Fast Algorithm for Computer Simulation of Optical Systems with Partially Coherent Illumination

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Abstract: The fast algorithm for calculating the image intensity distribution in optical system with partially coherent illumination is proposed. The algorithm is based on the coherent-mode representation of cross-spectral density function of illumination. An example of the numerical simulation of a partially coherent optical system is given.

Key-Words: Computer simulation; Image calculation; Fast algorithm; Computational efficiency

1 Introduction

The computer simulation is an essential tool in the design of optical systems. In most cases such a simulation consists in numerical calculating the image intensity distribution for given object, certain characteristics of illumination and known configuration of optical system. For partially coherent illumination, that is the most general case, such a calculation represents a rather complicated computational problem and needs frequently an unacceptably long computer run time. In this paper we propose the fast algorithm for calculating the image intensity distribution based on the modal theory of partially-coherent optical systems [1]. We evaluate the computational efficiency of this algorithm and give an example of image intensity calculation.

2 Image Intensity calculation

The intensity distribution in the image plane of an optical system with partially coherent illumination, under certain conditions, is given by [2]

$$I(u,v)=\iiiint_{-\infty}^{\infty} t(x_1,y_1)t^*(x_2,y_2) \times W(x_1,y_1,x_2,y_2)H(u-x_1,v-y_1) \times H^*(u-x_2,v-y_2)dx_1dy_1dx_2dy_2, \quad (1)$$

where $t(x,y)$ is the complex amplitude transmittance of object, $W(x_1,y_1,x_2,y_2)$ is the cross-spectral density function of illumination, $H(u,v,x,y)$ is the amplitude spread function of optical system, and the asterisk denotes the complex conjugate. In the limit cases of completely coherent and completely incoherent illumination, Eq.(1) takes the forms, respectively,

$$I(u,v)=\left|\iint_{-\infty}^{\infty} t(x,y)H(u-x,v-y)dxdy\right|^2, \quad (2)$$

$$I(u,v)=\iint_{-\infty}^{\infty} |t(x,y)|^2 |H(u-x,v-y)|^2 dxdy. \quad (3)$$

Let us evaluate the computational complexity of calculation in accordance with the general expression (1). The dominant portion of such calculation is the multiplication of four 4-D functions $t(x_1,y_1)t^*(x_2,y_2)$, $W(x_1,y_1,x_2,y_2)$, $H(u,v,x_1,y_1)$ and $H(u,v,x_2,y_2)$. To realize the numerical multiplication of these functions, it is necessary to multiply their samples for all possible combinations of sampling points taken one by one in each of three planes (u,v), (x_1,y_1) and (x_2,y_2). Hence, assuming that the illumination field, object and amplitude spread function have each been adequately represented by $N \times N$ sampling points, one finds that the total number of operations required to compute $I(u,v)$ is proportional to

$$C = (N^2)^3 = N^6. \tag{4}$$

The magnitude of this number can easily result in an unacceptably long computational time. Thus, for example, when $N = 100$ and the computational speed is 10^6 operations per second, the computer run time needed for calculation of $I(u,v)$ is about 300 h. Clearly, an alternative approach to the calculation of intensity distribution is desired as a way to reduce the computational effort.

3 Fast Algorithm

According to Wolf′s theory [3], the cross-spectral density function of a wide class of sources may be represented in the form of the Mercer expansion, i.e.,

$$W(x_1, y_1, x_2, y_2) = \sum_{n=0}^{\infty} \lambda_n \Phi_n(x_1, y_1) \Phi_n^*(x_2, y_2), \tag{5}$$

where λ_n are the eigenvalues and $\Phi_n(x, y)$ are the orthonormal eigenfunctions of the homogeneous Fredholm integral equation

$$\int_{-\infty}^{\infty} W(x_1, y_1, x_2, y_2) \Phi_n(x_2, y_2) dx_2 dy_2 = \lambda_n \Phi_n(x_1, y_1). \tag{6}$$

The expansion (5) represents the cross-spectral density function of the illumination field as a superposition of spatially coherent mutually uncorrelated elementary modes. Substituting for W from Eq. (5) into Eq. (1), after a straightforward calculation we obtain

$$I(u,v) = \sum_{n=0}^{\infty} I_n(u,v), \tag{7}$$

where

$$I_n(u,v) = \lambda_n \left| \iint_{-\infty}^{\infty} t(x,y) \Phi(x,y) H(u-x, v-y) dxdy \right|^2 \tag{8}$$

represents the intensity distribution formed by the n-th coherent mode of illumination field. The eigenvalues λ_n may be arranged in a converging sequence, and hence, it is possible to truncate the summation in Eq. (7) to a finite number M of expansion terms that ensures the admissible value of the relative error of approximation. In Ref. [4] the concept of the effective number $\aleph$ of uncorrelated modes, needed to represent the illumination field, is introduced, and its upper bound is defined by the following inequality:

$$\aleph \le \frac{\left[\iint_{-\infty}^{\infty} W(x,y,x,y) dxdy \right]^2}{\iiiint_{-\infty}^{\infty} \left| W(x_1, y_1, x_2, y_2) \right|^2 dx_1 dy_1 dx_2 dy_2}. \tag{9}$$

This number may be used to establish on optimal point for truncating the orthogonal representation of the intensity distribution.

Let us evaluate the computational complexity of intensity calculation in accordance with the proposed method. The dominant portion of intensity calculation from Eqs. (7) and (8) is the consecutive multiplication of 4-D function $H(u,v,x,y)$ by 2-D function $(\lambda_n)^{1/2} t(x,y) \Phi_n(x,y)$, followed by the calculation of a square absolute value of the product for every n-th expansion term. To realize the numerical calculation of every expansion term in Eq. (7), it is necessary to multiply the samples of this functions for all possible combinations of sampling points taken one by one in each of the planes (u,v) and (x,y), and then to multiply the obtained product by its conjugate value. Hence, using again $N \times N$ sampling points and truncating the summation in Eq. (7) to the effective number $\aleph$ of uncorrelated modes, one finds that the number of operations needed to compute $I(u,v)$ by the proposed algorithm is proportional to

$$C = \aleph \left[(N^2)^2 + N^2 \right] = \aleph N^2 (N^2 + 1), \tag{10}$$

or, for rather large N,

$$C \approx \aleph N^4. \tag{11}$$

For a completely coherent illumination, $\aleph$=1, and the computational effort C decrease to N^4. For a partially coherent illumination, C increases linearly with $\aleph$, i.e., the computational effort is lager the more incoherent the illumination. For sufficiently large values of $\aleph$, the illumination may be generally considered to be completely incoherent. In this case, the image intensity distribution can be calculated in accordance with Eq. (3) and the computational effort C reduces again to N^4. Comparison of the computational efficiency of the direct calculation and the proposed algorithm for different values of $\aleph$ is illustrated by schematic picture in Fig. 1. It is evident from this figure that the fast algorithm can be efficiently employed to calculate the image intensity distribution when $\aleph \leq N$. For the same values of N and the computational speed that are in the example of the previous section, the computer run time needed for calculation of $I(u,v)$ from Eqs. (7) and (8) takes from 2 min to 3 h, depending on the degree of coherence of the illumination.

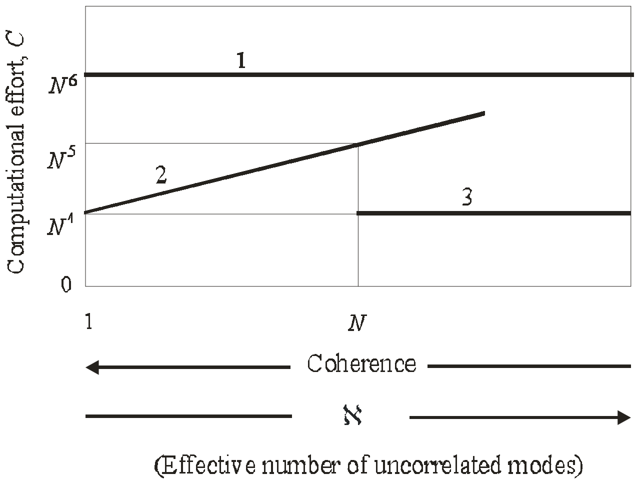

Fig. 1 Estimation of the computational effort C as a function of coherence (effective number $\aleph$ of uncorrelated modes of illumination) for: 1- the direct method in accordance with Eq. (1); 2- the fast algorithm; 3- the direct method in accordance with Eq. (3).

4 Example

To illustrate an application of the proposed algorithm, we calculated the intensity distribution in the image of object, which can be well approximated by the 1-D Dirac comb function, i.e.,

$$t(x) = \sum_k \delta(x - kx_0)\,. \tag{12}$$

Such a choice of the object allows the result of integrating in Eqs. (1) and (8) to be obtained in an explicit analytic form, a fact that gives us a chance to evaluate an accuracy of the proposed algorithm.

Taking into account the 1-D character of our object, and for the sake of simplicity, as an illumination field, we consider the secondary 1-D Gaussian Schell-model source [4] that is characterized by a cross-spectral density function of the form

$$W(x_1, x_2) = I_0 \exp\left(-\frac{x_1^2 + x_2^2}{4\sigma_I^2}\right)\exp\left[-\frac{(x_1 - x_2)^2}{2\sigma_\mu^2}\right], \tag{13}$$

where I_0, σ_I^2 and σ_μ^2 are positive constants. This type of source was chosen because it exhibits the essential features of many sources encountered in practice and yet it can be analyzed mathematically with relative easy. For this source

$$\lambda_n = I_0\left(\frac{\pi}{a+b+c}\right)^{1/2}\left(\frac{b}{a+b+c}\right)^n \tag{14}$$

and

$$\Phi_n(x) = \left(\frac{2c}{\pi}\right)^{1/4}\frac{1}{(2^n n!)^{1/2}} H(x\sqrt{2c})\exp(-cx^2)\,, \tag{15}$$

where

$$a = \frac{1}{4\sigma_I^2},\ b = \frac{1}{2\sigma_\mu^2},\ c = (a^2 + 2ab)^{1/2}, \tag{16}$$

and H_n (...) is the Hermite polynomial of order n.

At last, we considered that the amplitude spread function of an optical system is given by [2]

$$H(\rho) = \alpha \exp\left(i\frac{\pi}{\lambda z_2}\rho^2\right)\frac{J_1(\pi R\rho/\lambda f)}{\pi R\rho/\lambda f}, \tag{17}$$

where $\rho = (u^2 + v^2)^{1/2}$, $J_1(...)$ is the first-order Bessel function, λ is the wave length, f is the focal distance, R is the radius of output pupil, and α, here and further on, is a dimensionless coefficient.

On substituting from Eqs. (12), (13) and (17) into the 1-D version of Eq. (1) and making use of the sifting property of the Dirac function, it is straightforward matter to obtain the following expression for the theoretical image intensity distribution:

$$I(u) = \alpha I_0 \sum_{m,l} A_{ml}\frac{J_1[\pi R(u + mx_0)/\lambda f]}{\pi R(u + mx_0)/\lambda f} \times \frac{J_1[\pi R(u + lx_0)/\lambda f]}{\pi R(u + lx_0)/\lambda f}, \tag{18}$$

where

$$A_{ml} = \exp\left[-\frac{x_0^2}{4\sigma_I^2}\left(m^2 + l^2\right)\right]\exp\left[-\frac{x_0^2}{2\sigma_\mu^2}(m-l)^2\right]. \tag{19}$$

By analogy, but this time using the fast algorithm, with due regard for the truncation of summation in Eq. (7), we obtain the following approximation of the image intensity distribution (18):

$$\hat{I}(u) = \alpha I_0 \sum_{n=0}^{M-1} B_n \left[\sum_k C_{nk} \frac{J_1[\pi R(u + kx_0)/\lambda f]}{\pi R(u + kx_0)/\lambda f} \right]^2, \quad (20)$$

where

$$B_n = \frac{1}{2^n n!} \left(\frac{b}{a+b+c} \right)^n, \quad (21)$$

and

$$C_{nk} = H_n(kx_0 \sqrt{2c}) \exp(-ck^2 x_0^2). \quad (22)$$

To evaluate the quality of our approximation, we realized numerical calculation of the intensity distribution $I(u)$ in accordance with Eqs. (12), (18) and (20). When calculating we put $x_0 = 2.44\lambda f/R$, which is twice greater than the Rayleigh limit of resolution for our optical system, and $\sigma_I = 2\sigma_\mu = 10x_0$, which correspond to the case of true partial coherence. We truncated the summation over indexes k,m,l to nine central Dirac impulses in the object and varied the number M of the terms in the modal expansion. The results of calculation are shown in Fig. 2. As can be seen in this figure, with the increase of number M the approximate intensity distributions come closer to the theoretical curves. When the number M is equal to the effective number $\aleph$ of uncorrelated modes of illumination (in our example $\aleph=4$), the relative error of the fast algorithm makes up approximately 1% and, when $M = 2\aleph$, it becomes negligible.

4 Conclusions

The proposed fast algorithm allows to reduce considerably the computational effort needed for calculating the image intensity distribution in partially coherent optical system and its efficiency is larger the more coherent the illumination. It must be noted that the application of this algorithm requires the knowledge of the coherent-mode representation of illuminating field (eigenvalues λ_n and eigenfunctions Φ_n). In general case, the evaluation of coherent modes entails the numerical solution of the integral equation (6), that is not an easier computational task than the proper calculation of the image. However, it should be taken into account that once Φ_n and λ_n have been calculated for given illumination, they can be stored and applied to the calculation of image for any object and any optical system. Thus, the fast algorithm can be considered as an indispensable tool for the analysis and computer simulation of optical system with partially coherent illumination.

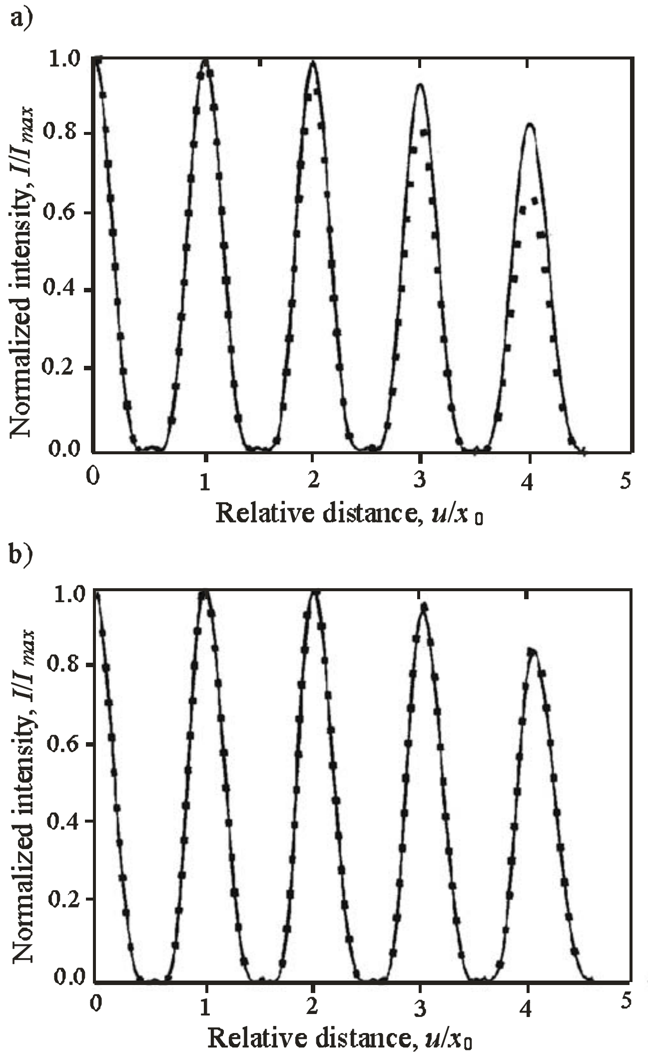

Fig. 2 Results of calculating the image intensity distribution in accordance with Eq.(20) for: a) M = 1; b) M = 4. Theoretical intensity distribution, obtained according to Eq.(18), is shown by solid curves.

References:

[1] A.S. Ostrovsky, O. Ramos-Romero, G. Martínez-Niconoff and J.C. Ramírez-San-Juan, On the modal representation of partially coherent imagery, *Revista Mexicana de Física*, , Vol. 47, No. 5, 2001, pp. 404-407.

[2] J. W. Goodman, *Statistical Optics*, Wiley, 1985

[3] E. Wolf, New theory of partially coherence in the space-frequency domain. Part 1: spectra and cross spectra of steady state sources, *J. Opt. Soc. Am.*, Vol. 72, No. 3, 1982, pp. 343-351.

[4] A. Starikov, Effective number of degrees of freedom of partially coherent sources, *J. Opt. Soc. Am.,* Vol. 72, No. 11, 1982, pp. 1538-1544.