A Simplified Nonlinear State Estimator for Power Systems

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Abstract: - This paper presents a simplified nonlinear observer for power systems by exploring the special features of the nonlinear power system model. Rather than directly applying the existing nonlinear observer theory to the 6^{th} order nonlinear power system model, the model is first decoupled into a 3^{rd} order nonlinear subsystem and a 3^{rd} order linear subsystem. Low order observers are designed for each subsystem separately. Simulation results show that the proposed observer scheme can estimate the state of a power system with good transient response and zero steady state error.

Key-Words: - Nonlinear observer, state estimation, power systems, single machine, infinite bus.

1. Introduction

It is well known that the control of generator terminal voltage through a conventional automatic voltage regulator (AVR) and excitation system has the potential to introduce negative damping into the system dynamics. A power system stabilizer (PSS) is designed to improve the system damping performance via the utilization of a supplementary damping signal through generator excitation.

Power system stabilizers based on optimal control or pole assignment techniques have been developed to damp power system dynamics oscillations. The implementation of these control techniques requires knowledge of the entire states, though they are not always directly measurable. The development of an estimator to construct the necessary states is required.

Linear Observers based on linearized power system model have been designed to estimate the state variables of a power system. Such observers will result in steady state estimation error when applied to nonlinear power systems. The error may be so large that the estimation results are useless in application. Hence the nonlinear features should be taken into consideration in the design of observers for power systems.

Recent development in nonlinear observer theory provides the possibility to the design of nonlinear observers for power systems. However, the derivation of a nonlinear observe is a tedious tasks and even the most recent results in nonlinear observer theory [1][2] are only suitable for low order nonlinear systems. As a synchronous machine model is usually sixth order or higher, observer derivation directly based on the nonlinear observer theory will be extremely complicated, if not impossible.

In this paper we will design a nonlinear observer for synchronous machines by combining the nonlinear observer theory in [1] with the linear observer theory in [4]. Rather than directly applying the existing nonlinear observer theory to the 6^{th} order nonlinear power system model, the model is first decoupled into a 3^{rd} order nonlinear subsystem and a 3^{rd} order linear subsystem. A low order nonlinear observer and a low order linear observer are designed for the two subsystems respectively. The resulted observer can estimate the state of a nonlinear power system with good transient performance and zero steady state error. Th paper is organized as follows: Section 2 gives a brief review on the nonlinear observer theory. Section 3 presents the proposed hybrid observer scheme for power systems. Section 4 shows some simulation results.

2. Nonlinear Observer Theory

In this section, we will review some of the well-developed results in nonlinear observer theory [1][2][3]. Consider the single-input single-output nonlinear system

$$\dot{x}(t) = f(x(t)) + g(t)u(t) \qquad x(0) = x_o \tag{1a}$$

$$y(t) = h(x(t)) \tag{1b}$$

where $x(t) \in \Re^n$ is the state, $f, g \in C^\infty$ are real valued vectors, and $h \in C^\infty$ is a real valued function. The class U of real-valued inputs is constituted of uniformly bounded functions, i.e. $U = \{u: \ |u(t)| \le M, \ \forall t \ge 0\}$

The Lie derivative [3] is defined as

$$L_f h = \frac{\partial h}{\partial x} f(x) \qquad L_f^i h = L_f(L_f^{i-1} h) \,. \tag{2}$$

The observability matrix of the system (1) is defined as

$$Q(x) = \frac{d\phi(x)}{dx} \tag{3}$$

$$\text{where } \phi(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{n-1} h(x) \end{bmatrix}. \tag{4}$$

The Hölder condition [1] is defined as

$$\| Q(x_1) - Q(x_2) \| \le \gamma_Q \| x_1 - x_2 \|^{\delta_Q} \tag{5}$$

γ_Q and δ_Q in Equation (5) are the Hölder constants associated with Q and $\delta_Q \in (0,1]$.

The global Hölder condition can be satisfied by most physically meaningful dynamic systems such as power systems.

Theorem [1]: Let $Q(x)$ be the observability matrix associated with the pair $(f(x),h(x))$ of functions in (1). Assume $u(t) \in U$ for all $t \ge 0$, and the triple $(f(x),g(x),h(x))$has relative degree n (see [3]). If

- $Q(x)$ has full rank for all $x \in \Re^n$ and
- $$\sup_{\|u\| \le M} \| L_f^n h(x_1) + uL_g L_f^{n-1} h(x_1) - L_f^n h(x_2) \ldots - uL_g L_f^{n-1} h(x_2) \| \le \gamma_Q \| x_1 - x_2 \|^{\delta_Q} ,$$

then there exists a finite gain vector $K \in \Re^n$ such that the solution of the following system equations

$$\dot{\hat{x}}(t) = f(\hat{x}(t)) + g(\hat{x}(t))u(t) + \ldots \\ [Q(\hat{x}(t))]^{-1} K[y(t) - h(\hat{x}(t))] , \qquad \hat{x}(0) = \bar{x} \tag{6}$$

have the following properties

- for $\delta_Q \in (0,1]$, $\varepsilon > 0$ and $\hat{x}(0) \in \Re^n$, $\lim_{t\to\infty} \| \hat{x}(t) - x(t) \| \le \varepsilon$;
- for $\delta_Q = 1$ and $\hat{x}(0) \in \Re^n$, $\lim_{t\to\infty} \| \hat{x}(t) - x(t) \| = 0$.

The parameter K is chosen to enable (A-KC) stable where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $C = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$.

3. Nonlinear Observer Design For Power Systems

We will consider a single-machine infinite-bus power system shown in Fig. 1. We will first make the following definitions:

v_t: terminal voltage; v_o: infinite bus voltage; E_{fd}: field voltage; e'_q: q-axis component of voltage behind transient reactance; T_m: mechanical input; δ: torque angle; ω: angular speed; x_d: d-axis reactance of the generator; x_q: q-axis reactance of the generator; x'_d: d-axis transient reactance of the generator; x_e: reactance of transmission line; T'_{do}: d-axis transient open circuit time constant; P_{eo}: active power output; Q_{eo}: reactive power output.

The generator is represented by a third order dynamic model:

$$\dot{\omega}(t)=\frac{1}{2H}[T_m - T_e(t)] \tag{7a}$$

$$\dot{\delta}(t)=\omega_b\omega(t) \tag{7b}$$

$$\dot{e}'_q(t)=\frac{1}{T'_{do}}[E_{fd}(t)-e'_q(t)-(x_d-x'_d)I_d(t)] \tag{7c}$$

where

$$T_e(t)=I_d v_d + I_q v_q,$$
$$v_d(t)=x_q I_q,\ v_q(t)=e'_q - x'_d I_d,$$
$$I_d(t)=\frac{e'_q - v_o\cos\delta}{x_e+x'_d},\ I_q(t)=\frac{v_o\sin\delta}{x_e+x_q}.$$

The excitation system is an IEEE Type ST1 exciter shown in Fig. 2. For simplicity, we assume $T_c=0$. Then the dynamic equations of the exciter can be written as:

$$\dot{E}_{fd}(t)=-\frac{1}{T_A}E_{fd}(t)+\frac{K_A}{T_A}x_B(t), \tag{8a}$$

$$\dot{x}_B(t)=-\frac{1}{T_B}x_B(t)-\frac{1}{T_B}x_F(t)+\frac{1}{T_B}[v_{ref}-v_t(t)], \tag{8b}$$

$$\dot{x}_F(t)=-\frac{K_F}{T_F T_A}E_{fd}(t)+\frac{K_F K_A}{T_F T_A}x_B(t)-\frac{1}{T_F}x_F(t), \tag{8c}$$

where $v_t(t)=\sqrt{v_d^2+v_q^2}$.

As the above power system is 6th order, direct application of the nonlinear observer theory summarized in the previous section will be extremely tedious. Also the resulted nonlinear observer is very complicated and makes real-time applications very difficult. To simplify the design, some practical aspects of power systems can be considered. For power systems, E_{fd}, δ and v_t are easily measurable variables. These will enable us to decouple a power system model into two parts: a nonlinear subsystem and linear subsystem. The order of the nonlinear subsystem will be only 3rd order. To design a nonlinear observer for this part of the system will be much simpler. The state of the linear subsystem will be estimated using a linear observer.

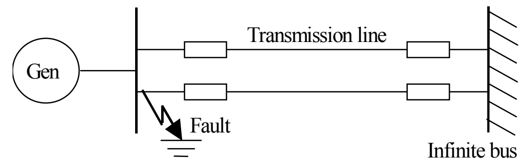


Fig. 1 Generator connected to infinite bus through a double transmission line

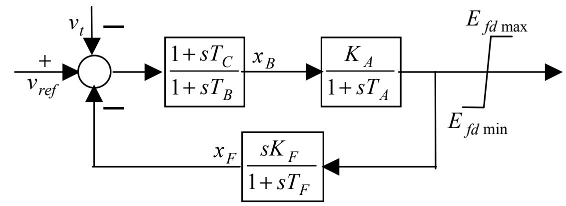


Fig. 2 IEEE Type ST1 excitation system

3.1 Decoupling

As E_{fd} and δ are measurable, we can write Equations (7) into the following form

$$\dot{x}_N(t)=f_N(x_N)+g_N E_{fd} \tag{9a}$$

$$y_N(t)=h_N(x_N)=\delta \tag{9b}$$

where

$$x_N=[\omega\quad \delta\quad e'_q]^T,$$

$$f_N=\begin{bmatrix}\frac{1}{2H}[T_m-T_e(t)]\\ \omega_b\omega(t)\\ \frac{1}{T'_{do}}[-e'_q(t)-(x_d-x'_d)I_d(t)]\end{bmatrix} \text{ and } g_N=\begin{bmatrix}0\\0\\ \frac{1}{T'_{do}}\end{bmatrix}.$$

As E_{fd} and v_t are measurable, we can write Equations (8) as

$$\dot{x}_L(t)=A_L x_L + B_L u_L \tag{10a}$$

$$y_L(t)=C_L x_L = E_{fd} \tag{10b}$$

where

$$x_L=[E_{fd}\quad x_B\quad x_F]^T,\ u_L=v_{ref}-v_t(t),$$

$$A_L = \begin{bmatrix} -\frac{1}{T_A} & \frac{K_A}{T_A} & 0 \\ 0 & -\frac{1}{T_B} & -\frac{1}{T_B} \\ -\frac{K_F}{T_F T_A} & \frac{K_F K_A}{T_F T_A} & -\frac{1}{T_F} \end{bmatrix}, \ B_L = \begin{bmatrix} 0 \\ \frac{1}{T_B} \\ 0 \end{bmatrix}$$

and $C_L = [1 \quad 0 \quad 0]$.

Then the design of a nonlinear observer for the nonlinear subsystem of Equations (9) can be carried out separately from the design of a linear observer for the linear subsystem of Equations (10), if E_{fd}, δ and v_t are measurable. Hence the nonlinear part and the linear part of a power system are totally decoupled from each other.

3.2 Observer Design for Nonlinear Subsystem

By applying the nonlinear observer theory summarized in Section 2, the nonlinear transform for the nonlinear subsystem (Equations 9) is obtained as

$$\phi(x_N) = \begin{bmatrix} h_N \\ L_{f_N} h_N \\ L_{f_N}^2 h_N \end{bmatrix} = \begin{bmatrix} \delta \\ \omega_b \omega \\ c_1 T_m + c_2 \sin 2\delta + c_3 e_q' \sin\delta \end{bmatrix}$$

where

$$c_1 = \frac{\omega_b}{2H},$$

$$c_2 = -c_1 \frac{(x_d' - x_q) v_o^2}{2(x_q + x_e)(x_d' + x_e)} \text{ and}$$

$$c_3 = -c_1 \frac{v_o}{x_d' + x_e}.$$

The observability matrix (see Equation 3) for this subsystem is

$$Q(x_N) = \begin{bmatrix} 0 & 1 & 0 \\ \omega_b & 0 & 0 \\ 0 & 2c_2 \cos 2\delta + c_3 e_q' \cos\delta & c_3 \sin\delta \end{bmatrix}$$

and its inverse can be easily derived as

$$Q^{-1}(x_N) = \begin{bmatrix} 0 & \frac{1}{\omega_b} & 0 \\ 1 & 0 & 0 \\ -\frac{2c_2 \cos 2\delta + c_3 e_q' \cos\delta}{c_3 \sin\delta} & 0 & \frac{1}{c_3 \sin\delta} \end{bmatrix}.$$

Then the observer for the nonlinear subsystem is

$$\dot{\hat{x}}_N(t) = f_N(\hat{x}_N) + g_N E_{fd} + \ldots$$
$$Q^{-1}(\hat{x}_N) K_N (y_N - \hat{h}_N(\hat{x}_N))$$

where K_N is chosen to enable $(A_N - K_N C_N)$ stable with $A_N = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B_N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $C_N = [1 \quad 0 \quad 0]$.

3.3 Observer Design for Linear Subsystem

With v_t available for measurement, a third order full order Luenberger observer for the linear subsystem (Equations 10) can be designed by applying linear observer theory in [4]. The observer is in the following form

$$\dot{\hat{x}}_L(t) = A_L \hat{x}_L + B_L u_L + K_L (y_L - \hat{y}_L)$$

where K_L is chosen to enable $(A_L - K_L C_L)$ stable.

To further simplify the observer structure, a reduced order observer [4] can also be designed for this linear subsystem.

4. Simulation Studies

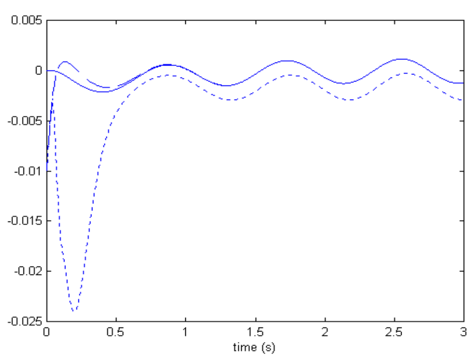
The following data is used in the simulation:

f=50 Hz, $\omega_b = 2\pi f$, $T_{do}' = 5s$, H=5, $x_d = 1.6$, $x_d' = 0.32$, $x_q = 1.55$, $x_e = 0.2$, $T_C = 0$, $T_B = 0.05s$, $K_A = 100$, $T_A = 0.05s$, $K_F = 0.025$, $T_F = 1s$.

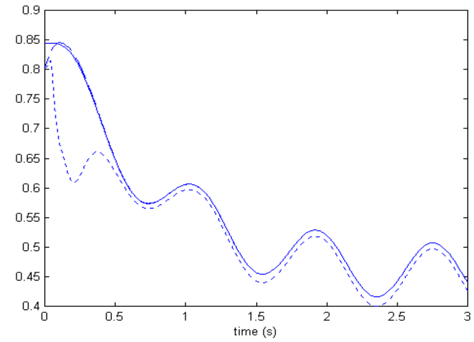
The pre-fault condition is: output power: $P_{eo} = 0.8pu$, $Q_{eo} = 0.5pu$; terminal voltage: $v_{to} = 1.05pu$.

4.1 Comparison With A Linear Observer

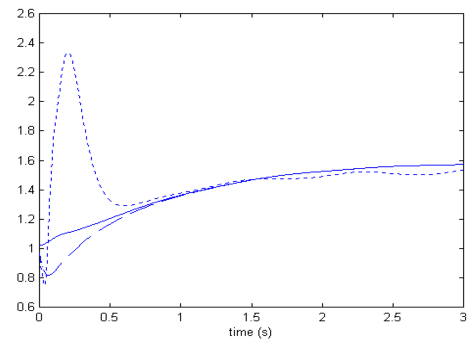
The performance of the proposed nonlinear observer for the above power system is shown in Fig. 3a-f by the dashed lines with the true state shown by the solid lines. To demonstrate the superior performance of the proposed hybrid observer over a linear observer, a linear sixth order Luenberger observer is also designed based on the fully linearized model of this power system. The measurement for this full order linear observer is E_{fd} and δ . This linear observer and the proposed hybrid observer are tested on the same nonlinear power system through simulation. It is shown that under a small step change of 0.01 pu in v_{ref} , both the proposed hybrid observer and the full order linear observer can follow the true state of the power system accurately. Both observers have the same initial values. However, when a large step increase of 0.2 pu in v_{ref} is applied, the difference in the performance of the two observers becomes obvious. The hybrid observer can still follow the true state with zero steady state error, while the linear observer fail to estimate the state accurately. The estimation from the linear observer for $\Delta v_{ref} = 0.2pu$ is shown in Fig. 3a-f by the dotted lines. The inability of the linear observer to follow the true state is because that a linearized model approximates the nonlinear power system accurately only in a small range.



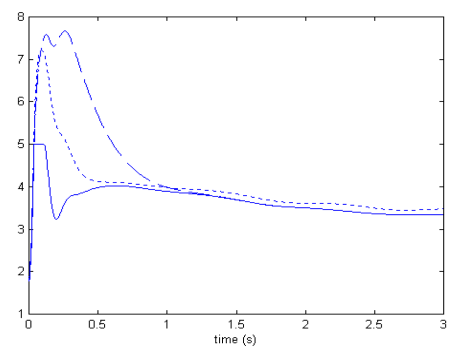
(a)



(b)



(c)



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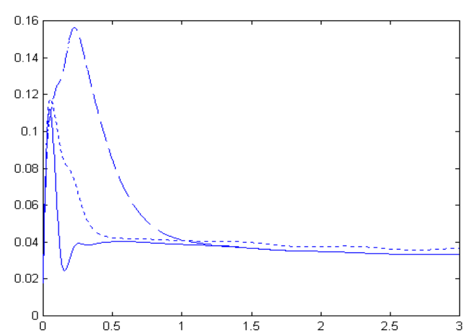
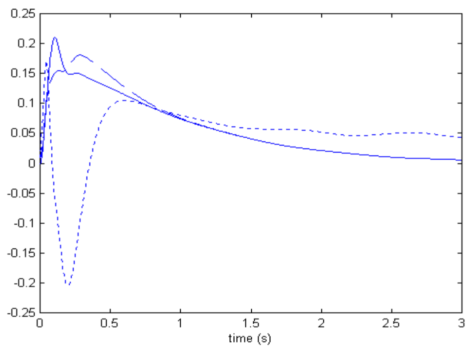


(e)



(f)

Fig. 3 Estimation of the state of a nonlinear power systems for $\Delta v_{ref} = 0.2pu$. Solid lines: the true state; dashed lines: estimation by the proposed observer; Dotted lines: estimation by linear observer. (a) Estimation of ω; (b) Estimation of δ; (c) Estimation of e'_q; (d) Estimation of E_{fd}; (e) Estimation of x_B; (f) Estimation of x_F.

4.2 Nonlinear Observer Based Control

Because the proposed nonlinear observer can estimate the state of a power system more accurately, it gives improved performance over a linear observer when combined with linear feedback control. A PSS based on linearized power system model is designed by applying the pole assignment technique. The state required by the feedback-control-law is estimated using the proposed nonlinear observer. This linear control in conjunction with the nonlinear observer scheme is applied to the simulated nonlinear power system under fault condition. The fault is a three-phase short circuit at one of the transmission lines near the generator and starts at *t=0.2s*. During fault, the terminal voltage is dropped to zero. The fault is cleared through isolating the faulted line 0.1 second after the fault started. The transient response of the machine speed without the PSS is shown in Fig. 4 by the dashed line. The speed response using the linear PSS based on the estimation from the proposed nonlinear observer is shown by the solid line. The speed response using the linear PSS based on the true state is also shown by the dotted line for comparison. It can be seen that a linear PSS based on the estimation from the proposed nonlinear observer can damp out power system oscillation very effectively. A linear control based on the estimation from a linear observer that is designed for the linearized power system model is also tested under the same fault condition. The resulted speed response is unstable.

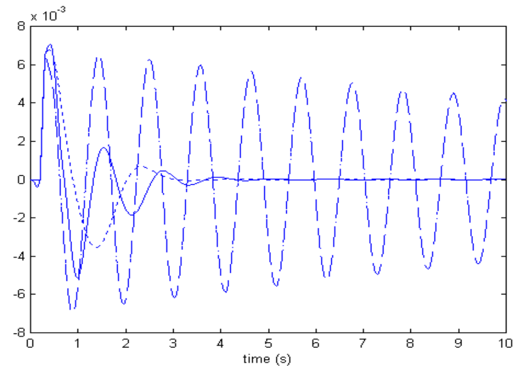


Fig. 4 Speed response under fault condition. Dashed line: without PSS; Solid line: with PSS based on estimation from the proposed nonlinear observer; Dotted line: with PSS based on the true state.

5. CONCLUSION

A simplified nonlinear observer for power systems has been proposed in this paper by combining nonlinear observer theory with linear observer design. The power system model is first separated into lower order nonlinear and linear subsystems by considering the practical features of a power system. Observers are then designed for each subsystem separately. The proposed hybrid observer is simple in design and gives superior performance over conventional linear observers under large disturbances.

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