# A Dynamical System Approach to the Simulation of Coach Passenger Variation

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*Abstract:* - In this paper, we for the first time propose a dynamical system (DS) approach to describe the time variation of the coach travel. This dynamical study for the analysis and modeling is of several advantages. First, it can be used for dynamical analysis, such as quantitative, stability, and equilibrium analysis. Second, it can be further applied to analyze the competition behaviors among various operators in the coach industry. Under some proper assumptions, the ordinary differential equation (ODE) model is formulated together with the collected data to describe the passenger variation. We solve this first order ODE with an analytical technique, and find the result fits the collected data. Some concluding remarks and future works are also suggested in this work.

*Key-Words:* - DS, ODE, Deregulation, Scheduled Express Service

## 1 Introduction

The problem relating to bus and coach industry usually is rather complicated. Traditionally, the methods of qualitative social science, statistics and economics are often applied to study these kinds of issue. For example, the effect of bus and coach deregulation has been largely discussed during the last two or more decades by using the method of qualitative social science (see [1-5] and references therein), statistics [6] and economics [7-13]. In fact, to apply the dynamical model to describe this kind of problems could be a feasible alternative. Dynamical system has long been successfully used in the various areas, including biology [14-15], economics [15-17], semiconductor [18-19] and others. In our experience, we have reviewed many literatures, such as SCI, SSCI and IEEE databases, and haven't found any literature used the dynamical modeling method to study the time variation of bus and coach travel so far. The purpose of this study is to consider the effect of the competitive market for commercial coach services six years after the start of deregulation in 1995 in Taiwan. We will study the scheduled express services operated on the route of the Taipei - Hsinchu corridor and focus on the travel trend of the public-own incumbent operator – Taiwan Motor Transport Company (TMTC), which was affected to a great extent by the other new entrants entering the coach market at different time after deregulation. Metropolitan Taipei is the political and economic center of Taiwan, and Hsinchu is the high-tech city about 80*km* to the south of Taipei. There are large-scale journey between these two cities, therefore the market opened up for coach competition in 1995. The coach travel of TMTC was affected by many factors. Some are exogenous (uncontrollable), such as the aggregate trend of the use in the public transportation, the number and the entry time of new entrants, the overall market frequencies, fare level and service quality. Others are endogenous (controllable), for example, the organization reform, the occupancy of frequency, accessibility and service quality as shown in Table 1. Under some simplified assumptions, this paper applies the first order differential equation to model, describe and solve these complicated phenomena.

This paper is organized as follows. In Section 2, we present the assumptions and construct the model. In Section 3, we state the solving method. In Section 4, we clarify the solution to the equation and relating analysis, then compare the solution to our collected data. Section 5 draws the conclusion.

## 2 A Dynamical Model for Passenger Trips Growth

### 2.1 Problem description

The purpose of this study is to consider the effect of the competitive market for commercial coach services six years after the start of deregulation in 1995 in Taiwan. We begin by examining the scheduled express services offered on the route of the Taipei - Hsinchu corridor and focus on the travel trend relating to the incumbent operator TMTC.

Table 1. Analysis of the factors affecting the Taipei-Hsinchu passenger trips of TMTC after deregulation.

| Type of Factors | Factors | Discription |
|---|---|---|
| Exogenous Factors | 1. Increasing use in public transportation<br>2. New entrants<br>3. Frequencies<br>4. Fares<br>5. Accessibility<br>6. Service quality | · Due to the parking difficulty and driver fatigue problems, there were increasing use in public transportation.<br>· A consortium of two independent operators entered firstly the market in Oct. 1996.<br>· The third independent operator entered in Feb. 1997.<br>· The fourth independent operator entered in Apr. 1999.<br>· The overall frequencies of the four new entrant accounted for 28% of the whole market from Oct. 1996 up to 83% by Jan. 2001.<br>· The use of various commercially discounted tickets was generally cheaper and more convenient, especially for students who account for large ratio of travel.<br>· New designed routes and pick-up/set-down points between termini had greater accessibility.<br>· New fleet.<br>· Lower seating densities (with 19-35 seats). |
| Endogenous Factors | 1. Organization reform<br>2. Frequencies<br>3. Fares<br>4. Service quality | · TMTC narrow-downed the organization and downsized employees before and after deregulation in order to avoid loss-making<br>· The frequencies of TMTC accounted for 72% of the whole market from Oct. 1996 and then declined sharply to 17% by Jun. 2001.<br>· The discount rate of the commercially discounted tickets are normally higher than new entrants.<br>· Student tickets were not available.<br>· Ageing fleet.<br>· Higher seating densities (with 44 seats). |

Source: [1], [20]

From our collected data provided by Department of Transportation and Communications, we notice that there exists a strange phenomenon in between, that is, total trips rose continuously by June 2001, while a steady decline occurred in incumbent operator's passenger trips simultaneously. This can contribute to a lot of factors, including exogenous and endogenous as mentioned in Sec. 1. If we want to use a model to describe this kind of phenomenon, it would be quite complicated. The main reason is that there were many new entrants entered the coach market at different entry time. For this reason, in this paper we would like to apply simple model and merely from the points of view of the incumbent itself to analyze the impact of new entrants brought to incumbent operator during different time periods, i.e., by constructing a single dynamical model to describe the decline trend of incumbent's passenger trips. Suppose that $x = x(t)$ represents the number of passenger trips of the incumbent operator at time $t$. Then for competition, management, and other practical reasons it may be desirable to predict future values of $x$. Knowing the function relating $x$ to $t$ would provide this capability. However, if we start with an assumption relating the rate of change of $x$ with respect to $t$ to the dependent variable $x$, then we do not know immediately the functional relationship between $x$ and $t$. Instead we must determine the function $x(t)$ subject to the condition that $x(0) = x_0$ is the starting passenger trips. We now carefully examine this coach travel trend problem.

According to our observation from collected data, the factors affecting the growth of passenger trips in coach services are similar to those of found by [1]. But further analysis shows that there are two more key factors involved. One is the increasing use of public transportation, which would probably influence the coach trips. The other is that whenever a new entrant entered the market with higher frequencies, lower fare and quality improvement, there would be a significant impact to the market structure, especially to the incumbent operator's passenger trips in the study route. This is the reason why we will focus this study from the points of view of the incumbent operator itself. In addition, we have to introduce time factor into our modeling process, since most of the factors affecting the passenger trips are of time variation.

## 2.2 Assumptions

For the DS model, it would be difficult to keep the mathematical model solvable and still consider many factors. In this study we consider a simple model first. Initially we will assume that (1) the increasing use in public transportation, and the number and the entry time of the new entrants as the two dominant factors affecting the passenger trips of the incumbent operator, and (2) the pre-entry action and post-entry reactions of the incumbent and new entrants are operator is neglected. Then we will model both the increasing use in public transportation and the number and the entry time of the new entrants.

Let Δx denote the change in incumbent passenger trips during a given time interval $[t, t+\Delta t]$ in the study corridor:

$$\Delta x = x(t+\Delta t) - x(t). \tag{1}$$

Although continuous functions may not desirable actual coach travel in strictest sense, they do provide sufficiently small values of $\Delta t$, a certain variation of the passenger trips is produced during the interval $[t+\Delta t]$.

## 2.3 Model Formulation

If we let $k > 0$ be a constant that represents the variation rate of the passenger trips per unit time, we have the following model:

$$x(t+\Delta t) = x(t) + kx(t)\Delta t. \tag{2}$$

Note that since *k* is a variation rate per unit time, we have multiplied *k* by both *x* and Δ *t*. Applying the definition (1) and dividing Δ *x* by Δ *t* in Eq. (2), we obtain the average time rate of change of *x* over any interval $[t+\Delta t]$:

$$\Delta x / \Delta t = kx. \tag{3}$$

Eq. (3) states that the average change $\Delta x / \Delta t$ is proportional to the amount present *x*. At this point we need to ask whether it makes sense in the physical situation to allow Δ *t* to become arbitrarily small. Even though many passenger trips vary in discrete time periods, a reasonable approximation to Eq. (3) in some instances can be obtained by using the continuous model:

$$dx / dt = \lim \Delta x / \Delta t = kx, \tag{4}$$

where $dx / dt$ represents the instantaneous rate of change *x* with respect to *t*.

## 2.4 Empirical Study

Eq. (4) is the basic formulation of our model, this model is so-called the first order ODE. The complication of the model lies in many affecting factors incurred at different point of time. These factors, including exogenous and endogenous, are shown in Table 1. The data period is from Nov. 1995 to Jan. 2001, i.e. one year before deregulation and six years after deregulation. The items include passenger trips, frequencies and fares of the TMTC and other new entrants. According to the table, we find that the increasingly use in public transportation and the new entrants are the two key dominant factors. Then we can combine the model and these two dominant factors and present a dynamical equation to describe the variation of the passenger trips of the incumbent operator over the study period as follows.

# 3 Analytical Solution of the Model

The General Linear Equation can be written as

$$\frac{dx}{dt} - k(t)x = f(x). \tag{5}$$

Firstly, let *f* (*x*) = 0 to simply the derivation process. Next, let us multiply both sides of the equation by some function *u* (*x*) so that the left side is the derivation of the product *ux*. That is

$$\begin{aligned} u(t)\left[\frac{dx}{dt} - k(t)x\right] &= \frac{d}{dt}[u(t)x] \\ &= u(t)x' + u'(t)x \end{aligned} \tag{6}$$

The function *u*(*t*) is not yet known, but from the last equation it must satisfy

$$u(t) \cdot [-k(t)]x = u'(t)x. \tag{7}$$

or

$$\frac{u'(t)}{u(t)} = -k(t). \tag{8}$$

We seek only one function *u*(*t*) in our procedure, so assume that *u*(*t*) is positive over the interval. Then integrating Eq. (8) gives us

$$\ln u(t) = \int -k(t)dt. \tag{9}$$

or, exponentiating both sides

$$u(t) = e^{-\int k(t)dt}. \tag{10}$$

That is, Eq. (10) determines precisely a function *u*(*t*) that will work for our procedure. Note that *u*(*t*) defined by Eq. (10) is indeed positive. The function *u*(*t*) is called an integrating factor for the general linear first-order Equation. Now multiplying Eq. (5)

through by the integrating factor (10) results in

$$u(t)\left[\frac{dx}{dt}-k(t)x\right]=u(t)f(t)\,. \tag{11}$$

Since $f(t) = 0$, Eq.(11) then becomes

$$u(t)\left[\frac{dx}{dt}-k(t)x\right]=0\,. \tag{12}$$

In order to solve Eq. (12), simply integrate both sides:

$$u(t)x = c\,. \tag{13}$$

or

$$x=\frac{1}{u(t)}\cdot c\,. \tag{14}$$

From Eq.(14), the solution is

$$x = e^{\int k(t)dt}$$

Then the dynamical equation derived in previous section 2 can be solved by using Eq.(6) as following:

$$\dot{x}=\begin{cases}-0.0025x, & 0<t\le 10\\ -0.06x, & 10<t\le 15\\ -0.03x, & 15<t\le 30\\ 0.015x, & 30<t\le 40\\ -0.015x, & 40<t\le 60\\ 0, & 60<t\end{cases},\quad x=\begin{cases}162, & t=0\\ 158, & t=10\\ 177, & t=15\\ 75, & t=30\\ 87, & t=40\\ 64, & t=60\end{cases} \tag{15}$$

The estimated coefficients; i.e., $k$ value, of the above equations and the initial value are all derived from our collected data.

The $k$ values represent the rates of the growth and decline concerning the travel trend of the TMTC, and can be plotted as Fig. 1.

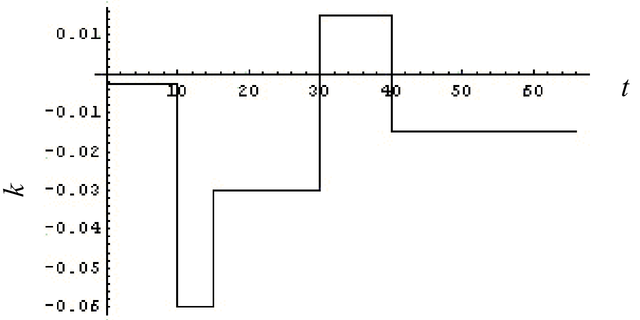

Fig. 1 The cause and effect plot concerning the travel trend of TMTC

There still needs a lot of detailed discussion concerning the model. And the model will be formulated more realistic further. Next the solution to the equation as shown in Fig. 2 and it's behavior will be discussed and compared to our collected data.

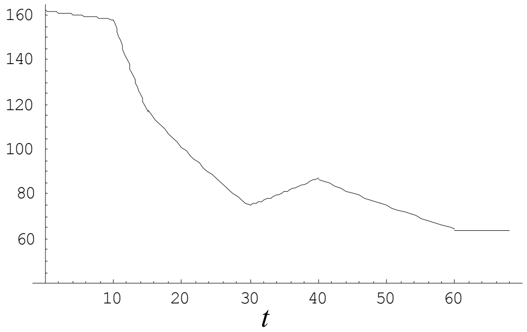

Fig. 2 A plot of the solved solution for the passenger trips variation problems

The $t$ in Fig. 2 represents a dimensionless time scale, we study this model problem after Nov. 1995. To model the problem with proper time scale, we define the time scale $t$ as dimensionless variable.

## 4 Results and Discussions

To explore the cause and effect represented by $k$ value in Fig. 1, we have outlined a number of exogenous and endogenous factors and shown in Table 1. For more convenient discussion, some assumptions have been made further in section 2. We then will discuss firstly two key factors influencing the travel trend of the TMTC, i.e. the variation around aggregate trend and the new entrants in this paper and study other important factors in future works.

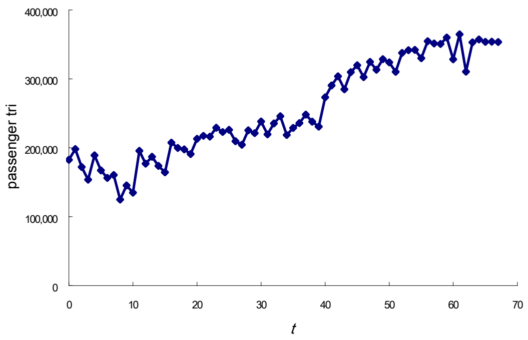

Fig. 3 Travel Trend in the Study Route

Fig. 3 summarizes the aggregate trend in the study route, total trips rose substantially by about 2 times by June 2001. This was associated with increasing use in public transportation, the entry of new entrants, together with higher frequencies, price competition (the use of various commercially discounted tickets), faster timings making better use of the national freeway, greater accessibility due to convenient stops, quality improvements such as new-fleet and lower seating densities, and an increased public awareness of coach travel. However, from the point of view of the TMTC, most of the innovations mentioned above could be regarded as the exogenous factors (Table 1), which led significant impact to the passenger trips of the TMTC. The growth pattern of the aggregate trend is strictly increased function of time. In effect, the growth in aggregate trend seems to have had faster pace than the growth derived from new entrants, and hence this provided as a positive factor to the increase of the TMTC passenger trips. However, as shown in the 30 $< t \leq 40$ time of period, the market force brought by new entrants increasingly reduced the effect of $k$ value, therefore the positive impact maintain just a short period of time.

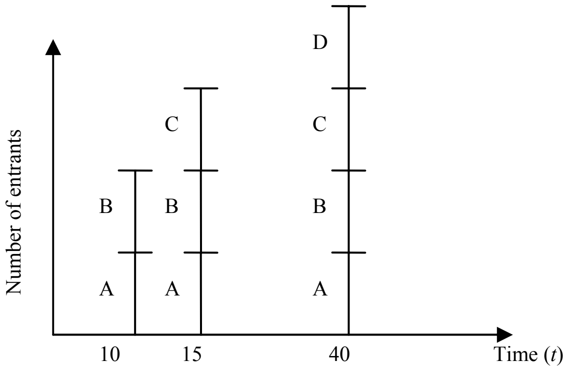

Fig. 4 The number of entrants and the entry time

Fig. 4 outlines the number of new entrants and their entry point of time. Before coach deregulation in 1995, TMTC was the monopolizer in Taipei - Hsinchu route. From Oct. 1996 after deregulation, a rival consortium of two independent (private) operators, termed as A & B bus companies, introduced competing services on the Taipei - Hsinchu route, and was able to provide the alternative coach service other than TMTC. The A & B bus companies, traditionally a local bus operator in the Taipei and Hsinchu area respectively, were setting up a entirely new service but almost parallel route (by way of the number 1 national freeway but more convenient stops in the two cities) with that of the TMTC. They matched higher frequency of service that TMTC was unable to offer, then issued various commercially discounted tickets (especially for students who account for large ratio of the overall trips), and used new fleet with lower seating densities. To judge from the travel pattern in Fig. 2, the A & B companies took a greater role on the Taipei - Hsinchu route and brought about the most significant impact to TMTC. As for the C Company, which entered the market in Feb. 1997, was setting up a new route (by way of the number 2 national freeway) and a different terminal (in the east district of Taipei City different from those of the A & B Company and TMTC) from the point of view of market segment. C company thus exercises remarkable dominance in the deregulated market, as shown in Fig. 2. The D company began their coach service and paralleled with A & B route in Apr. 1999. Despite the market tendency of the use of commercially discounted tickets, the D company offered luxury coaches with lowest seating densities but highest fare to attract high-income travelers who emphasize the in-vehicle comfort rather than the price. The overall frequency of the four new entrants accounted for 28% of the whole market from Oct. 1996 up to 83% by Jun. 2001, and the overall occupancy of passenger trips from 19% to 74%. Obviously, the extent of the inter-operator coach competition was more than expected, and the dramatic impact to TMTC has been seen.

During the early 1990s, the nationwide public-own TMTC was the only coach operator in the study route and suffered a long-period loss making. As shown in Fig. 5, about one year before deregulation in 1995, the TMTC passenger trips had been declined slowly. After deregulation, a steady decline had occurred, just 80 thousand trips in the end of 1998 itself. The first half of 1999 saw a growth, peaking at about 86 thousand trips in Apr. 1999, and then subsequently falling to 67 thousand trips in the end of 2000, and 64 thousand trips in Jun. 2001. The decline trend was largely explained by the exogenous and endogenous factors as shown in Table 1. The former had been mentioned in the aggregate trend above. The latter have occurred not only due to the worsening policy-making (roistering and scheduling concerning frequency) but also because of lack of innovation (discriminating pricing). And the peak in Apr. 1999 could have been the effect of the positive factor produced by the growth in aggregate trend mentioned above.

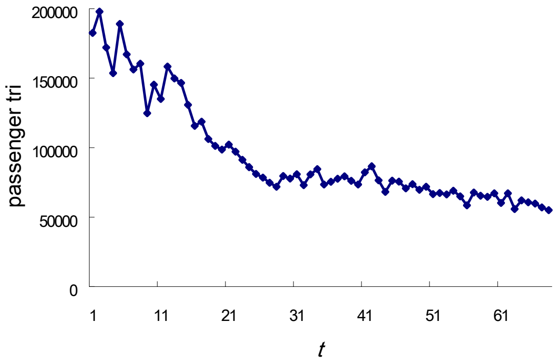

Fig. 5 The trend of the TMTC passenger trips (Nov. 1995-Jun. 2001)

According to our observation from Figs. 3, 4, 5 and relating analysis, a detail had been obtained to clarify the cause and effect about the travel trend of the TMTC showed in Fig. 2.

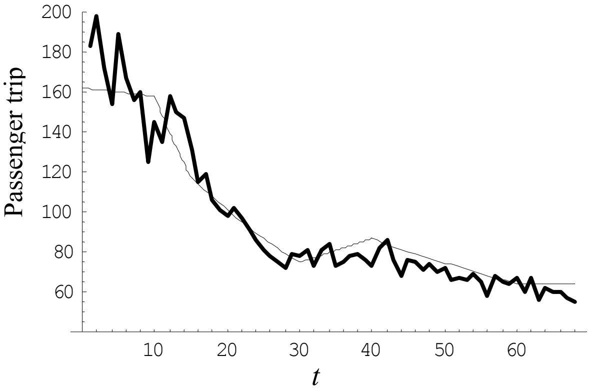

Fig. 6 Comparison between collected historical data and calculated result (The thin line indicates the calculated result and the thick line indicates the collected historical data)

As shown in the Fig. 6, these is a very good agreement between the calculated result and historical data, this indicates that the dynamical model for passenger trips growth can describe the phenomena completely under the simplified assumptions.

## 5 Conclusions

In this paper, based on the dynamical system theory, the passenger trips variation has been modeled and solved systematically. To the best of our knowledge, this approach for the first time has provided a novel alternative for studying the dynamic of the passenger trip variation successfully. Compared with the collected historical data in Taiwan, the computer solution of the model has a good agreement in the passenger trips variation over the past years. However, the assumption applied in this model are rather simple, to model more complicated transportation phenomena, some exogenous and endogenous factors should be taken into account in the future works.

*References:*

[1] P. White, “The Experience of Bus and Coach Deregulation in Britain and in other countries”. International Journal of Transport Economics, Vol XXIV-No. 1, Feb. 1997. pp. 35-52.

[2] D. K. Kobbins and P. R. White, “The Experience of Express Coach Deregulation in Great Britain”. Transportation 13: 359-384, 1986.

[3] P. White, “Deregulation of Local Bus Services in Great Britain: An Introductory Review”. Transport Review, Vol. 15, No.2, pp. 185-209, 1995.

[4] G. Alexandersson et al, “The Effects of Competition in Swedish Local Bus Services”. Journal of Transport Economics and Policy, Vol.32, Part 2, pp. 203-219, 1997.

[5] D. Banister, “Competition Regimes with the European Bus Industry: Theory and Practice”. Transportation Research, Vol. 26A, No.2, pp. 167-178, 1992.

[6] M. Fairhurst and D. Edwards, “Bus Travel Trends in the U. K.”. Transport Reviews, Vol. 16, No. 2, pp. 157-167, 1996.

[7] M. L. Manheim, “Fundamentals of Transportation Systems Analysis”. Vol. 1&2, The MIT Press, 1979.

[8] P. Mackie et al, “Bus Deregulation: Ten Years On”. Transport Reviews, Vol. 15, No. 3, pp. 229-251, 1995.

[9] R. Darbera, “Deregulation of Urban Transport in Chile: What have we learned in the decade 1979-1989?”, Transport Reviews, Vol. 13 No. 1 pp. 45-49, 1993.

[10] A. Evans, “Hereford: A Case Study of Bus Deregulation”. Journal of Transport Economics and Policy, pp. 283-306, Sep. 1988.

[11] A. Evans, “Competition and the Structure of Local Bus Markets”. Journal of Transport Economics and Policy, pp. 255-281, Sep. 1990.

[12] A. Evans, “A Theoretical Comparison of Competition with other Economics Regimes for Bus Services”. Journal of Transport Economics and Policy, pp. 7-36, Jan. 1987.

[13] S. Glaister, “Competition on Urban Bus Route”, Journal of Transport Economics and Policy, pp. 65-81, Jan. 1985.

[14] A. T. Bradshow and L. L. Moseley, “Dynamics of Competiting Predator- Prey Species”, Physica A, 261, pp. 107-114, 1998.

[15] P. N. V. Tu, “Dynamical System: An Introduction with Applications in Economics and Biology”. Springer- Verlag, 1994.

[16] G. Feichtinger, “Limit Cycles in Dynamic Economic Systems”. Annals of Operations Research Vol. 37, pp. 313-344, Aug. 1992.

[17] V. der Ploeg, “Classical Growth Cycles”. Metroeconomica 37(2), pp. 221-230, 1985.

[18] Y. Li, et al., “A New Parallel Adaptive Finite Volume Method for the Numerical Simulation of Semiconductor Devices”, CCP 2000 Adv. Prog. Tech. Digest, pp.138, 2000.

[19] D. L. Schafetter, H. K. Gummel, “Large-Signal Analysis of a Silicon Read Diode Oscillator”, IEEE Trans. Elec. Dev., Vol. ED-16, pp.66-77, 1969.

[20] Y. J. Chou and W. Jen, “Applying the Strategy Theory Addressed by Porter the Planning and Designing a Balanced Scorecard- Taking a Bus Company as an Example”. Master Thesis, Institute of Transportation Engineering and Management, National Chiao Tung University, 1999.