Experimental Realization of Binary Signals Transmission Based on Synchronized Lorenz circuits

Cornelio Posadas-Castillo.
Faculty of Engineering Mechanic and Electrical (F.I.M.E.),
Nuevo León Autonomous University (U.A.N.L.),
Pedro de Alba s/n Cd. Universitaria San Nicolas de los Garza Nuevo León, MÉXICO.

César Cruz-Hernández and Ricardo Nuñez-Pérez.
Electronics and Telecommunications Department,
Scientific Research and Advanced Studies of Ensenada (CICESE),
Km. 107, Carretera Tijuana-Ensenada, Ensenada, B. C., 22860, MÉXICO.

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Abstract

In this paper, an experimental study of practical realization to transmit binary signals using chaos is presented. In particular, we use a Generalized Hamiltonian systems approach to synchronize two unidirectionally coupled Lorenz circuits. Experimental results are in close accordance with the simulations.

Key-Words: Synchronization, chaotic systems, Lorenz system, passivity based observers, private communication.

1 Introduction

Since Pecora and Carroll reported the discovery of synchronized chaos in [Pecora & Carroll, 1990], the possibility of secure communications using chaos has received considerable attention. Synchronization of chaotic systems has in recent years become an area of active research. Different approaches are proposed and being pursued (see, e.g., [Chua *et al.*, 1992; Kocarev *et al.*, 1992; Ogorzalek, 1993; Wu & Chua, 1993; Ding & Ott, 1994; Feldmann, *et al.*, 1996; Nijmeijer & Mareels, 1997; Special Issue, 1997, 2000; Cruz & Nijmeijer, 2000; Sira-Ramírez & Cruz, 2000; 2001; Pikovsky *et al.*, 2001] and the references therein).

Data encryption using chaotic dynamics was reported in the early 1990's as a new approach for signal encoding that differs from the conventional methods

that use numerical algorithms as the encryption key. Thereby, the synchronization of chaotic systems plays an important role in chaotic communications. In particular, several techniques, such as chaotic masking [Cuomo *et al.*, 1993], chaotic switching [Parlitz *et al.*, 1992; Cuomo *et al.*, 1993; Dedieu *et al.*, 1993] and chaotic parameter modulation [Yang & Chua, 1996] have been developed.

Our objective in this paper consists in describing an experimental study of practical realization to transmit binary signals based on synchronized chaotic systems. In particular, we use a Generalized Hamiltonian systems approach developed in [Sira-Ramírez & Cruz, 2000; 2001] to synchronize two unidirectionally coupled Lorenz circuits. We enumerate several advantages over the existing methods:

- it enables synchronization be achieved in a systematic way and clarifies the issue of deciding on the nature of the output signal to be transmitted;

- it can be successfully applied to several well-known chaotic oscillators;

- it does not require the computation of any Lyapunov exponent;

- it does not require initial conditions belonging to the same basin of attraction.

This paper is organized as follow. In Section 2, we present the system description to transmit binary information by chaotic switching. In Section 3, we obtain the synchronization of two Lorenz systems through a Generalized Hamiltonian systems approach. In Section 4, we present the stability analysis related to the synchronization process. In Section 5, we show the physical system implementation and experimental results: 5.1 synchronization and 5.2 binary communication. Finally, in Section 6, we give some concluding remarks.

2 System description

In this section, we describe the communication system based on synchronized chaos and its operating principles. Figure 1 shows a block diagram to transmit binary messages by chaotic switc We will use the Lorenz system as chaos generator. The state equations are given by [Lorenz, 1963]

$$
\begin{aligned}
\dot{x}_1 &= \sigma\left(x_2 - x_1\right), \\
\dot{x}_2 &= rx_1 - x_2 - x_1 x_3, \\
\dot{x}_3 &= x_1 x_2 - bx_3,
\end{aligned}
\tag{1}
$$

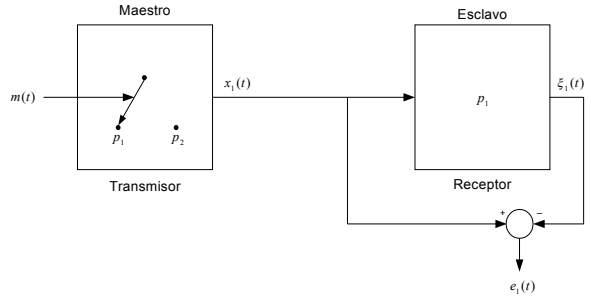with $\sigma = 10$, $r = 28$, and $b = 8/3$ the Lorenz system exhibits chaotic behavior.

Figure 1: Block diagram to transmit binary messages by chaotic switching.

For communication purposes here we apply the Hamiltonian synchronization of Lorenz circuits to chaotic switching. In this technique, the message $m(t)$ is supposed to be a binary message, and is used to modulate one or more parameters of the (switching) transmitter, i.e. $m(t)$ controls a switch whose action changes the parameter values of the transmitter. Thus, according to the value of $m(t)$ at any given time t, the transmitter has either the parameter set value p or the parameter set value p'. At the receiver, $m(t)$ is decoded by using the synchronization error to decide whether the received message corresponds to one parameter value, or the other (it can be interpreted as an one or zero). In the next section we describe the Generalized Hamiltonian systems approach to synchronize two unidirectionally coupled Lorenz circuits. Synchronization is thus between the transmitter and the dynamics with the receiver being given by an observer.

3 Synchronization of Lorenz system

Consider the following n-dimensional autonomous system

$$
\dot{x} = f(x), \qquad x \in \mathbb{R}^n,
\tag{2}
$$

which represents a circuit exhibiting a *chaotic* behavior. Following the approach provided in [Sira-Ramírez & Cruz, 2001], many physical systems described by Eq. (2) can be written in the following "Generalized Hamiltonian" Canonical form,

$$
\dot{x} = \mathcal{J}(x)\frac{\partial H}{\partial x} + \mathcal{S}(x)\frac{\partial H}{\partial x} + \mathcal{F}(x),
\tag{3}
$$

where $H(x)$ denotes a smooth *energy function* which is globally positive definite in $\mathbb{R}^n$. The column *gradient vector* of H, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We frequently use quadratic energy function $H(x) = 1/2\ x^T \mathcal{M} x$ with $\mathcal{M}$ being a,

constant, symmetric positive definite matrix. In such case, $\partial H/\partial x = \mathcal{M} x$. The square matrices, $\mathcal{J}(x)$ and $\mathcal{S}(x)$ satisfy, for all $x \in \mathbb{R}^n$, the following properties, which clearly depict the *energy managing* structure of the system, $\mathcal{J}(x) + \mathcal{J}^T(x) = 0$ and $\mathcal{S}(x) = \mathcal{S}^T(x)$. The vector field $\mathcal{J}(x) \, \partial H/\partial x$ exhibits the *conservative* part of the system and it is also referred to as the *workless* part, or *work-less* forces of the system; and $\mathcal{S}(x)$ depicting the *working* or *nonconservative* part of the system. For certain systems, $\mathcal{S}(x)$ is *negative definite* or *negative semidefinite*. In such cases, the vector field is addressed to as the *dissipative* part of the system. If, on the other hand, $\mathcal{S}(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global and local *destabilizing* part of the system. In the last case, we can always (although nonuniquely) descompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite matrix $\mathcal{R}(x)$ and a symmetric positive semidefinite matrix $\mathcal{N}(x)$. And where $\mathcal{F}(x)$ represents a *locally destabilizing* vector field.

We consider a special class of Generalized Hamiltonian systems with destabilizing vector field and linear output map , y, given by

$$\dot{x} = \mathcal{J}(y)\frac{\partial H}{\partial x} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial x} + \mathcal{F}(y), \quad x \in \mathbb{R}^n,$$
$$y = \mathcal{C}\frac{\partial H}{\partial x}, \quad y \in \mathbb{R}^m, \tag{4}$$

where $\mathcal{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathcal{I}$ is a constant skew symmetric matrix. The vector variable y is referred to as the system *output*. The matrix $\mathcal{C}$ is a constant matrix.

We denote the *estimate* of the state vector x by ξ, and consider the Hamiltonian energy function $H(\xi)$ to be the particularization of H in terms of ξ. Similarly, we denote by η the estimated output, computed in terms of the estimated state ξ. The gradient vector $\partial H(\xi)/\partial \xi$ is, naturally, of the form $\mathcal{M}\xi$.

A dynamic nonlinear state observer for the system (4) is readily obtained as

$$\dot{\xi} = \mathcal{J}(y)\frac{\partial H}{\partial \xi} + (\mathcal{I} + \mathcal{S})\frac{\partial H}{\partial \xi} + \mathcal{F}(y) + K(y - \eta),$$
$$\eta = \mathcal{C}\frac{\partial H}{\partial \xi}, \tag{5}$$

where K is a constant vector, known as the *observer gain*.

The state estimation error, defined as $e = x - \xi$ and the output estimation error, defined as $e_y = y - \eta$, are governed by

$$\dot{e} = \mathcal{J}(y)\frac{\partial H}{\partial e} + (\mathcal{I} + \mathcal{S} - K\mathcal{C})\frac{\partial H}{\partial e}, \quad e \in \mathbb{R}^n,$$

$$e_y = \mathcal{C}\frac{\partial H}{\partial e}, \quad e_y \in \mathbb{R}^m, \tag{6}$$

where the vector, $\partial H/\partial e$ actually stands, with some abuse of notation, for the gradient vector of the *modified* energy function, $\partial H(e)/\partial e = \partial H/\partial x - \partial H/\partial \xi = \mathcal{M}(x - \xi) = \mathcal{M}e$. Below, we set, when needed, $\mathcal{I} + \mathcal{S} = \mathcal{W}$. We say that the receiver system (5) synchronizes with the transmitter system (4), if $e(t) \to 0$ as $t \to \infty$.

Lorenz system. To synchronize the Lorenz circuit using the Generalized Hamiltonian systems approach we the Eq. (1). The set of state equations describing Lorenz system in Hamiltonian Canonical form with a destabilizing field (**master**) is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -x_1 \\ 0 & x_1 & 0 \end{bmatrix} \frac{\partial H}{\partial x} \tag{7}$$
$$+ \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial x} + \begin{bmatrix} 0 \\ rx_1 \\ 0 \end{bmatrix},$$

taking as the Hamiltonian energy function the scalar function

$$H(x) = \frac{1}{2}\left[\frac{1}{\sigma}x_1^2 + x_2^2 + x_3^2\right]. \tag{8}$$

The destabilizing vector field evidently calls for $y = x_1 = \begin{bmatrix} \sigma & 0 & 0 \end{bmatrix} \partial H/\partial x$ to be used as the output of the master (7). The matrices $\mathcal{C}$, $\mathcal{S}$ and $\mathcal{I}$, are given by

$$\mathcal{C} = \begin{bmatrix} \sigma & 0 & 0 \end{bmatrix}, \quad \mathcal{S} = \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix},$$
$$\mathcal{I} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The pair of matrices $(\mathcal{C}, \mathcal{S})$ already constitutes a pair of detectable, but nonobservable, matrices. Even though the addition of the matrix $\mathcal{I}$ to $\mathcal{S}$ does not improve the lack of observability, the pair $(\mathcal{C}, \mathcal{W}) = (\mathcal{C}, \mathcal{S} + \mathcal{I})$ remains, nevertheless, detectable. In this case, the dissipative structure of the system is fully "damped" due to the negative definiteness of the matrix $\mathcal{S}$. Then, there is no need for an output estimation error injection for complementing, or enhancing, the system's natural dissipative structure. The **slave** is designed as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -x_1 \\ 0 & x_1 & 0 \end{bmatrix} \frac{\partial H}{\partial \xi}$$

$$+ \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial \xi} \qquad (9)$$

$$+ \begin{bmatrix} 0 \\ rx_1 \\ 0 \end{bmatrix},$$

and the synchronization error is therefore governed by the globally asymptotically stable system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -x_1 \\ 0 & x_1 & 0 \end{bmatrix} \frac{\partial H}{\partial e}$$
$$+ \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial e}.$$

If the negative real part of the observable eigenvalues, related to the constant dissipation structure matrix of the error dynamics, must be enhanced, one can still use the above observer but now including an output reconstruction error injection term. The resulting observer (**slave**) is given by

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma & 0 \\ -\frac{1}{2}\sigma & 0 & -y \\ 0 & y & 0 \end{bmatrix} \frac{\partial H}{\partial \xi}$$
$$+ \begin{bmatrix} -\sigma^2 & \frac{1}{2}\sigma & 0 \\ \frac{1}{2}\sigma & -1 & 0 \\ 0 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial \xi} \qquad (10)$$
$$+ \begin{bmatrix} 0 \\ ry \\ 0 \end{bmatrix} + \begin{bmatrix} K_1 \\ K_2 \\ K_3 \end{bmatrix} e_1.$$

The asymptotically stable reconstruction error dynamics is then governed by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2}\sigma(1+K_2) & \frac{1}{2}\sigma K_3 \\ -\frac{1}{2}\sigma(1+K_2) & \frac{1}{2}\sigma K_3 & -y \\ -\frac{1}{2}\sigma K_3 & y & 0 \end{bmatrix} \frac{\partial H}{\partial e} \quad (11)$$
$$+ \begin{bmatrix} -\sigma(\sigma+K_1) & \frac{1}{2}\sigma(1-K_2) & -\frac{1}{2}\sigma K_3 \\ \frac{1}{2}\sigma(1-K_2) & -1 & 0 \\ -\frac{1}{2}\sigma K_3 & 0 & -b \end{bmatrix} \frac{\partial H}{\partial e}.$$

One may now specify the values of k_i, $i = 1, 2, 3$ in order to guarantee a faster asymptotic stability to zero of the state reconstruction error trajectories.

4 Synchronization stability analysis

In this section, we examine the stability of the synchronization error (11) between the Lorenz system (7)

and observer (10). A necessary and sufficient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

Theorem 1 (Sira-Ramírez and Cruz, 2001)
The state x of the nonlinear system (7) can be globally, exponentially, asymptotically estimeted, by the state ξ of the observer (10) if and only if there exists a constant matrix K such that the symmetric matrix

$$[\mathcal{W} - K\mathcal{C}] + [\mathcal{W} - K\mathcal{C}]^T = [\mathcal{S} - K\mathcal{C}] + [\mathcal{S} - K\mathcal{C}]^T$$
$$= 2\left[\mathcal{S} - \frac{1}{2}\left(K\mathcal{C} + \mathcal{C}^T K^T\right)\right]$$

is negative definite.

5 Physical system implementation and experimental results

In this section, we describe the physical implementation of the synchronization and communication schemes. Figure 2 shows the analog circuit implementation of the Lorenz equations (7)

Figure 2: Analog circuit implementation of the Lorenz system.

5.1 Synchronization

The experimental set-up of Generalized Hamiltonian systems approach to synchronize the Lorenz circuits is shown in Figure (3). The basic idea of this scheme is as follows: a voltage $\pm 15\ Vdc$. is injected into the Lorenz circuit (transmitting system) which modifies

the voltage x_1. This voltage x_1 is then used as driving signal on the Lorenz circuit (receiving system) across $I.C.\ AD734$. The signal error $e_1 = x_1 - \xi_1$, is injected into the block of $U8$. The choice of gains $k_1 = 50$ and $k_2 = k_3 = 0$ was used. Figure 4 shows
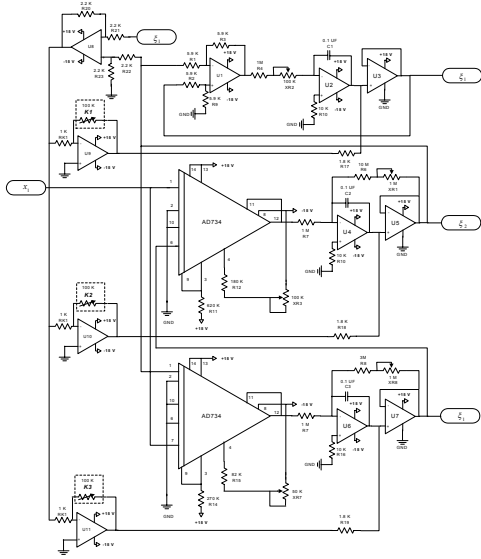

Figure 3: Experimental set-up of Generalized Hamiltonian systems approach to synchronize the Lorenz circuit.

the quality of synchronization between the transmitter and receiver. The horizontal axis corresponds to x_1 of the transmitter and the vertical axis to ξ_1 of the receiver.
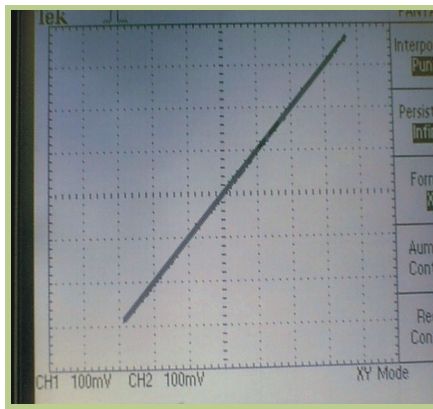
Figure 4: Synchronization between the master and slave: x_1 of the master versus ξ_1 of the slave.

5.2 Binary communication

The experimental set-up to transmit binary signals by chaotic switching is shown in Figure 5. The basic idea of this scheme is as follows: We generate two chaotic attractors of the Lorenz circuit (Eq. (1)), utilizing a square signal of $7\,Vdc.$ and $3\,Vdc.$ With these values, the Lorenz circuit exhibits two different but qualitatively similar chaotic attractors (to encoding "1" or "0"). The Figure 6 shows the transmission of
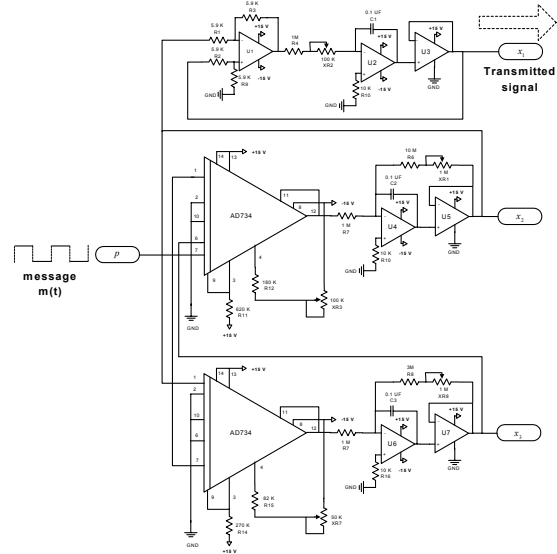

Figure 5: Experimental set-up to transmit binary signals by chaotic switching.

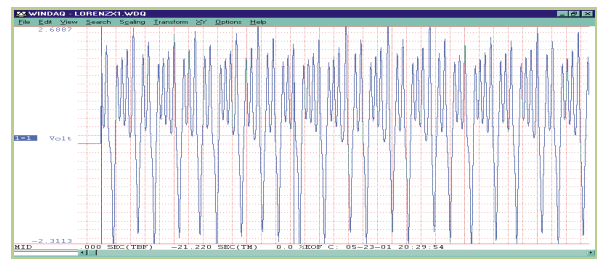an alternating sequence of "1" and "0" (transmitted signal x_1). The Figure 7, top of figure: shows binary

Figure 6: Transmitted signal x_1.

signal to be transmitted. Bottom of figure shows the original information is recovered by synchronization error detection at the receiver.

6 Concluding remarks

In this paper, we have developed an experimental study of practical realization to transmit binary sig-
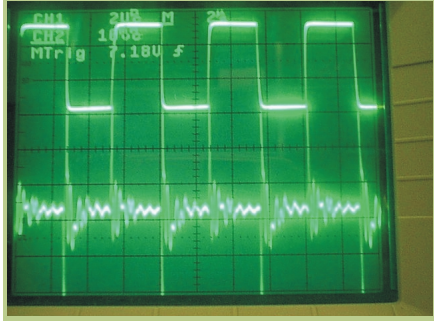
Figure 7: Top of figure: Binary signal to be transmitted. Bottom of figure: Synchronization error detection $e_1 = x_1 - \xi_1$.

nals using synchronized chaos. Synchronization of two Lorenz circuits was obtained from the perspective of Generalized Hamiltonian systems including dissipation and destabilizing terms. The approach allows to give a simple design procedure for the receiver system and clarifies the issue of deciding on the nature of the output signal to be transmitted. We have shown that the experimental results are in close accordance with the simulations.

In a forthcoming article we will be concerned with a physical implementation of the method combining with conventional cryptographic schemes to enhance the security of low-dimensional chaos-based secure communication systems.

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