Dynamically Stable Optical Solitons

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Abstract

The existence of a dynamically stable soliton in optical fibers is established by virtue of the multiple scale perturbation technique applied in a new way to the perturbed Nonlinear Schrödinger’s equation. We show that, by introducing a proper definition of the phase of the soliton, one can obtain the corrections to the pulse where the standard soliton perturbation approach fails.

The propagation of solitons through an optical fiber, governed by the perturbed Nonlinear Schrödinger’s equation (NLSE), in the dimensionless form is

\[ i\dot{q} + \frac{1}{2} qtt + |q|^2 q = i\epsilon R \quad (1) \]

where

\[ R = -\delta |q|^{2n} q + \sigma \int_{-\infty}^{t} |q|^2 q \, d\tau \quad (2) \]

Here, in (1) \( t \) is the time on the coordinate moving with group velocity, \( q \) is the normalized electric field and \( z \) the distance along the fiber normalized by the dispersion distance. The perturbation parameter \( \epsilon \), where \( 0 < \epsilon \ll 1 \), arises due to quasi-monochromaticity [6]. For the perturbation terms in (1), \( \delta \) represents the coefficient of nonlinear damping while \( \sigma \) represents the coefficient of saturable amplifiers. In (1), if \( \epsilon = 0 \), the NLSE is recovered.
that is integrable by the method of Inverse Scattering Transform [1, 6].
The 1-soliton solution of (1) has the form

\[ q(z, t) = \frac{\eta}{\cosh[\eta(t - vz - t_0)]} e^{(-i\sigma + i\omega z + i\sigma_0)} \]

(3)

with

\[ \kappa = -v \]

(4)

and

\[ \omega = \frac{\eta^2 - \kappa^2}{2} \]

(5)

Here, \( \eta \) is the amplitude (or the inverse width) of the soliton, \( v \) is its velocity, \( \kappa \) is the soliton frequency while \( t_0 \) and \( \sigma_0 \) are the center of the soliton and the center of the soliton phase respectively.

The main issue of this work is to implement a perturbation scheme by modifying the solution (1) to

\[ q = \dot{q}(\theta, X, T; \epsilon) e^{i\omega[\epsilon(x, t; \epsilon)]} \]

(6)

where

\[ \frac{\partial \theta}{\partial t} = 1, \frac{\partial \theta}{\partial z} = 0 \]

(7)

and

\[ X = \epsilon z, \ T = \epsilon t \]

(8)

Here, \( \theta \) is a fast variable while \( X \) and \( T \) are the slow variables in space and time respectively. When the perturbation terms of the NLSE are turned on, the soliton parameters \( \eta \) and \( \kappa \), are slowly varying functions of space and time, namely \( \eta = \eta(X, T) \) and \( \kappa = \kappa(X, T) \).

Substituting (6) in (1) and expanding

\[ \dot{q} = \dot{q}^{(0)} + \epsilon \dot{q}^{(1)} + \epsilon^2 \dot{q}^{(2)} + \cdots \]

\[ \rho = \rho^{(0)} + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \cdots \]

\[ v = v^{(0)} + \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \cdots \]

to get at the leading order

\[ \dot{q}^{(0)} = \frac{\eta}{\cosh \eta(\theta - \bar{\theta})} \]

(9)

where

\[ \frac{d\theta}{dz} = v \]

(10)

Here, \( \bar{\theta} \) represents the center position of the soliton. At \( O(\epsilon) \), decomposing \( \dot{q}^{(1)} = \dot{\phi}^{(1)} + i\dot{\psi}^{(1)} \) into its real and imaginary parts yields the following differential equations

\[ -\frac{\eta^2}{2} \dot{\phi}^{(1)} + \frac{1}{2} \frac{\partial^2 \dot{\phi}^{(1)}}{\partial \theta^2} + 3 \left( \dot{q}^{(0)} \right)^2 \dot{\phi}^{(1)} = \left\{ \rho_X^{(1)} + v^{(0)} \rho_T^{(1)} \right\} \dot{q}^{(0)} - \frac{\partial^2 \dot{q}^{(0)}}{\partial \theta \partial T} \]

(11)

and

\[ -\frac{\eta^2}{2} \dot{\psi}^{(1)} + \frac{1}{2} \frac{\partial^2 \dot{\psi}^{(1)}}{\partial \theta^2} + \left( \dot{q}^{(0)} \right)^2 \dot{\psi}^{(1)} = - \frac{\partial \dot{q}^{(0)}}{\partial X} - v^{(0)} \frac{\partial \dot{q}^{(0)}}{\partial T} - \left\{ \rho_T^{(1)} + \sigma - v^{(1)} \right\} \frac{\partial \dot{q}^{(0)}}{\partial \theta} \]

\[ + (\sigma \eta - \rho_{TT}^{(0)}) \dot{q}^{(0)} - \delta \left( \dot{q}^{(0)} \right)^2 \]

(12)
In order to avoid the secular terms or resonances of the perturbation expansion the Fredholm’s Alternative (FA) is applied to (11) gives

\[
\frac{\partial \eta}{\partial T} = 0
\]

(13)

\[
\dot{\psi}^{(1)} = \frac{\partial \phi}{\partial X \cosh \phi}
\]

and

\[
\rho_X^{(1)} + v^{(0)} \rho_T^{(1)} = 0
\]

(14)

Applying FA to (12) and setting \( \rho_T^{(0)} \) to zero, to eliminate frequency chirp gives

\[
\frac{\partial \eta}{\partial X} = -\delta \sqrt{\pi \eta^{2m+1}} \frac{\Gamma(m+1)}{\Gamma(m+\frac{3}{2})} + 2\sigma \eta^2
\]

where \( \phi = \eta(\theta - \bar{\theta}) \). Finally, the \( O(\varepsilon) \) solution of (1) is

\[
q \approx A e^{i\psi}
\]

(15)

and

\[
\rho_T^{(1)} = v^{(1)} - \sigma
\]

(16)

\[
A = \dot{q}^{(0)}
\]

(22)

Thus, implementing these conditions, and the second order equations for \( \dot{\phi}^{(1)} \) and \( \dot{\psi}^{(1)} \) respectively reduce to

\[
-\eta^2 \ddot{\phi}^{(1)} + \frac{1}{2} \frac{\partial^2 \phi^{(1)}}{\partial \theta^2} + 3 \left( \dot{q}^{(0)} \right)^2 \dot{\phi}^{(1)} = 0
\]

(17)

\[
B(\theta) = \dot{\psi}^{(1)} / \dot{q}^{(0)}
\]

(24)

Thus, (25) is a Dynamically Stable Soliton (DSS) that can be used as a very efficient bit carrier in an optical communication system. The solitons that start with a range of amplitudes and frequencies emerge with a fixed

\[
\begin{align*}
\dot{\phi}^{(1)} &= 0 \\
\frac{\partial \phi}{\partial X \cosh \phi} &= 0
\end{align*}
\]

(19)

and

\[
-1 \frac{\partial \eta}{\partial X} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{1}{2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{2 \sigma \eta^2}{\eta^{2m+1}} \frac{\Gamma(m+1)}{\Gamma(m+\frac{3}{2})} - \delta \frac{\partial q^{(0)}}{\partial x} = 0
\]

(18)
amplitude and frequency at the terminal end of the optical fiber that is given by $(\eta, \kappa) = (\tilde{\eta}, 0)$ where

$$\tilde{\eta} = \left\lfloor \frac{2\sigma \Gamma \left( m + \frac{3}{2} \right)}{\delta \sqrt{\pi \Gamma} (m + 1)} \right\rfloor^{\frac{1}{2n-1}}$$  \hspace{1cm} (25)

References


