State observation for Nonlinear Hybrid Automata

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Abstract: - This paper presents a state observer for nonlinear hybrid systems characterized by a finite number of discrete modes, in which the switching between different modes is commanded by an external function. It is assumed that at each mode the system is globally drift-observable and either is unforced or has full relative degree. Exponential decay of the observation error is ensured provided that there exists an upper bound on the input amplitude and a lower bound on the time between two consecutive switches. Computer simulations have shown good performances of the proposed observer.

Key-Words: - Nonlinear Systems, State Observation, Hybrid Automata.

1 Introduction

Many recent papers deal with control problems on hybrid systems (see e.g. [3,4,6,11,12] and references therein). On the other hand, smaller attention has been devoted to the problem of state observation for hybrid systems in a deterministic setting. The problem of state estimation for switching linear systems in a stochastic framework has been investigated first in [1] for discrete time-systems. Recent papers on the topic are [13], for the continuous-time case, and [7] for the discrete-time case. The case of nonlinear stochastic hybrid systems has been investigated in [9], where the optimal state estimator is presented in terms of a Zakai-type equation that gives the evolution of the conditional density of the state variable.

In the stochastic setting, the switching between discrete modes is commanded by a finite-state Markov chain. In the deterministic setting the switching times and the corresponding discrete modes are assumed known. The state observation problem for deterministic switching discrete-time linear systems is investigated in [2], in which a Luenberger switching observer with scheduled gains is proposed.

This paper studies the problem of state observation, in a deterministic setting, for a class of hybrid automata described by switching nonlinear differential equations. A switching observer is proposed and exponential decay of the observation error is proved under suitable assumptions. The observer gain is easy to compute and the observation algorithm is easy to implement. Before to present the observer, some preliminaries taken from [8,5] are reported. The paper closes with some simulation results on a simplified model of an internal combustion engine, that demonstrate the high effectiveness of the proposed algorithm.

This work is supported by *MIUR* (Ministry of Education, University and Research) and by *CNR* (National Research Council).

2 Hybrid Automata

Let $\mathcal{V} = \{1, 2, \ldots, N\}$, and let f, g, h be vector functions

$$f, g : \mathcal{V} \times I\!R^n \mapsto I\!R^n, \quad h : \mathcal{V} \times I\!R^n \mapsto I\!R, \tag{1}$$

smooth with respect to the variable in $I\!R^n$. In this paper we consider the class of hybrid automata characterized by N discrete modes, described by nonlinear differential equations of the type

$$\begin{aligned} \dot{x}(t) &= f(v, x(t)) + g(v, x(t))u(t), \quad x(0) = x_0 \\ y(t) &= h(v, x(t)). \end{aligned} \tag{2}$$

For each $v \in \mathcal{V}$ the system (2) describes the continuous state flow in the v-th discrete mode. The discrete variable v is piecewise constant, so that the system (2) is piecewise smooth. When a change occurs in the variable v (jump) also the continuous state $x(t)$ may undergo a discontinuous change. We assume the existence of a function

$$\begin{aligned} &R : (\mathcal{V} \times \mathcal{V})^\star \times I\!R^n \mapsto I\!R^n \\ &\text{where } (\mathcal{V} \times \mathcal{V})^\star = \{(i, j) \in \mathcal{V} \times \mathcal{V} : \ i \neq j\} \end{aligned} \tag{3}$$

which assigns new values to the continuous state variable when a jump occurs from one discrete mode i to another discrete mode j, $(i,j) \in (\mathcal{V} \times \mathcal{V})^\star$.

We assume that the mode changes in the differential equation (2) are commanded by a piecewise constant right-continuous function $\sigma : I\!R^+ \to \mathcal{V}$, that is available in real time.

Denoting with t_j, $j = 1, 2, \ldots$ the switching times for $\sigma(t)$, the hybrid automaton (2) satisfies the equations

$$\begin{aligned} \dot{x}(t) &= f(\sigma(t), x(t)) + g(\sigma(t), x(t))u(t), \ t \neq t_j, \\ x(t_j) &= R\big(\sigma(t_{j-1}), \sigma(t_j), x(t_j^-)\big), \\ y(t) &= h(\sigma(t), x(t)), \end{aligned} \tag{4}$$

where

$$x(t_j^-) = \lim_{t \to t_j^-} x(t), \tag{5}$$

Note that $\sigma(t)$ piecewise constant right-continuous is such that $\sigma(t_{j-1}) = \sigma(t_j^-) \neq \sigma(t_j)$.

3 An Observer for Smooth Nonlinear Systems

The observer for nonlinear switching systems proposed in this paper is an extension of the one presented in [8, 5] for smooth nonlinear systems, and therefore it is useful to recall in short the relevant notations and results.

When considering the problem of state observation for nonlinear systems of the type

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + g(x(t))u(t), \\ y(t) &= h(x(t)), \end{aligned} \qquad t \geq 0 \tag{6}$$

where $x(t) \in I\!R^n$, $u(t)$, $y(t) \in I\!R$, and the vector functions f, g and h are C^∞, a key-role is played by the so called *drift-observability map*, defined as

$$z = \big[h(x) \ L_f h(x) \ \cdots \ L_f^{n-1} h(x)\big]^T = \Phi(x), \tag{7}$$

where the symbol $L_f^k h(x)$ denotes the k-th order repeated Lie derivative of the function h along the vector field f.

Definition 1. *The system* (6) *is said to be globally drift-observable if the function $z = \Phi(x)$ is a diffeomorphism in all $I\!R^n$.* ∎

The main assumption needed in the paper is the following:

H_1: *The system* (6) *is globally drift-observable, and the diffeomorphism $z = \Phi(x)$ and its inverse $x = \Phi^{-1}(z)$ are globally uniformly Lipschitz in $I\!R^n$.*

This means that in all $I\!R^n$

$$\begin{aligned} \|\Phi(x_1) - \Phi(x_2)\| &\leq \gamma_\Phi \, \|x_1 - x_2\|, \\ \|\Phi^{-1}(z_1) - \Phi^{-1}(z_2)\| &\leq \gamma_{\Phi^{-1}} \|z_1 - z_2\|. \end{aligned} \tag{8}$$

(γ_Φ and $\gamma_{\Phi^{-1}}$ denote the Lipschitz constants of the maps $\Phi(x)$ and $\Phi^{-1}(z)$).

If system (6) is globally drift-observable, the Jacobian

$$Q(x) = \frac{\partial \Phi(x)}{\partial x}, \tag{9}$$

is nonsingular for all $x \in I\!R^n$, and $z = \Phi(x)$ defines a global change of coordinates. Defining the Brunowski triple (A_b, B_b, C_b) (see [8, 5]), by definition of the map Φ it follows

$$Q(x)f(x) = A_b \Phi(x) + B_b L_f^n(x), \quad h(x) = C_b \Phi(x). \tag{10}$$

Differentiating $z(t) = \Phi\big(x(t)\big)$ w.r.t. time, using properties (10) we obtain

$$\begin{aligned} \dot{z}(t) &= A_b z(t) + P(x, u), \\ y(t) &= C_b z(t), \end{aligned} \tag{11}$$

$$\text{where} \quad P(x,u) = B_b L_f^n h(x) + Q(x) g(x) u. \tag{12}$$

The system (6) is written in z-coordinates as

$$\begin{aligned} \dot{z}(t) &= A_b z(t) + \widetilde{P}(z(t), u(t)), \\ y(t) &= C_b z(t) \end{aligned} \tag{13}$$

$$\text{where} \quad \widetilde{P}(z,u) = P(\Phi^{-1}(z), u), \tag{14}$$

The second assumption needed to ensure exponential convergence is the following:

H_2: *$\widetilde{P}(z,u)$ is globally uniformly Lipschitz with respect to z, and the Lipschitz coefficient $\gamma_{\widetilde{P}}$ is a non decreasing function of $|u|$.*

$$\|\widetilde{P}(z_1, u) - \widetilde{P}(z_2, u)\| \leq \gamma_{\widetilde{P}}(|u|) \|z_1 - z_2\|. \tag{15}$$

The third assumption needed in this paper requires the following definition:

Definition 2. *The triple $\big(f(x), g(x), h(x)\big)$ is said to have observation relative degree equal to r in a set $\Omega \subseteq I\!R^n$ if*

$$\begin{aligned} &\forall x \in \Omega \ \ L_g L_f^k h(x) = 0, \quad k = 0, 1, \ldots, r-2, \\ &\exists x \in \Omega \ : L_g L_f^{r-1} h(x) \neq 0. \end{aligned} \tag{16}$$

The third assumption is:

H_3: *for system* (6) *the triple* $\big(f(x), g(x), h(x)\big)$ *has observation relative degree equal to* n *in all* $I\!R^n$.

It is easy to verify that in the case of observation relative degree equal to n the function $P(x,u)$ defined in (12) can be written as

$$P(x,u) = B_b \left(L_f^n h(x(t)) + L_g L_f^{n-1} h(x(t)) u(t) \right). \tag{17}$$

The observer studied in [5, 8] has the following structure

$$\begin{aligned} \dot{\hat{x}}(t) =& f(\hat{x}(t)) + g(\hat{x}(t))u(t) \\ &+ Q^{-1}(\hat{x}(t)) K \big(y(t) - h(\hat{x}(t))\big), \end{aligned} \tag{18}$$

The following theorem [8] can be given:

Theorem 3. *For system* (6) *assume that hypotheses* H_1, H_2, H_3 *are satisfied and that there exists* u_M *such that* $|u(t)| \le u_M$, *for* $t \ge 0$. *Then, for any positive* α *there exists a gain vector* K *for the observer* (18) *and a constant* $\tilde{\mu}$ *such that*

$$\|x(t) - \hat{x}(t)\| \le \tilde{\mu}\, e^{-\alpha t} \|x(0) - \hat{x}(0)\|, \tag{19}$$

■

4 A Switching Observer for Hybrid Automata

When dealing with hybrid automata of the type (4), all functions of the state that are defined in the previous section depend also on the discrete mode $v \in \mathcal{V}$. Hence, in the following the Lie derivatives of h along f will be denoted $L_f^k h(v,x)$ and the map (7) and its inverse will be denoted $z = \Phi(v,x)$ and $x = \Phi^{-1}(v,z)$, respectively. Also the functions P and $\widetilde{P}$ defined in (12) and (14) will be denoted $P(v,x,u)$ and $\widetilde{P}(v,x,u)$, respectively.

The proposed observer for the hybrid automaton (4) has the following structure

$$\begin{aligned} \dot{\hat{x}}(t) =& f(\sigma(t), \hat{x}(t)) + g(\sigma(t), \hat{x}(t))u(t) \\ &+ Q^{-1}(\sigma(t), \hat{x}(t)) K \big(y(t) - h(\sigma(t), \hat{x}(t))\big), \\ & t \ge 0,\ t \ne t_j,\ j = 1, 2, \ldots \\ \hat{x}(t_j) =& R(\sigma(t_{j-1}), \sigma(t_j), \hat{x}(t_j^-)) \end{aligned} \tag{20}$$

$$\text{where} \qquad Q(\sigma, x) = \frac{\partial \Phi(\sigma, x)}{\partial x}. \tag{21}$$

The only design degree of freedom is the constant observer gain K, that is responsible of the observer performance.

5 Convergence Results

In this section we show that the assumptions (H_1–H_3) are sufficient to guarantee existence of a gain vector K in the observer (20) that ensures asymptotic observation error decay. This result is based on the capability to assign large eigenvalues to the matrix $A_b - KC_b$ while keeping small the norm of suitable matrices (note that the pair A_b, C_b is observable and that matrix $A_b - KC_b$ has a companion structure). Let us define a set $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ of real eigenvalues and let us denote with $K(\Lambda)$ the gain such that $\text{eig}\big(A_b - K(\Lambda)C_b\big) = \Lambda$. In [8] it is shown that if the eigenvalues λ_j, $j = 1, \ldots, n$ are distinct, then $V(\Lambda)\big(A_b - K(\Lambda)C_b\big)V^{-1}(\Lambda) = \text{diag}(\Lambda)$, where $V(\Lambda)$ is a Vandermonde matrix.

The following Lemma is fundamental in the proof of exponential decay of the observation error.

Lemma 4. *For any positive reals* c_1, c_2, c_3, α, *there exists a set* Λ *of* n *reals* λ_j, *with* $\lambda_n < \lambda_{n-1} < \ldots < \lambda_1 < 0$, *such that*

$$\begin{aligned} &\lambda_1 + c_1 \cdot \|V^{-1}(\Lambda)\| \\ &\quad + c_2 \cdot \ln(c_3 \cdot \|V^{-1}(\Lambda)\| \cdot \|V(\Lambda)\|) = -\alpha. \end{aligned} \tag{22}$$

Lemma 4 is a non trivial extension of Lemma 1 in [8], and can be proved following the same lines.

The assumptions needed in this paper to work out the proof of exponential decay of the observation error are the following:

Hp_1: for each $v \in \mathcal{V}$, the triple $f(v,\cdot), g(v,\cdot), h(v,\cdot)$ satisfies hypotheses H_1, H_2 and H_3 of previous section (the largest among the Lipschitz constants of the functions Φ, Φ^{-1} and $\widetilde{P}$ over $\mathcal{V}$ will be denoted γ_Φ, $\gamma_{\Phi^{-1}}$ and $\gamma_{\widetilde{P}}$, respectively);

Hp_2: The function $R(\cdot,\cdot,\cdot)$ defined in (3) is globally Lipschitz with respect to the variable in $I\!R^n$, with Lipschitz constant γ_R, i.e.

$$\begin{aligned} &\|R(v_1, v_2, x_1) - R(v_1, v_2, x_2)\| \le \gamma_R \|x_1 - x_2\| \\ &\forall (v_1, v_2) \in (\mathcal{V} \times \mathcal{V})^\star,\ \forall x_1, x_2 \in I\!R^n. \end{aligned} \tag{23}$$

Hp_3: The piecewise constant right-continuous switching function $\sigma(t)$ is characterized by a minimum time interval $T_{\min}$ between two consecutive switches, i.e. $|t_{j+1} - t_j| \ge T_{\min}$.

Lemma 5. *Under assumption Hp_1, after the switching change of coordinates $z(t) = \Phi(\sigma(t), x(t))$, the hybrid automaton* (4) *can be rewritten as*

$$\begin{aligned} \dot{z}(t) &= A_b z(t) + B_b \widetilde{P}((\sigma(t), z(t), u), t \neq t_j, \\ z(t_j) &= \widetilde{R}(\sigma(t_{j-1}), \sigma(t_j), z(t_j^-)), \end{aligned} \tag{24}$$

where the function $\widetilde{R} : (\mathcal{V} \times \mathcal{V})^\star \times I\!R^n \mapsto I\!R^n$ is defined as

$$\widetilde{R}(\sigma_h, \sigma_k, z) = \Phi(\sigma_k, R(\sigma_h, \sigma_k, \Phi^{-1}(\sigma_h, z)). \tag{25}$$

Proof. The computations that led to system (13) can be repeated for the automaton (4) between two adjacent switching times. At a switching time t_j it is $z(t_j^-) = \Phi(\sigma(t_{j-1}), x(t_j^-))$. On the other hand the change of coordinates imposes $z(t_j) = \Phi(\sigma(t_j), x(t_j))$, so that the equation that gives the jump of the z-coordinates is obtained as

$$\begin{aligned} z(t_j) &= \Phi(\sigma(t_j), x(t_j)) \\ &= \Phi\Big(\sigma(t_j), R(\sigma(t_{j-1}), \sigma(t_j), x(t_j^-))\Big). \end{aligned} \tag{26}$$

Since $x(t_j^-) = \Phi^{-1}(\sigma(t_{j-1}), z(t_j^-))$, taking into account the definition (25) the second of (24) follows. ∎

Lemma 6. *Under the assumption Hp_1 the observer* (20) *can be rewritten in the form*

$$\begin{aligned} \dot{\hat{z}}(t) &= A_b \hat{z}(t) + B_b \widetilde{P}((\sigma(t), \hat{z}(t), u) \\ &\qquad + K(y(t) - C_b \hat{z}(t)) \quad t \neq t_j, \\ \hat{z}(t_j) &= \widetilde{R}(\sigma(t_{j-1}), \sigma(t_j), \hat{z}(t_j^-)) \\ \hat{x}(t) &= \Phi^{-1}(\sigma(t), \hat{z}(t)) \qquad \forall t \geq 0, \end{aligned} \tag{27}$$

where the function $\widetilde{R}$ is defined in (25).

Proof. The lemma states a result which is a slight modification of equation 2.11 in [8], with the addition of the function $\widetilde{R}$. The same lines of the proof of Theorem 5 can be followed to derive equations (27). ∎

Theorem 7. *Consider the system* (2). *Assume that hypotheses Hp_1, Hp_2, Hp_3 are satisfied and that there exists u_M such that $|u(t)| \leq u_M$, for $t \geq 0$. Then, for any positive α there exists a gain vector K for the observer* (20) *and a constant $\tilde{\mu}$ such that for $t \geq 0$*

$$\|x(t) - \hat{x}(t)\| \leq \tilde{\mu}\, e^{-\alpha t} \|x(0) - \hat{x}(0)\|. \tag{28}$$

Proof. Consider the system (4) and the observer (20) in z-coordinates, as given by Lemmas 5 and 6, and define the observation errors

$$\eta(t) = z(t) - \hat{z}(t), \quad \zeta(t) = \|V(\Lambda)\eta(t)\|. \tag{29}$$

Following the proof of Theorem 1 in [8], it follows that in any interval $[t_{j-1}, t_j)$ we have

$$\zeta(t) \leq e^{\left(\lambda_1 + \sqrt{n}\gamma_{\widetilde{P}}(u_M)\|V^{-1}(\Lambda)\|\right)\cdot(t - t_{j-1})} \zeta(t_{j-1}). \tag{30}$$

At the switching times t_j the following inequality holds

$$\zeta(t_j) \leq \|V(\Lambda)\|\gamma_\Phi \gamma_{\Phi^{-1}} \gamma_R \|V^{-1}(\Lambda)\|\zeta(t_j^-). \tag{32}$$

Substituting in (32) the inequality (30) for $t = t_j^-$ one has

$$\zeta(t_j) \leq \mu_1(\Lambda) e^{\mu_2(\Lambda)(t_j - t_{j-1})} \zeta(t_{j-1}), \tag{31}$$

$$\begin{aligned} \mu_1(\Lambda) &= \|V(\Lambda)\| \cdot \|V^{-1}(\Lambda)\| \gamma_\Phi \gamma_{\Phi^{-1}} \gamma_R, \\ \mu_2(\Lambda) &= \lambda_1 + \sqrt{n}\gamma_{\widetilde{P}}(u_M)\|V^{-1}(\Lambda)\| \end{aligned} \tag{33}$$

Repeated application of inequality (31) for $j = 1, 2, \ldots, k$ and using (30) for $j = k + 1$ yields the following inequality that holds for $t \in [t_k, t_{k+1})$

$$\begin{aligned} \zeta(t) &\leq \mu_1^k(\Lambda) e^{\mu_2(\Lambda)t} \zeta(0) \\ &\leq e^{k \ln \mu_1(\Lambda) + \mu_2(\Lambda)t} \zeta(0). \end{aligned} \tag{34}$$

Since, by assumption Hp_3, if $t \in [t_k, t_{k+1})$ it follows that $k \leq t/T_{\min}$, then (34) can be rewritten as

$$\zeta(t) \leq e^{\left(\mu_2(\Lambda) + \frac{1}{T_{\min}}\mu_1(\Lambda)\right)t} \zeta(0). \tag{35}$$

Note that (35) is satisfied for $t \in [t_k, t_{k+1})$ for all $k = 0, 1, 2, \ldots$, and therefore it holds for all $t \geq 0$. Using Lemma 4, for any chosen positive α a vector Λ of eigenvalues can be found such that

$$\mu_2(\Lambda) + \frac{1}{T_{\min}}\mu_1(\Lambda) = -\alpha, \tag{36}$$

so that for this choice of Λ, the gain vector $K(\Lambda)$ in the observer (20) ensures that $\zeta(t) \leq e^{-\alpha t}\zeta(0)$. Recalling the definition of $\zeta = \|V(\Lambda)\eta\|$ it follows

$$\|V(\Lambda)\eta(t)\| \leq e^{-\alpha t} \|V(\Lambda)\eta(0)\| \tag{37}$$

Since $\eta(t) = z(t) - \hat{z}(t)$, we have

$$\|z(t)-\hat{z}(t)\| \leq \|V(\Lambda)\|\|V^{-1}(\Lambda)\|e^{-\alpha t}\|z(0)-\hat{z}(0)\|, \tag{38}$$

and from this inequality (28) (the thesis) follows with

$$\tilde{\mu} = \|V(\Lambda)\| \cdot \|V^{-1}(\Lambda)\|\gamma_\Phi\gamma_{\Phi^{-1}} \tag{39}$$

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6 Simulation Results

This section presents simulation results on a simplified model of an Internal Combustion Engine (ICE) to which a mechanical load can be connected and disconnected by means of a stiff friction clutch. This system can be described by a hybrid automaton with two discrete modes, characterized by the engagement/disengagement of the clutch to the load shaft. The simplified ICE model is the following:

$$\begin{aligned} \dot{m}_{ac} &= -\frac{1}{\tau(\omega)}\Big(m_{ac} - m_{ss}(\omega,\alpha)\Big) \\ \dot{\omega} &= \frac{1}{J(\sigma)}\Big(\beta(\omega, m_{ac}) - f(\sigma)\omega - T(\sigma)\Big). \end{aligned} \tag{40}$$

$$\text{where} \quad \begin{aligned} J(\sigma) &= J_E + \sigma J_L, \\ f(\sigma) &= f_E + \sigma f_L, \\ J(\sigma) &= T_E + \sigma T_L. \end{aligned} \tag{41}$$

In (40) m_{ac} is the air mass in the cylinder, ω is the speed of the engine shaft (measured output) and α the throttle angle (control input). (J_E, J_L), (f_E, f_L) and (T_E, T_L) are the inertias, the viscous friction coefficients and the resistive torques at the engine and load shafts, respectively. The parameter σ selects the functioning mode ($\sigma = 0$: clutch disengaged; $\sigma = 1$: clutch engaged)

The first equation provides a simple dynamic model of the air mass m_{ac} inducted per intake stroke in the cylinder. The function $m_{ss}(\omega, \alpha)$ gives the steady state air mass through the throttle to the cylinder as a function of the engine speed ω and of the throttle angle α. The time constant of the transient, denoted $\tau(\omega)$, decreases with the engine speed.

The second equation models the speed transient. The total inertia at the engine shaft is J_E when the clutch is disengaged ($\sigma = 0$) and $J_E + J_L$ when the clutch is engaged ($\sigma = 1$). The same considerations hold for the viscous friction torque coefficient $(f_E + \sigma f_L)$ and for the resistive torque $(T_E + \sigma T_L)$. The term $\beta(\omega, m_{ac})$ provides the combustion torque as a function of of the speed ω and of the air mass in the cylinder, m_{ac} (the aspired fuel mass is a function of m_{ac}).

In the simplified model (40) the time constant $\tau(\omega)$, the functions $m_{ss}(\omega, \alpha)$ and $\beta(\omega, m_{ac})$ are chosen as follows

$$\begin{aligned} &\tau(\omega) = \frac{1}{p\omega}, \qquad m_{ss}(\omega,\alpha) = (m_0 + m_1\omega)\alpha + q_0, \\ &\beta(\omega, m_{ac}) = \bar{\beta}\, m_{ac} \exp\left(-\gamma(\omega - \bar{\omega})^4\right), \end{aligned} \tag{42}$$

where the constants $p, m_0, m_1, q_0, \bar{\beta}, \gamma, \bar{\omega}$ are parameters that characterize the ICE.

It is assumed that when the clutch is engaged the mechanical load is at zero speed. Moreover, it is assumed that the clutch is sufficiently stiff to induce an instantaneous speed change on the engine shaft, thus preserving the angular velocity momentum. On the other hand, when the clutch is disengaged the engine shaft speed is equal to the load shaft speed, so that the angular velocity momentum is automatically preserved. These conditions are expressed as follows:

$$\begin{aligned} &\left.\begin{aligned}\sigma(t_j^-) &= 0, \\ \sigma(t_j) &= 1,\end{aligned}\right\} \Rightarrow \quad (J_E + J_L)\omega(t_j) = J_E\omega(t_j^-), \\ &\left.\begin{aligned}\sigma(t_j^-) &= 1, \\ \sigma(t_j) &= 0,\end{aligned}\right\} \Rightarrow \quad \omega(t_j) = \omega(t_j^-). \end{aligned} \tag{43}$$

It is also assumed that the constant load torques T_E and T_L are unknown, so that we can try to reconstruct them by modeling the resistive torque as a third state variable with null derivative (the ratio $T_E/(T_E + T_L)$ is assumed known).

Defining the state variables as follows: $x_1 = m_{ac}$, $x_2 = \omega$, $x_3 = T_E + \sigma T_L$, and the input $u = \alpha$, the hybrid model for the ICE is the following:

$$\begin{aligned} \dot{x}_1 &= -\frac{1}{\tau(x_2)}\Big(x_1 - m_{ss}(x_2, u)\Big) \\ \dot{x}_2 &= \frac{1}{J(\sigma)}\Big(\beta(x_2, x_1) - f(\sigma)x_2 - x_3\Big), \\ \dot{x}_3 &= 0, \\ y &= x_2. \end{aligned} \tag{44}$$

plus the jump function $x(t_j) = R\big(\sigma(t_j^-), \sigma(t_j), x(t_j^-)\big)$, defined, thanks to conditions (43), as

$$R(0,1,x) = \begin{bmatrix} x_1 \\ \frac{J_E}{J_E+J_L}x_2 \\ \frac{T_E+T_L}{T_E}x_3 \end{bmatrix}, \tag{45}$$

$$R(1,0,x) = \begin{bmatrix} x_1 \\ x_2 \\ \frac{T_E}{T_E+T_L}x_3 \end{bmatrix}. \tag{46}$$

Fig.'s 1 and 2 report some results on the reconstruction of the air mass in the cylinder and of the load torque.

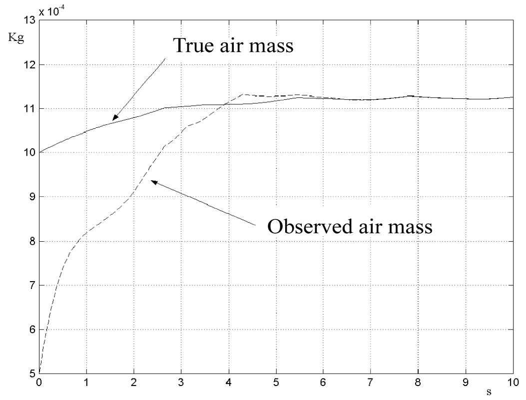

Fig. 1. Observed and true air mass in the cylinder.

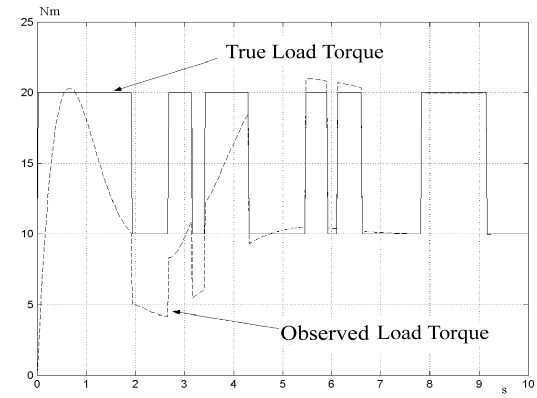

Fig. 2. Observed and true resistive load torque.

7 Conclusions

A state observer for nonlinear hybrid systems with a finite number of discrete modes is presented in this paper. The switching between different modes is commanded by an external function, known in real time. At each discrete mode the system is assumed globally drift-observable with full relative degree. The observer gain is easy to compute and the observation algorithm is easy to implement. Exponential decay of the observation error is ensured provided that there exists a minimum time interval between consecutive switches. Computer simulations on a simple model of an Internal Combustion Engine have shown good performances of the proposed observer.

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