Internal Model Controller for Chaotic Systems

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Abstract: In the last years there have been several works dealing with the solution of two particular problems: regulation and tracking of chaotic systems. In this paper, a new idea for controlling this kind of systems is proposed. Robust control has been used for minimize the effect due to disturbances or parametric uncertainties over the dynamic behavior of the controlled system, particularly over its stability characteristics. The idea of introducing an Internal Model Controller (IMC) for regulation of a chaotic system is developed here. Simulation results are also shown for the Lorenz system

Key-Words: - Chaotic Behavior, Nonlinear control, Regulation, Robustness.

1 Introduction

Recently, chaos control has been explored from several disciplines, these efforts may be classified in order to describe the nature of these methods according with [2] in: parametric perturbation methods, automatic control methods, entrainment and migration controls, external force controls and some others using intelligent computation techniques.

The technique used here may be classified into the class of automatic control techniques. This method is used mainly in order to reduce the effect of disturbances or parametric uncertainties, the so-called Internal Model Control [5] has shown to be effective solving this problem and consists basically in the inclusion of a plant model in which it is not strictly needed the exact knowledge of the parameters values, this is the internal model. A linear filter for the plant-model error and a control law stabilizing some global system are added.

The paper is organized as follows, in the next section the Internal Model Controller technique is introduced, section 3 deals with the implementation of the IMC for the Lorenz system, some results are shown section 4 and finally concluding remarks are given.

2 Internal Model Control

A design of a controller for nonlinear systems in chaotic regime is presented. This controller has an structure called internal model controller, which provides fine characteristics of performance and stability for systems under disturbances or when the values of the plant parameters are not well known, i.e. just the input-output plant response is known.

The IMC scheme has shown to be very effective under these conditions [5], and it was first developed for linear systems. The approach used here gives conditions for applying the IMC technique for continuous nonlinear systems, which are used in this paper in order to apply the method for the well-known Lorenz system.

Systems treated here are SISO and completely linearizable via coordinate transformation and state feedback. The performance of the plant converges to a constant reference under the presence of parameter uncertainties, such that the closed loop system has an asymptotically stable equilibrium point. This kind of systems do not need an explicit design of a nonlinear observer, instead states of the internal model are used.

Consider a system dynamics represented by:

$$\begin{aligned} \dot{z} &= f(z) + g(z)u \\ y &= h(z) \end{aligned} \qquad (1)$$

where $z(t) \in Z$ is the state, $Z \subset \Re^n$ and contains the origin; $u(t) \in \Re$ is the control input; f and g are smooth vector fields defined on Z, and $h: Z \rightarrow \Re$ is a smooth function. It is assumed that the origin is an equilibrium point of the autonomous system.

Let us define the Lie Derivative of a scalar function h along the vector field f as:

$$L_f h \overset{def}{=} \left(\frac{\partial h}{\partial z} \right) f$$

and

$$L_f^j h \overset{def}{=} \left(\frac{\partial L_f^{j-1}}{\partial z} \right) f \qquad (2)$$

Let us assume that (1) has strong relative degree n, i.e.,

$$L_g L_f^j h(z) = 0 \qquad (3)$$

for $j = 0,1,\ldots,n$-2 and

$$L_g L_f^{n-1} h(z) \neq 0 \qquad (4)$$

If condition (3) is satisfied, then the map $x^P = T(z) = \left(h, L_f h, \ldots, L_f^{n-1} h\right)(z)$ is a coordinate transformation [4]. System (1) may be given by its so-called normal form using the state x^P.

$$\begin{aligned} \dot{x}_i^P &= x_{i+1}^P, (i = 1,\ldots, n-1) \\ \dot{x}_n^P &= f^P\left(x^P\right) + g^P\left(x^P\right)u \\ y^P &= x_1^p \end{aligned} \qquad (5)$$

where $x^P \in X^P$=T(Z), $f^P\left(x^P\right) = L_f^n h\left(T^{-1}\left(x^P\right)\right)$ and $g^P\left(x^P\right) = L_g L_f^{n-1} h\left(T^{-1}\left(x^P\right)\right)$.

Thus, nonlinear plants described by (5) are considered here. Now let us assume that an approximate model of (5) is available

$$\begin{aligned} \dot{x}_i^M &= x_{i+1}^M, (i = 1,\ldots, n-1) \\ \dot{x}_n^M &= f^M\left(x^M\right) + g^M\left(x^M\right)u \\ y^M &= x_1^M \end{aligned} \qquad (6)$$

where $x^M = \left(x_1^M,\ldots,x_n^M\right) \in X^M \subset \Re^n, X^P \subset X^M$, f^M and g^M are smooth functions. Also let us assume that (6) has strong relative degree equal to n; *i. e.* $g^M(x^M) \neq 0$ for all $x^M \in X^M$ and $f^M(0)$=0.

Let e^{PM} be the plant-model output error:

$$e^{PM} = y^P - y^M \qquad (7)$$

and consider a linear system in order to filter this error:

$$\begin{aligned} \dot{x}_i^F &= x_{i+1}^F, (i = 1,\ldots, n-1) \\ \dot{x}_n^F &= -a^F \cdot x^F + a_1^F e^{PM}, \\ y^F &= x_1^F \end{aligned} \qquad (8)$$

where $x^F = \left(x_1^F,\ldots,x_n^F\right) \in \Re^n$ and $a^F = \left(a_1^F,\ldots,a_n^F\right) \in \Re^n$ is such that $\lambda^n + a_n^F \lambda^{n-1} + \ldots + a_1^F$ is strictly Hurwitz.

A schematic diagram of this control structure is shown in figure 1. It is assumed that the only available plant signal is its output y^P. The control goal is that the output of the plant follows a smooth signal y* which exponentially converges to a constant $\bar{y}*$.

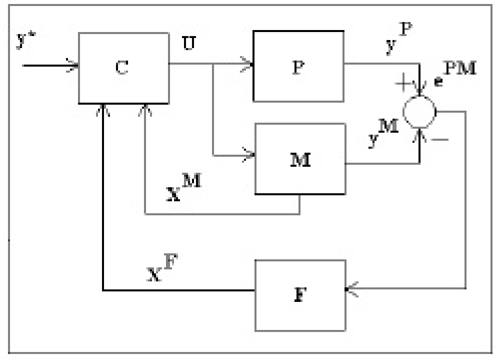

Fig. 1. Internal Model Controller

A control law that linearizes the global input output relationship is proposed now. Let us define the following auxiliary variables:

$$y_{i+1} = \frac{d^i y^M}{dt^i} + \frac{d^i y^F}{dt^i} - \frac{d^i y^*}{dt^i} \qquad (9)$$

for $i = 0,\ldots,n$-1, and a global output $y_g \equiv y_1$. Thus, the following control law is proposed [1]:

$$u = -\frac{f^M\left(x^M\right) - a^F \cdot x^F + a_1^F e^{PM} - d^n y^* / dt^n + a \cdot y}{g^M\left(x^M\right)} \qquad (10)$$

where $y = (y_1,\ldots,y_n)$ and $a = (a_1,\ldots,a_n)$ is a real vector such that $\lambda^n + a_n \lambda^{n-1} + \ldots + a_1$ is strictly Hurwitz as well. This control law linearizes the dynamics of the global output y_g in such a way that:

$$\dot{y} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_1 & -a_2 & \cdots & -a_n \end{bmatrix} y \qquad (11)$$

3 Implementation of IMC for the Lorenz System

Let us consider a forced Lorenz system [3] whose dynamics are described by:

$$\begin{aligned} \dot{x}_1^P &= \sigma(x_2^P - x_1^P) \\ \dot{x}_2^P &= \rho x_1^P - x_2^P - x_1^P x_3^P \\ \dot{x}_3^P &= -\beta x_2^P + x_1^P x_2^P + u \\ y &= x_1^P \end{aligned} \qquad (12)$$

The chaotic regime is given when σ=10, β=8/3 and ρ=28 and a projection of the attractor in the x_1-x_3 plane is shown in figure 2. The normal form, which stands for the model, is given by:

$$
\begin{aligned}
\dot{x}_1^M &= x_2^M \\
\dot{x}_2^M &= x_3^M \\
\dot{x}_3^M &= f^M\left(x^M\right) + g^M\left(x^M\right)u
\end{aligned}
\qquad (13)
$$

where

$$
\begin{aligned}
f^M\left(x^M\right) &= (\rho-1)\sigma x_2^M - (\sigma+1)x_3^M \\
&- \left(x_1^M\right)^2 (x_2^M + \sigma x_1^M) \\
&- (\beta x_1^M - x_2^M)\left[\frac{\sigma(1-\rho)x_1^M + (\sigma+1)x_2^M + x_3^M}{x_1^M}\right]
\end{aligned}
\qquad (14)
$$

and

$$
g^M\left(x^M\right) = -\sigma x_1^M \qquad (15)
$$

A projection of the model attractor in the z_1- z_2 plane is shown in figure 3. In order to observe the performance of the controller under parametric uncertainties, the parameter ρ for the model is fixed to ρ=25, which also leads to a chaotic regime in the model. The filter is proposed as a linear system as in (8) with a^F = (0.125, 0.75, 1.5). The control law in (10) is applied using vector a = (1000,300,30).

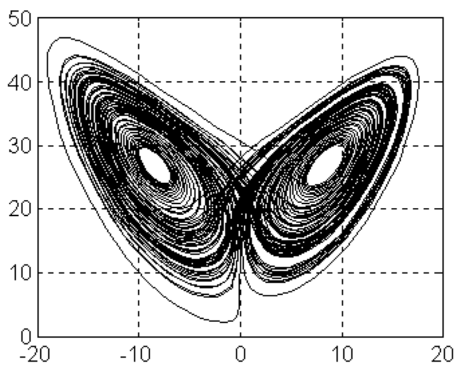

Fig. 2. Projection in the x_1- x_3 plane of the plant attractor

Finally, simulation results for the closed loop system, are shown in figure 4. The reference used here was to y^*=-5.

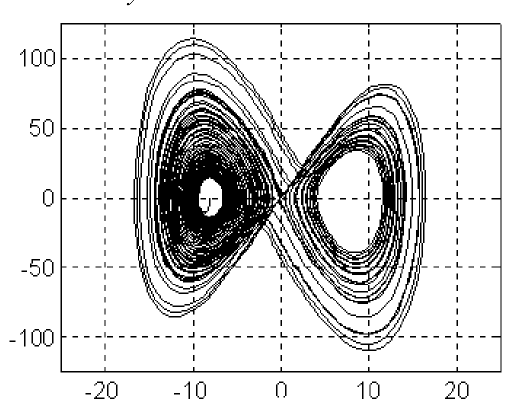

Fig. 3. Projection of the model attractor in the z_1- z_2 plane

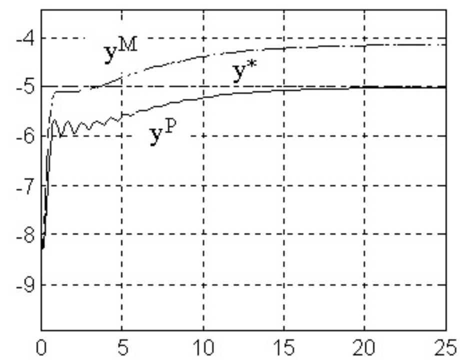

Fig. 4. Plant output converges y^P to y^*=-5

4 Concluding Remarks

An Internal Model Controller structure has been used for a class of nonlinear systems in chaotic regime. A chaotic system must be completely linearizable via coordinate transformation and state feedback, in order to allow an IMC implementation. A couple of important features of this scheme are that it does not need an explicit design of a nonlinear observer and that a difference between plant and model is tolerated.

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