Trigonometric Saturated Controller for Robot Manipulators*

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Abstract: A new position controller with gravity compensation for robot manipulators in joint space is proposed in this paper. The goal of position control problem for global asymptotically stability is achieved by using Lyapunov 's direct method and LaSalle 's invariance principle over the full non–linear mutivariable robot closed–loop system. In adition, this paper also presents experimental results on a three – degrees–of– freedom direct drive robot.
Keywords: Robot manipulators, Satured controller, Lyapunov's method, Asymptotic stability.

1 Introduction

The nonlinear and multivariable phenomena present on robot manipulators are well known and not are fully characterized problems. Those nonlinearities present on the robot dynamics can lower controller performance and lead to malfunction. The saturation phenomenon is a common problem present when the amplitude of the control law overruns the actuator linear range; as a result, the torque supplied by the actuator will differ from the control law.

Besides these problems, current industrial manipulators are equipped with regulators such as the proportional derivative (PD) or proportional integral derivative (PID); these controllers are effective at controlling position [1]. However, they are not robust against the saturation problem – because they assume that the actuator is always able to supply the requested torque. Furthermore, the static friction present in the physical system may hamper the controller performance.

The practical interest of developing a new control algorithm with global asymptotic stability of the closed-loop system is our main motivation. In addition, the new saturated controller is capable of improving the performance of the TANH controller[2] and bypass the actuator saturation problem. This new controller is based on the energy shaping methodology[3] and consists of three parts: a trigonometric saturated type function for the proportional part of the controller, a saturated derivative term and gravitational compensation.

This paper is organized as follows. section 2 a brief exposition of the robot dynamics and its

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useful properties. In section 3, the new controller is presented along with its stability analysis. Section 4, summarizes the main components of the experimental set–up. The experimental results on a direct drive arm are in section 5. Finally, a conclusion is offer in section 6.

2 Robot Dynamics

The dynamics of a serial n-link rigid can be written as[4]:

$$M(q)\ddot{q} + C\left(q,\dot{q}\right)\dot{q} + g(q) = \tau \qquad (1)$$

where q is the n x 1 vector of joint displacements, $\dot{q}$ is the n x 1 vector of joint velocities, τ is the n x 1 vector of input torques, $M(q)$ is the n x n symmetric positive definite manipulator inertia matrix, $C\left(q,\dot{q}\right)$ is the n x n matrix of centripetal and Coriolis torques, and $g(q)$ is the n x 1 vector of gravitational torques obtained as the gradient of the robot potential energy $\mathcal{U}(q)$ due to gravity:

$$g(q) = \frac{\partial \mathcal{U}(q)}{\partial q} \qquad (2)$$

We assume that the robot links are joined together with revolute joints. Although the equation of motion (1) is complex, it has several fundamental properties which can be exploited to facilitate the design of control systems.

Property 1. The matrix $C\left(q,\dot{q}\right)$ and the time derivate $\dot{M}(q)$ of the inertia matrix satisfy:

$$\dot{q}\left[\frac{1}{2}\dot{M}(q) - C\left(q,\dot{q}\right)\right]\dot{q} = 0, \forall q, \dot{q} \in \mathcal{R}^n. \qquad (3)$$

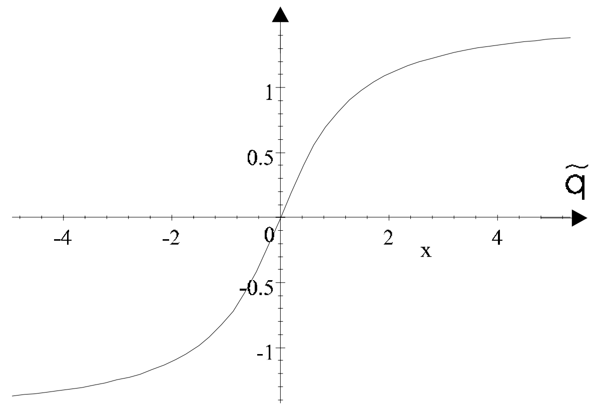

Figure 1: Arctan function.

3 Trigonometric Satured Controller

Based on the energy shaping metodology, the stability analysis of the new controller is shown here. This new controller has a saturated structure with gravity compensation and it is given by: 1) Saturation-proportional term based on the arctangent function, 2) Saturation-derivative term which is also based on the arctangent function and 3) The gravity compensation term. The equation for the new controller is:

$$\tau = K_p \arctan\left(\widetilde{q}\right) - K_v \arctan\left(\dot{q}\right) + g(q) \qquad (4)$$

where $\widetilde{q}_i = q_{di} - q_i$, $i = 1...n$ denote position errors, q_{di} are desired constant joint positions, $K_p \in \mathcal{R}^{nxn}$ is a diagonal positive definite proportional gain matrix, $K_v \in \mathcal{R}^{nxn}$ is a positive definite matrix derivative gain matrix.

The position control problem can be solved by: Selecting the design matrices K_p, K_v in such a way that the position error $\widetilde{q} = q_d - q$ diminishes asymptotically towards a zero value, i.e. $\lim_{t\to\infty} \widetilde{q}(t) = 0$ and keeping the applied torques inside the prescribed limits of the robot actuators. To solve the problem, the following proposition is formulated:

Proposition. Considering the robot dynamic model (1) together with the control law (3), then the closed-loop system is globally asymptotically stable and the desired position –where $\lim_{t\to\infty} q(t) = q_d$ – is achieved.

Proof. Consider the following closed–loop ecuation.

$$\frac{d}{dt}\begin{bmatrix}\widetilde{q}\\ \dot{q}\end{bmatrix} = \begin{bmatrix} -\dot{q} \\ M^{-1}(q)\left[K_p \arctan(\widetilde{q})\right. \\ \left. -K_v \arctan\left(\dot{q}\right) - C\left(q,\dot{q}\right)\right] \end{bmatrix} \quad (5)$$

Where equation (5) is an autonomous differential equation and the state space origin is its unique equilibrium point.

The stability analysis is carried out by Lyapunov's direct method and for that purpose, the following Lyapunov function candidate is proposed:

$$V\left(\widetilde{q},\dot{q}\right) = \frac{1}{2}\dot{q}^T M(q)\,\dot{q} + \sum_{i=1}^{n} k_{pi}[\widetilde{q}_i \arctan(\widetilde{q}_i) - \frac{1}{2}\ln\left(1+\widetilde{q}_i^2\right)] \quad (6)$$

Lyapunov's direct method requires that any function candidate should be positive definite, in this case the first term is positive definite respect to $\dot{q}$ since it is a cuadratic form, with a positive definite matrix $M(q)$ associated. The term associated to $\widetilde{q}$ is positive definite since all k_{pi} are positive values; inspecting Fig(2).

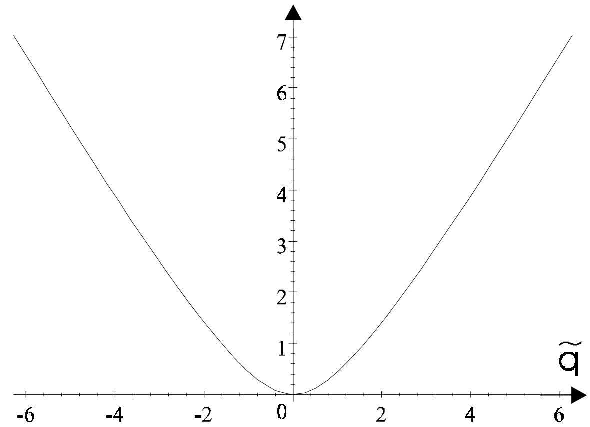

Figure 2: $\widetilde{q}\arctan(\widetilde{q}) - \frac{1}{2}\ln\left(1+\widetilde{q}^2\right)$

The Lyapunov function candidate is then time–derivated (6) along the trayectories of the closed-loop equation (5). After some algebra using property 1, the time derivate can the written as follows:

$$\dot{V}\left(\widetilde{q},\dot{q}\right) = -\dot{q}^T K_v \arctan\left(\dot{q}\right) \leq 0 \quad (7)$$

The time derivate of the Lyapunov function candidate is a globally negative semidefined function and therefore only stability of the point of equilibrium is concluded. To ensure asymptotic stability, LaSalle's theorem is applied (7).

$$\Omega = \left\{\begin{pmatrix}\widetilde{q}\\ \dot{q}\end{pmatrix} \in \mathcal{R}^{2n} : \dot{V}\left(\widetilde{q},\dot{q}\right) = 0\right\} = \left\{\widetilde{q} \in \mathcal{R}^n, \dot{q} = 0 \in \mathcal{R}^n\right\} \quad (8)$$

Since the unique invariant $\widetilde{q} = 0, \dot{q} = 0$, all solutions of (5) will globally asymptotically converge to Ω as $t \to \infty$.

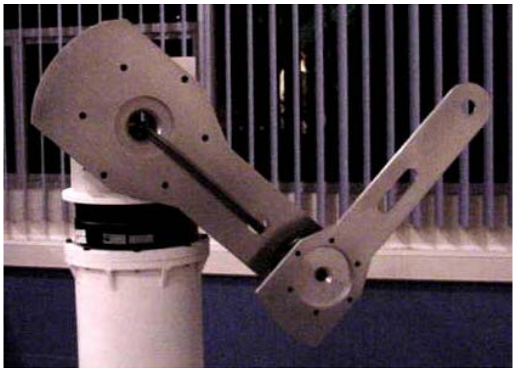

Figure 3: Experimental Robot.

4 Experimental Set-Up

The experiments are carried out on a purpose–built and designed "experimental system for research robot control algorithms" at The Benemérita Universidad Autónoma de Puebla. The robot is a direct drive manipulator with three degrees of freedom moving in three-dimensional space, as shown in Fig(3). The experimental robot consist of links made of 6061 aluminum, actuated by brushless direct drive servo actuators from Parker Compumotor. Some advantages of this type of robot are: capability of driving the joints without gear reduction, freedom of backlash and significantly lower joint friction compared to actuators with gear drives. The motors used in the robot are listed in Table I.

Table I

Link	Model	Torque[Nm]	p/rev
Base	DM1050A	50	1024000
Shoulder	DM1150A	150	1024000
Elbow	DM1115B	15	655360

Position information is obtained from incremental encoders located in the motors. The standard backwards difference algorithm applied to the joint position measurements generates the velocity signals. In addition to position sensors and motor drivers, it also includes a motion control board manufactured by Precision MicroDynamics Inc., used to obtain the joint position from the encoders. The control algorithm runs on a Pentium II (333Mhz) host computer. Knowledge of the robot's gravitational vector is the only necessary prerequisite for system implementation.

5 Experimental Results

The experimental evaluation of the controller must support the theoretical developments. Therefore, an extensive set of experiments were carried out between the ARCTAN controller and the TANH controller. During the experimental test, no friction compensation was modeled on the controller. The two implemented controllers are shown below; to distinguish them, an indetification subindex is added. The new controller equation is given by :

$$\begin{aligned}\tau_{arc} &= K_{parc}\arctan\left(\widetilde{q}_{arc}\right) - K_{varc}\arctan\left(\dot{q}_{arc}\right)\\ &\quad +g\left(q\right)\end{aligned}$$

The TANH controller is given by[2]:

$$\begin{aligned}\tau_{hip} &= K_{phip}\tanh\left(\widetilde{q}_{hip}\right) - K_{vhip}\tanh\left(\dot{q}_{hip}\right)\\ &\quad +g\left(q\right)\end{aligned}$$

The experimental robot gravitional torque vector is defined as follows:

$$g\left(q\right) = \begin{pmatrix} 0 \\ 38.45\sin\left(q_2\right) + 1.82\sin\left(q_2+q_3\right) \\ 1.82\sin\left(q_2+q_3\right) \end{pmatrix}$$

In both cases, the proportional parts are saturated functions; for this type of functions, the gains can be selected acording to $|\tau| \leq \tau_{\max}$ as shown in Table I. In the other hand, both controllers share same desired joint position for the base, shoulder and elbow:

$$q_d = \begin{pmatrix} 90 \\ 45 \\ 45 \end{pmatrix}$$

all the values are in degrees with initial conditions zero for the robot position and velocity. The following proportional and derivative tunning gains were selected for each controller:

$$K_{parc} = \begin{pmatrix} 22.293 & 0 & 0 \\ 0 & 82.8025 & 0 \\ 0 & 0 & 6.3694 \end{pmatrix}$$

$$K_{phip} = \begin{pmatrix} 35 & 0 & 0 \\ 0 & 130 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

$$K_{varc} = \begin{pmatrix} 22.293 & 0 & 0 \\ 0 & 63.6943 & 0 \\ 0 & 0 & 6.3694 \end{pmatrix}$$

$$K_{vhip} = \begin{pmatrix} 35 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

The units for the proportional gains are $Nm/degree$ and the units for the derivative gains are $Nm\text{–}degree/s$. The experimental results for the position error and applied torques for the ARCTAN controller are shown in Fig(4) and for TANH controller Fig(5). Notice that the applied torques are within the actuartor limits. The transient state in both cases is breaf but, in the ARCTAN controler the transient state is fast and smooth. The goal of position control is achived by reaching a small neighborhood of zero position error.

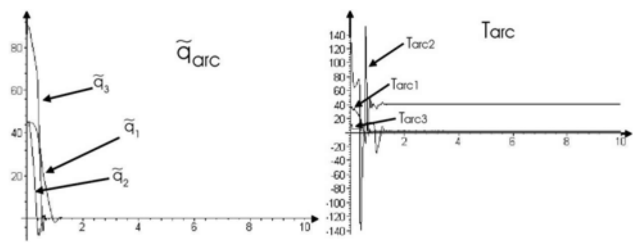

Figure 4: ARCTAN controller.

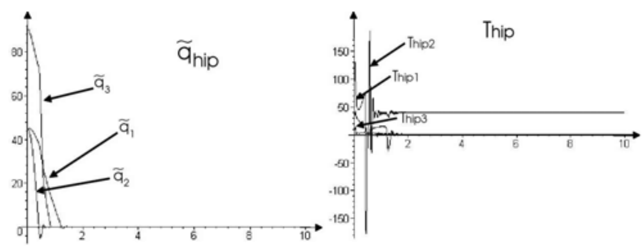

Figure 5: TANH controller.

5.1 Performance Indicators

The performance evaluation is solved implementing the scalar value $\mathcal{L}_2$ as an objetive numerical measure for te entire error curve. In other words, the $\mathcal{L}_2\,|\widetilde{q}|$ marks the compromise between velocity and presition of the moviment performed by the robot. The $\mathcal{L}_2\,|\widetilde{q}|$ norm measures the root–mean–square (RMS) of the position error and its given by the following formula:

$$\mathcal{L}_2 = \sqrt{\frac{1}{t - t_0} \int_{t_o}^{t} \|\widetilde{q}\|^2 \, dt}$$

where $t, t_0 \in \mathcal{R}$ are the initial and final time, repectively. A smaller $\mathcal{L}_2\,|\widetilde{q}|$ represents smaller position error, a fast transient state and a better performance of the evaluated controller. The comparison graph of the controller is shown in fig(6). As a result, the ARCTAN controller has a lower $\mathcal{L}_2$ norm wich means a better performance; considerating that final value of $\mathcal{L}_2\,|\tilde{q}|$ norm is an average of five experimental runs under the same operatin conditions.

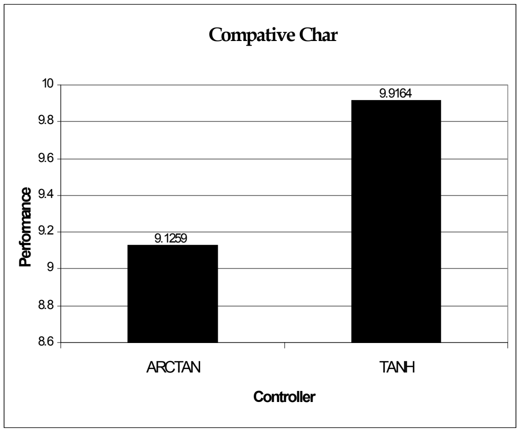

Figure 6: Performance Indices

6 Conclusions

In this paper, we have a new controller of position for robot manipulators. This controller is supported by a rigurous stability analysis. The proposed controller can resolve the saturating problem of the servo–motors. Experimental results on a tree degree–of–freedom direct drive robot manipulator have shown the asymptotic stability and performance. In our opinion, on the basis of the experimental results we cautiously conclude that, the new controller has a better performance than the TANH controller.

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