching techniques. In this scheme the space and time discretizations are formulated separately, in such a way that the steady state solution is independent of the time step, so that large time steps can be used without altering the solution. To capture stationary shocks, a blend of third and first order dissipation terms are added to the flux terms.

2.1 Governing Equations

The Euler equations for the two-dimensional flow of a compressible fluid may be expressed as:

$$\frac{\partial}{\partial t}\iint_{\Omega_P} W dx dy + \oint_{\Gamma_P} (F dy - G dx) = 0 \qquad (1)$$

where W is any conserved property and F and G are the fluxes. These may be written as:

$$W = \begin{Bmatrix} r \\ ru \\ rv \\ rE \end{Bmatrix} \quad , \quad F = \begin{Bmatrix} ru \\ ru^2 + P \\ ruv \\ ruH \end{Bmatrix} ,$$

$$G = \begin{Bmatrix} rv \\ ruv \\ rv^2 + P \\ rvH \end{Bmatrix} \qquad (2)$$

in which ρ is the density, u is the axial velocity, v the pitch-wise velocity, E is the total internal energy, H the total enthalpy and P is the pressure, whose relation between them will depend on the type of fluid employed.

2.2 Space Discretization

As described by Jameson, Schmidt and Turkel [16], the Euler equations are discretized by first dividing the computational domain into quadrilateral cells, and for a centred scheme a discrete approximation to the spatial terms of equation (1) yields:-

$$\frac{d(hW)}{dt} + Q(W) = 0 \qquad (3)$$

where h is the cell area and

$$Q(W) = x_{AB} + x_{BC} + x_{CD} + x_{DA} \qquad (4)$$

represents the net flux out of the cell. The quantities ξ are the fluxes through the sides of the cell.

2.3 Dissipation Terms

Central differencing schemes suffer from the tendency of decoupling between odd and even points. This tendency will introduce instabilities into the solution and can prevent convergence. To suppress the decoupling it is necessary to augment the finite volume scheme by the addition of artificial dissipative terms. These terms are third order and are formed by fourth differences of the dependent variables [16]. Thus equation (3.) takes the form:

$$\frac{d(hW)}{dt} + Q(W) - D(W) = 0 \qquad (5)$$

where

$$D(W) = Dx(W) + Dy(W) \qquad (6)$$

and

$$\begin{aligned} Dx(W) &= d_{i,j+1/2} - d_{i,j-1/2} \\ Dy(W) &= d_{i+1/2,j} - d_{i-1/2,j} \end{aligned} \qquad (7)$$

and for one of the terms of the right hand side of equation (7)

$$d_{i,j+1/2} = \frac{h_{i,j+1/2}}{\Delta t}[e^{(2)}_{i,j+1/2}(W_{i,j+1} - W_{i,j}) + e^{(4)}_{i,j+1/2}(W_{i,j+2} - 3W_{i,j+1} + W_{i,j} - W_{i,j-1})] \qquad (8)$$

where ε is a switch between second and four differences to be applied in regions of high and low pressure gradients respectively.

2.4 Time Stepping

Considering the grid to be held fixed in time so that the area h remains constant, as describe by Jamenson, equation (5) can be integrated following the scheme:

$$W^{(0)} = W^{(t)}$$
$$W^{(1)} = W^{(0)} - \frac{\Delta t}{4h}\left[Q(W)^{(0)} - D(W)^{(0)}\right]$$
$$W^{(2)} = W^{(0)} - \frac{\Delta t}{4h}\left[Q(W)^{(1)} - D(W)^{(0)}\right]$$
$$W^{(3)} = W^{(0)} - \frac{\Delta t}{4h}\left[Q(W)^{(2)} - D(W)^{(0)}\right]$$
$$W^{(4)} = W^{(0)} - \frac{\Delta t}{4h}\left[Q(W)^{(3)} - D(W)^{(0)}\right]$$
$$W^{(t+\Delta t)} = W^{(4)} \quad (9)$$

which is fourth order accurate in time.

2.5 Boundary Conditions

In a typical turbine blade cascade, four types of boundaries confining the flow region can be identified. These are the inlet flow boundary where the plane stagnation pressure, temperature and flow directions are specified; the outflow boundary in which the pressure is specified and the rest of the properties are found by extrapolation; the solid boundaries where there is no flux crossing the walls of the blades and only normal contributions of the pressure are accounted for; and finally the periodic boundaries which are considered as moving walls at which the net pressure is zero.

2.6 Treatment of Two-Phase Flow

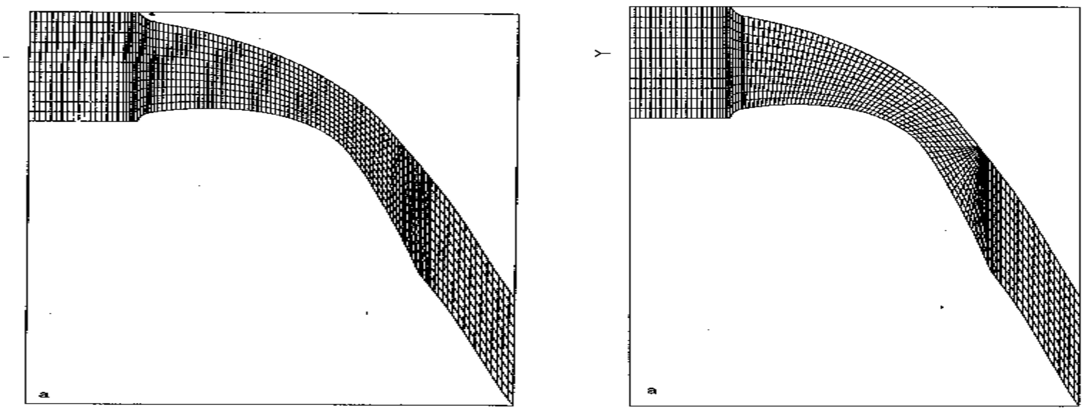
To treat the main flow field the governing equations are the conservation equations as above described, written for a two-phase mixture, which are then combined with equations describing droplet formation and interphase heat and mass transfer. Here because of the small size of droplets slip between phases is neglected. The general treatment is that of Tochai [21] and modified to be applied to Jameson's Scheme.

3. Results

As described by J.M. Rodriguez [22] the computer program behave perfectly well for perfect gas and superheated steam solutions using a coarse 12 X 115 H grid and will not be repeated here. For the present paper only the subsonic solutions are presented using the same 122 X 155 mesh arrangements. Here the general conditions are:

Ref.	Inlet Conditions			Pout/Po	W %	Limiting Degree Of Sup.	Droplet Radius 10^{-8} m		Wetness Loss %
	Po(bar)	To(^{O}K)	To-Ts(Po) (^{O}K)				Theory	Exp.	
Subsonic Wet	1.72	381.1	-8.7	0.57	0.036	38.6	2.31	3.8	4.6

Table 1. General Conditions for the Blade Cascade



(a) (b)

Figure 1. (a) Standar H Grid and (b) Special H grid.

The mesh arrangements employed in this work are shown in Figure 1. Figure 1a shows a standard H grid and Figure 1b special H grid designed to capture the position were the flow reverts to equilibrium.

Comparisons between theoretical and experimental mid-passage and surface pressure distributions are given in Figure 2 and 3. In Figure 2 a pressure rise at about 70 % of the axial cord is predicted earlier by the solution, than the rise observed experimentally which coincides with the start of the rapid condensation zone. It was reasoned that the discrepancy between theory and experiment could have been resulted from: (a) shortcomings of the nucleation theory and droplet grow laws; and (b) inaccuracy produced by the use of the shear H grid. Then the mesh arrangement of Figure 1b was employed and the solutions are shown in Figure 3. Here it will be seen that the first pressure rise is experienced at 71% of the axial cord. As in the previous solution this corresponds with the zone of rapid condensation. After this the flow accelerates again and a second pressure rise can be identified at 75 % of the axial cord.

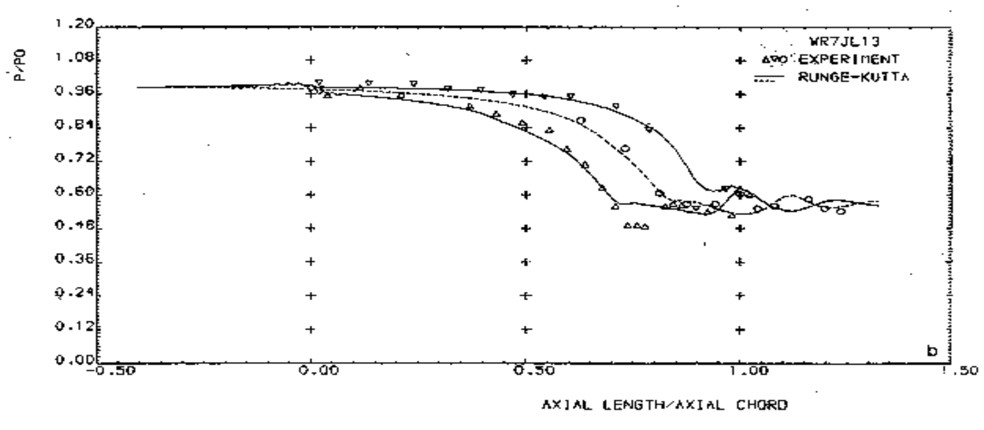


Fig. 2 Pressure Distribution shear H grid 12 X 115 mesh size.

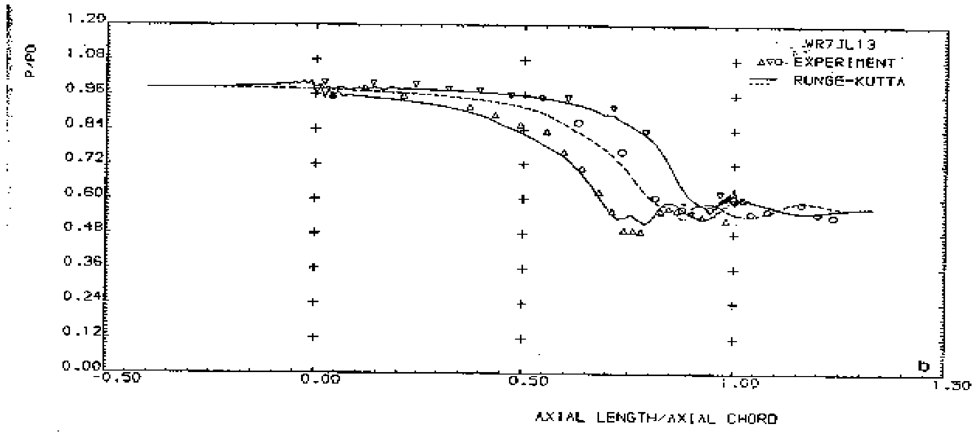


Fig. 3 Pressure distribution, special H grid 12 X 115 mesh size.

Although the solution was improved with the new grid, still this is smeared after the first pressure rise. At this stage, it was thought that the inaccuracy was caused by the insufficient resolution pitch-wise, thus the mesh size was set to 14 X 155. The solution carried out with this new mesh arrangements is shown in Figure 4. Here, although the experimental and theoretical results agree well the total pressure error increased at the trailing edge to 12 % compared to the 7.5 % obtained in the previous solution.

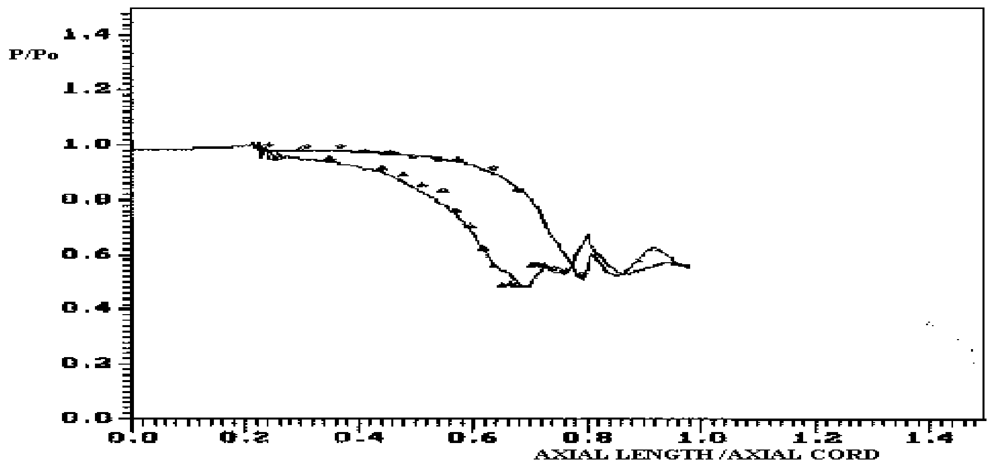


Fig. 4 Pressure distribution, special H grid 14 X 115 mesh size.

Better solutions may be obtained by refinement of the mesh in the zone of rapid condensation, however for the purpose of the present work the results may be considered as good.

4. Conclusions

A computer program based on Jameson's Scheme, for the analysis of nucleating flows of steam was developed and tested against experimental results.

The numerical errors resulting from the internal inconsistencies associated with the algorithm employed can be overcome by the use of a suitable grid arrangement.

The agreement between theoretical and experimental results reasonably good, and may also be improved by a better treatment of the outlet periodic boundaries.

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